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FRICTIONAL DAMPING OF THE TIDE IN THE GULF OF CALIFORNIA

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RESUMEN

Se presenta una estimación del efecto de la fricción del fondo sobre la marea en el Golfo de California, suponiendo que su cuenca es un canal con profundidad y anchura variables. Se demuestra que el efecto es pequeño. Esto permite incluir la interacción de las componentes de marea inferida por el modelo cuadrático de fricción suponiéndola como una perturbación sobre el movimiento base. Permite también reescribir la ecuación de movimiento en forma lineal pero en la que el coeficiente de fricción depende en cada punto, de la velocidad máxima y la amplitud relativa de la componente armónica que se esté considerando, además del valor dado al coeficiente de fricción $r = g/C^2$. Usando el valor de $r = .0036$, este modelo reproduce adecuadamente los registros que se tienen de las componentes diurna y semidiurna de la marea en el Golfo de California.

ABSTRACT

The effect of friction on the tide in the Gulf of California is evaluated, using a one dimensional schematization of the basin; it is found to be minor. This permits to take account of the interaction between the tidal components implied by the quadratic law of friction, by assuming it to be a perturbation on the basic motion and to rewrite the equation of motion in a form which is linear but in which the coefficient of friction depends on the maximum velocity at each point and on the relative amplitude of the harmonic component being considered, besides the value given to the friction coefficient $r = g/C^2$. A simple one dimensional model using the value $r = .0036$ reproduces adequately the measurements presently available on the semidiurnal and diurnal tides felt along the Gulf.

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INTRODUCTