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MULTIFREQUENCY ELECTROMAGNETIC RESPONSE OF A SHIELDED CONDUCTOR

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RESUMEN

Se ha obtenido la respuesta electromagnética de frecuencia múltiple de un sistema compuesto mena-roca encajonante, el cual se representa teóricamente por un modelo de dos superficies esféricas concéntricas (que no están en contacto galvánico). Se han investigado comparaciones cualitativas y cuantitativas entre la respuesta electromagnética total del sistema compuesto y las respuestas aisladas del cuerpo de mena (superficie interior) y del cuerpo de roca encajonante (superficie exterior), para combinaciones de parámetros diferentes. Las contribuciones relativas y/o diferenciales de la mena y de la roca encajonante se reflejan predominantemente en la representación de Argand y las curvas de variación de frecuencia contra gradiente. Los diagramas de Argand se han utilizado también para identificar las zonas de frecuencia que resultan en un encubrimiento completo de la mena y las que resultan en poco efecto de la roca encajonante.

Los resultados pueden resultar de utilidad en:

(i) entender el efecto de encubrimiento de las formaciones corticales superiores (en particular para delinear depósitos de sulfuros en los fondos oceánicos; explorar la distribución y alteración de menas de pórfidos cupríferos y proveer información útil para una selección adecuada de la perforación, minado y excavación, etc.).

(ii) interpretación de datos obtenidos por sistemas de inducción electromagnéticos de espectro múltiple (ejemplo 'Geoprobe EMR-14').

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ABSTRACT

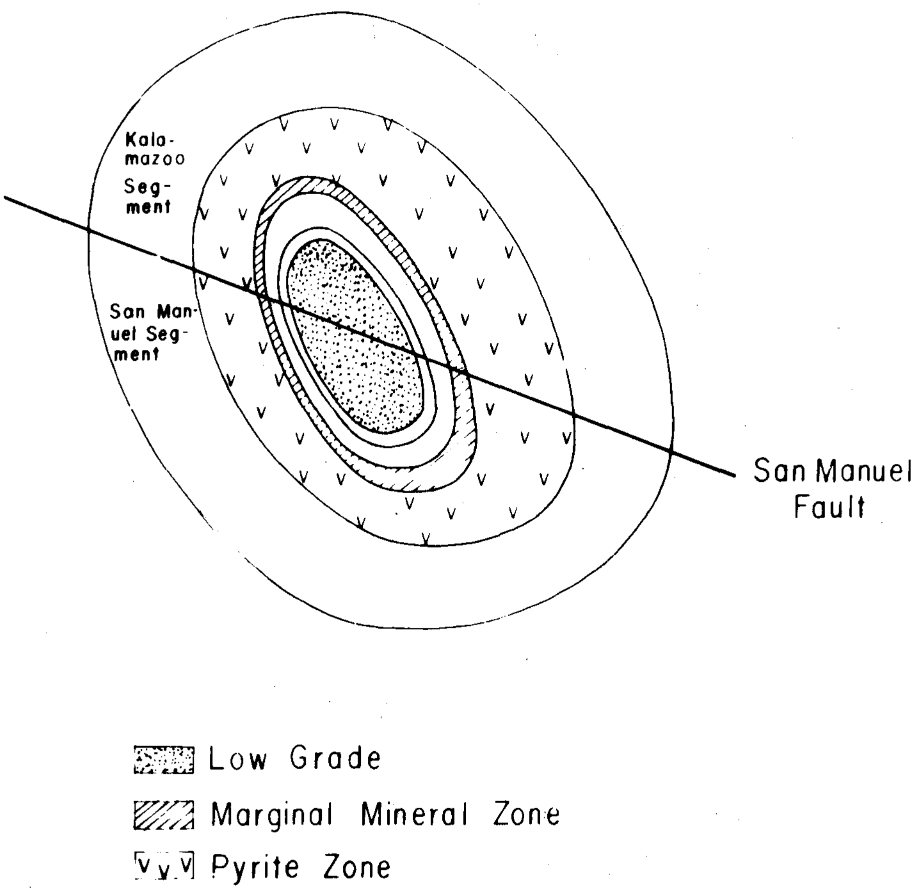
Multifrequency electromagnetic response of a composite overburden-ore system, theoretically represented by a concentric spherical two-shell model (in nongalvanic contact), has been obtained. Qualitative and quantitative comparisons of the total electromagnetic response of the composite system to the individual responses of the ore body (inner shell) and the overburden (outer shell) have been investigated for different parameteric combinations. Differential and/or relative contributions from the ore body and the overburden are predominantly reflected in the argand representation and gradient versus frequency variation curves. Argand diagrams have also been used to identify frequency zones characterizing complete shielding of the ore body and almost transparency of the overburden formations. Results may provide an aid:

(i) in understanding the shielding behaviour of the upper crustal formations (more particularly for delineating sulphur deposits buried in the ocean; exploring porphyry copper ores of alterations and mineralization zoning, and providing useful informations for the proper choice of the bore holes; mine openings, tunnellings, etc.) and

(ii) in the interpretation of the data collected by multispectral electromagnetic induction systems (viz. Geoprob EMR - 14).

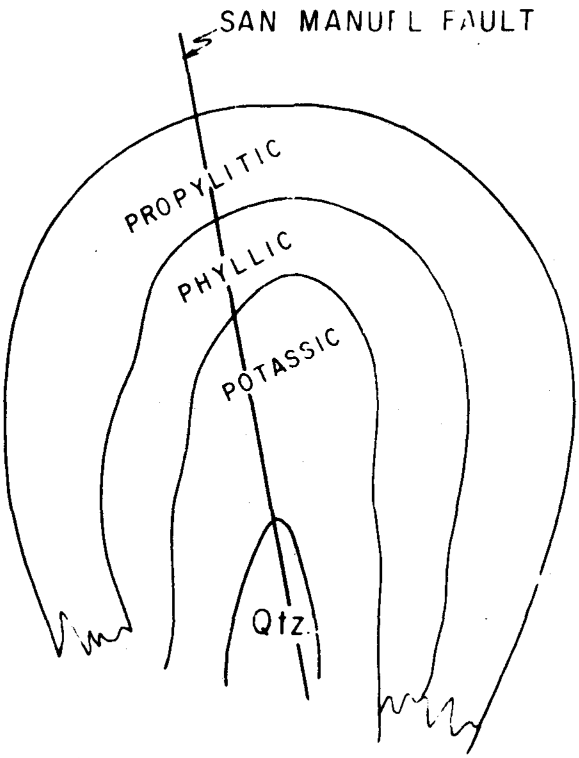
INTRODUCTION

Geoelectromagnetic methods have been proved a powerful tool for the exploration of mineral deposits occurring in the first few hundred meters of the upper crustal formations. Several case histories of exploration of base metal deposits covered under glaciated conditions of Scandinavia, Canada, Sweden, etc., are published in S.E. G. Mining Geophysics Volume I (1966). However, in low latitudes of Australia, India, etc., the conducting weathered rocks cause serious difficulties in searching the underlying mineral deposits. Such a problem of cover-target system has also gained importance in delineating sulphur deposits buried in the ocean (Francheteau *et al.*, 1979) and providing useful informations for proper choice of the sites for boreholes, mine-openings, tunnelings, etc. In geophysical prospecting overburden-ore problem has also received significant attention after the Raux's (1959) observation on the occurrence of disseminated conducting zones surrounding the massive sulphide ore deposits. Among such sub-surface situations one of the most interesting one has been reported by Lowell (1968) and Lowell and Gilbert (1970) in Kalamazoo and Sanmuel portions of the Arizona, revealing the coaxial symmetry of the alteration and mineralization zoning in porphyry copper deposits. These porphyry deposits, shown in figures 1 and 2, consist of copper and molybdenum sulphides emplaced in various host rocks altered by hydrothermal solutions into roughly concentric zonal patterns and contain pyrite, chalchopyrite, molybalenite, quartz with other alterations/ore minerals. Tarkhov (1965) also referred some usually occurring field observations in which the conducting grains are separated from one another by insulating minerals.



Schematic horizontal plan showing alteration and mineralization zones of the Kalamazoo ore body (after Lowell 1968)

Fig. 1. Schematic horizontal plane showing alternation and mineralization zones of Kalamazoo ore body.



Concentric alteration mineralization zones at San Manuel Kalamazoo, after Lowell and Guilbert (1970)

Fig. 2. Concentric alteration mineralization zone at San Manuel Kalamazoo.

Velekin and Bulgakov (1967) experimentally studied the transient response of a sphere covered by a conducting sheet and found that the earlier and later parts of the total response corresponded to individual responses due to sheet and sphere, respectively. Later, Negi and Verma (1972), Rao, Gupta and Raval (1972), Verma (1974), Nagendra *et al.* (1980-1981) etc., examined similar resolutions of time domain responses due to individual components (overburden and target) theoretically. However, it is realized that in spite of the present day technical advantages of the transient field methods, the ultimate advantages in selecting and identifying the effects of conducting overburden from the ore body, probably, rests with multifrequency electromagnetic techniques (Parasnis, 1974). In the last decade several theoretical works (Negi, Gupta and Raval, 1972a, 1972b, 1973; Fuller, 1971; Negi, Raval and Rao, 1976; Negi and Saraf, 1981, 1983 and Saraf and Negi, 1983) have also shown that the multifrequency techniques have the capabilities of detecting subsurface conditions under the conducting overburden.

Hence, in this paper an attempt is made to demonstrate theoretically, the quantitative comparison of the total response of the composite conductor to the individual responses of the ore body and overburden rocks, considering coaxial, spherical double shell model by multifrequency electromagnetic techniques.

FORMULATION

The physical system of the problem is described in Figure 3. It consists of two con-

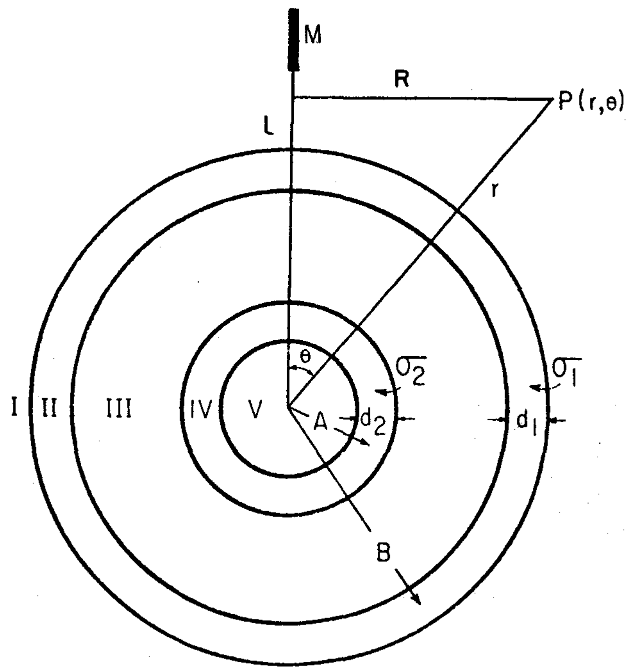


Fig. 3. Double shell model.

centric thin spherical shells at radial distances B and A; thicknesses d_1 and d_2 and conductivities σ_1 and σ_2 respectively. The source of the primary magnetic field is assumed to be a uniform magnetic current 'M' situated at (ℓ, 0, 0). Displacement currents are neglected, throughout the analysis. Field distribution at any point (P r, θ) can be derived from (Singh, 1972):

$$E\phi = (1/r) \quad (\delta u/\delta\theta), \tag{1}$$

$$H_r = [1/(i\omega\pi r^2 \sin^2\theta)] \frac{\delta}{\delta\theta} \sin\theta \frac{\delta u}{\delta\theta}) \tag{2}$$

and

$$H_\theta = (1/i\omega\mu r) \quad (\delta^2 u/\delta\theta \cdot \delta r), \tag{3}$$

where U is a scalar stream function.

The magnetic potential V is related to the magnetic field H through H = −grad V where

$$V = -(1/i\omega\mu) \quad (\delta u/\delta r) \tag{4}$$

Following Negi and Verma (1972), the total magnetic potential V in the outer region is given by:

$$V_{total} = (M/4\pi i\omega\mu) \; [\frac{1}{R} + \sum_{n=1}^{\infty} (\frac{n}{n+1})^*$$

$$* \frac{B^{2n+1}}{\ell^{n+1} \; r^{n+1}} \left\{ 1 - \frac{(2n+1)(1+\alpha)}{(2n+1)+r_{II} d_1 B(1+\alpha)} \right\}] \tag{5}$$

where

$r_{II} = i\omega\mu\sigma_1$;

$r_{IV} = i\omega\mu\sigma_2$; and

$$\alpha = - \left\{ \frac{r_{IV} \; d_2 \; A}{(2n+1) + r_{IV} \; d_2 \; A} \right\} (A/B)^{2n+1} \tag{6}$$

Following Wait (1969), reflection factors can be written as:

$$R_{total} + 1 - \left\{ \frac{(2n+1) \; (1+\alpha)}{(2n+1) \; + r_{II} B d_1 (1+\alpha)} \right\} \tag{7}$$

On putting n = 1, A/ℓ and $B/\ell << 1$ and using only the first term in the expression (viz., uniform field considerations) the corresponding values for the reflection factor defined in equation (7) can be re-written as:

$$R_T = 1 - \left\{ \frac{[3(1+p_H)]}{3+r_{II} B d_1 (1+p_H)} \right\} \quad (8)$$

where

$$p_H = -\left(\frac{r_{IV} A d_2}{3+r_{IV} A d_2} \right) \left(\frac{A}{B} \right)^3 \quad (9)$$

DISCUSSION OF THE NUMERICAL RESULTS

Reflection factor for the composite system, shown in equation 8, has been computed under quasi-static limits to illustrate the effects of changing conductivity and radius of outer shell on the variations of real, imaginary, amplitude and phase components of the response parameter with the frequency of the incident electromagnetic waves. Resolution of the overburden ore system in the multifrequency technique has also been presented through argand diagrams and gradient versus frequency variation curves. Individual responses of overburden or ore bodies (viz., a single shell model) have also been obtained for quantitative comparison with the total response of the composite-system.

A. *Variation with radius (B) of the overburden (Outer shell):*

In figure 4, components of the reflection factor have been plotted against the frequency content of the electromagnetic waves. Solid line curves represent the case of a single shell. One finds that:

(i) In the intermediate frequencies ($\simeq 10^4$ Hz) inflexation in the curves has been observed. Physically, this means that in this intermediate frequency range effects of both shells are observed, viz., the outer shell is partially transparent while the inner shell is partially shielded.

(ii) Two conducting regions can be distinguished from each other before and after this inflexation zone in the form of two peaks; and

(iii) Imaginary component of the reflection factor is more sensitive in resolving cover target system in comparison to the other components of the reflection factor.

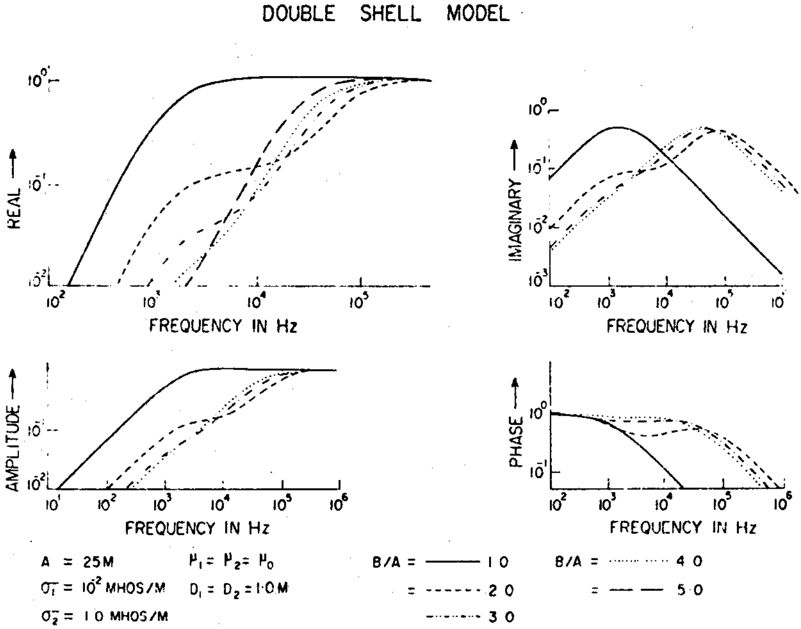


Fig. 4. Variations of Real, Imaginary, Amplitude and Phase components of the reflection factor against the frequency of the electromagnetic waves for B/A = 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0, $D_1 = D_2 = 10m$.

In figure 5, argand diagram (Real versus imaginary components of the response factor) has been plotted at several frequencies for various values of the radius of the cover (B), keeping the radius (A) of the target (ore body/inner shell) constant. Fre-

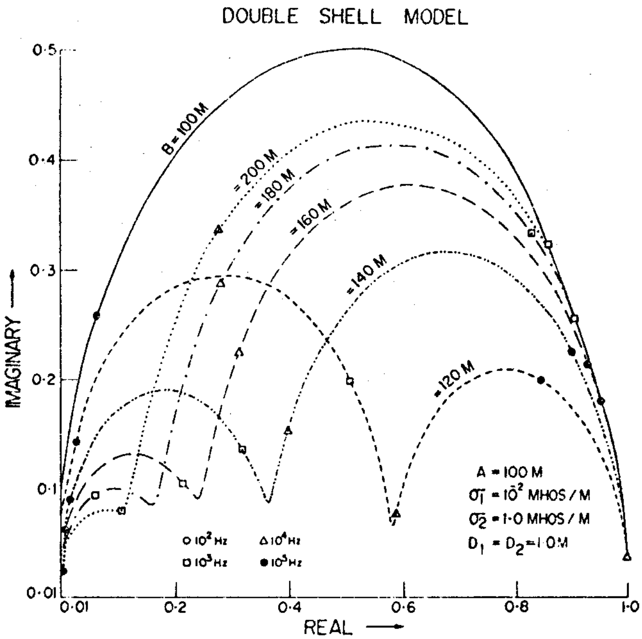


Fig. 5. Argand diagram (Real vs. Imaginary) for change in frequencies and B/A = 1.0, 1.2, 1.4, 1.6, 1.8 and 2.0, $D_1 = D_2 = 10m$.

quency ranges for resolving overburden and ore body are tabulated in Table 1 and can be classified as:

(a) Zone of complete transparency:

(i) In multifrequency variation pattern the first peak obtained in the low frequency range ($<10^3$ Hz) is due to the ore body (or inner shell), as the outer shell (or overburden) can be penetrated by such slowly varying fields.

(ii) Frequency range for resolving the target (or inner shell) increases by an order of magnitude, with the decrease in the radius of the cover (viz., in Table 1, resolution range of inner shell for B = 200 m, is from 10 Hz to 6 x 10^2 Hz; while the corresponding range for B = 120 m, is from 10 Hz to 6 x 10^3 Hz).

VARIATION WITH RADIUS OF THE OUTER SHELL

	FIRST PEAK (inner shell)				SECOND PEAK (outer shell)			
B	Real	Imaginary	Frequency	Frequencies for resolving inner shell	Real	Image	Frequency	Frequencies for resolving outer shell
100 M	0 5321	0·4986	4 X 10^2 Hz	10^1 to 10^6 Hz	—	—	—	—
120 M	0·3102	0·2940	4 X 10^2 Hz	10^1 to 6 X 10^3 Hz	0·776	0·21	7 X 10^4 Hz	6 X 10^3 Hz to 10^6 Hz
140 M	0·1960	0·1910	4 X 10^2 Hz	10^1 to 3 X 10^3 Hz	0·664	0·3173	4 X 10^4 Hz	3 X 10^3 H to 10^6 Hz
160 M	0·1320	0 1343	4 X 10^2 Hz	10^1 to 1 X 10^3 Hz	0 607	0 3776	3 X 10^4 Hz	10^3 Hz to 10^6 Hz
180 M	0 0936	0·101	4 X 10^2 Hz	10^1 to 1 X 10^3 Hz	0·490	0 403	3 X 10^4 Hz	10^3 Hz to 10^6 Hz
200 M	0·0933	0·824	6 X 10^2 Hz	10^1 to 6 X 10^2 Hz	0·529	0·436	2 X 10^4 Hz	10^2 Hz to 10^6 Hz

$\sigma_1 = 10^{-2}$ MHOS/M, $D_1 = D_2 = 1{\cdot}0$ M

$\sigma_2 = 1{\cdot}0$ MHOS/M, A = 100 M

Table 1

Variation with the radius of the outer shell ($\sigma_1 = 10^{-2}$ mhos/m, $\sigma_2 = 1.0$ mhos/m, $D_1 = D_2 = 10$ m, A = 100 m).

(b) Zone of complete shielding:

(i) The second peak is mainly due to the cover (or outer shell) as the high frequency electromagnetic fields (>6 x 10^3 Hz) have greater skin effects resulting in the high shielding behaviour of the cover. In this frequency zone, the ore body is completely shielded and hence the contribution of the conducting overburden is reflected well in the argand diagram.

(ii) The frequency range for resolving the cover (outer shell) increases with an increase in the cover radius from 120 m, to 200 m (viz., in Table 1, resolution range of outer shell for B = 120 m is 6 x 10^3 Hz to 10^6 Hz, while the corresponding range for B = 200 m is from 10^2 Hz to 10^6 Hz).

In figure 6, the resolving capabilities of the gradient ($\frac{d\,real}{dF}$) versus frequency curves (ie., the gradient of the real component is plotted against the frequency of the electromagnetic waves for various values of B/A ratios) have been examined. Some observations are:

(i) Contributions of overburden (second peak) and ore body (first peak) are identified clearly in higher and lower frequency bands,

(ii) For higher B/A ratios the frequency values for resolving the cover and the magnitude of the gradient values are significantly large, and

(iii) the frequency range of e.m. waves capable of resolving the ore body shifts, proportionately, towards higher frequencies with decreasing B/A ratios.

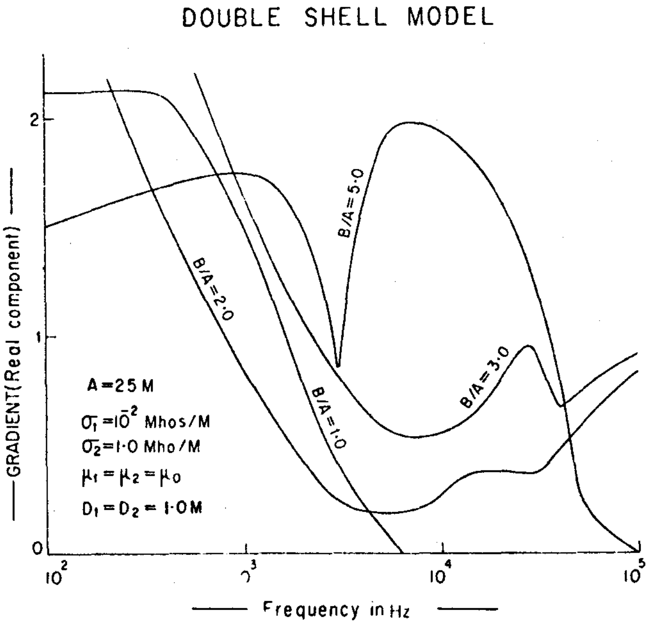


Fig. 6. $\frac{dR}{dF}$ (gradient of the real component) vs. (F) frequency for B/A = 1.0, 2.0, 3.0 and 5.0, $D_1 = D_2 = 10$m.

B. *Variations in the conductivity (σ_1) of the overburden (outer shell):*

In figure 7, components of the reflection factor have been plotted against the

DOUBLE SHELL MODEL

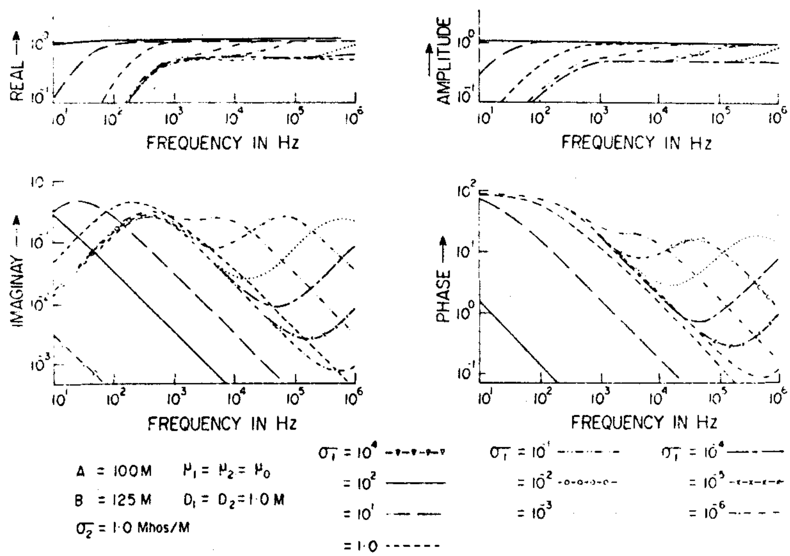


Fig. 7. Variations of real, imaginary, amplitude and phase components of the reflection factor against the frequency of the electromagnetic waves for $(\sigma_1/\sigma_2) = 10^4, 10^2, 10^1, 1.0, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}$, and 10^{-6}, $D_1 = D_2 = 10$m.

frequency of the electromagnetic waves for several values of the conductivity of the overburden. One finds that:

(i) The phase and the imaginary components are more sensitive to the conductivity variations than the corresponding real and amplitude components of the reflection factor,

(ii) Only for the moderate conductivity values ($\sigma_1 \sim 10^{-2}$, 10^{-3}mhos/m) of the overburden two characteristic and distinct peaks are clearly visualized in the phase and imaginary components, while in both the extreme conditions of highly conducting (~ 10 mhos/m) and highly insulating overburden or cover ($\sim 10^{-6}$mhos/m) the variation pattern resemble that of a single shell only,

(iii) with decreasing $\sigma_1$value, penetration of electromagnetic energy through the conducting cover increases resulting into more contributions from the ore body and this shifts the second peaks towards higher frequency sides.

Two conditions corresponding to the cases when the ore body is covered by more, and less conducting cover are illustrated in figure 8 and Table 2.

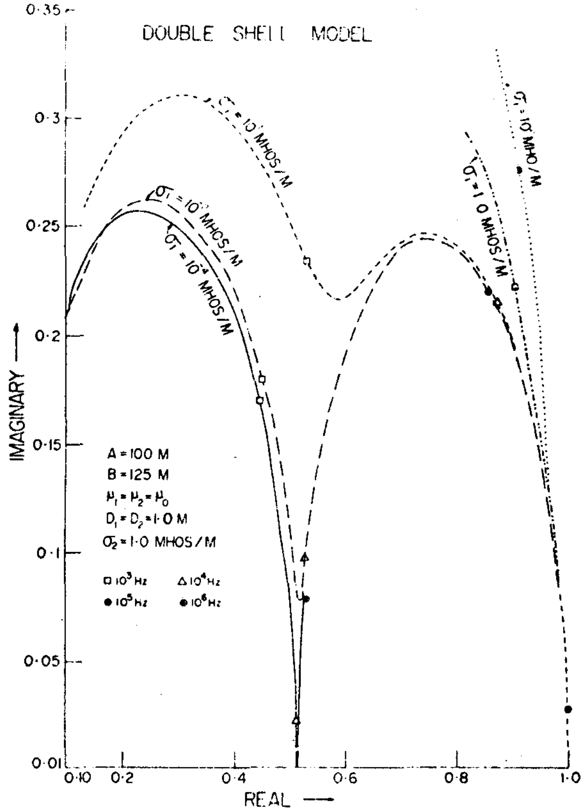


Fig. 8. Argand diagrams (real, vs. imaginary) for change in frequencies and $(\sigma_1/\sigma_2) = 10^{-4}, 10^{-2}, 10^{-1}$, 1.0, 10^2, $D_1 = D_2 = 10$m.

VARIATION WITH THE CONDUCTIVITY OF THE OUTER SHELL

	FIRST PEAK (inner shell)				SECOND PEAK (outer shell)			
σ_1	Real	Imaginary	Frequency	Resolving range	Real	Imaginary	Frequency	Resolving range
10^{-4} MHOS/M	0 269	0 255	4×10^2 Hz	10 to 5×10^4 Hz	0·524 (Max.)	0·767 (Max)	10^6 Hz	10^4 to 10^6 Hz
10^{-2} MHOS/M	0 275	0 261	4×10^2 Hz	10 to 5×10^3 Hz	0·749	0·243	6×10^4 Hz	10^4 to 10^6 Hz
10^0 MHOS/M	0 4590	0 4638	1×10^2 Hz	10 to 10^6 Hz	—	—	—	—
10^2 MHOS/M	0 916 (Max)	0 276 (Max)	10 Hz	10 to 10^6 Hz	—	—	—	—
10^4 MHOS/M	1 0	0 3×10^{-2}	10 Hz	10 to 10^6 Hz	—	—	—	—

(A = 100M , B = 125M , $D_1 = D_2 = 1{\cdot}0$M , σ_2 = 1MHO/M)

Table 2

Variation with the conductivity of the outer shell (A = 100 m, B = 125 m, $\sigma_2 = 1.0$ mhos/m, $D_1 = D_2 = 10$ m).

(i) *Case of perfect shielding:* Viz., the conductivity of the outer shell is more than the conductivity of the inner shell ($\sigma_1 > \sigma_2$). In the geophysical exploration problem such a situation is usually obtained in the areas where the highly conducting tertiary formations cover the less conducting bodies. Similar situations are also found where the conducting and insulating minerals are intermixed so that the insulating minerals are formed earlier (inner shell) and the conducting ones later (conducting cover). Such conducting minerals screen the inner layers (mostly the zones of sulphide enrichment). In this argand diagram, figure 8, one also finds that the inner shell is completely shielded for the cases when $\sigma_1 > \sigma_2$.

(ii) *Less conducting cover: ($\sigma_1 < \sigma_2$)* This is the situation which is important from the geophysical exploration point of view and resembles the actual field conditions when the conducting grains are formed earlier and then they are covered by insulating minerals. Minerals of oxidized zones come under this class of problem. In such situation contributions of both overburden as well as the ore body are reflected well in the argand diagram. In the total response of the system, contributions of ore body are observed more in the lower frequency band than the contribution of the cover (outer shell). With an increase in the conductivity of the cover (σ_1) from 10^{-4}mhos/m to 10^{-2}mhos/m the frequency range for resolving the ore body changes by an order of magnitude (viz., in Table 2, for $\sigma_1 = 10^{-4}$ mhos/m the resolution range is from 10 Hz to 5 x 10^4 Hz while for $\sigma_1 = 10^{-2}$ mhos/m the corresponding range is from 10 Hz to 5 x 10^3 Hz).

The first peak value, corresponding to target (ore body) is obtained at the frequency 4 x 10^2 Hz and this is unaltered by a change in the conductivity of the cover. While the second peak value (corresponding to the cover) shifts towards lower frequency sides with an increase in the σ_1 value. For $\sigma_1 > \sigma_2$, the overburden shields the ore body completely and this is true for any frequency value considered in this analysis.

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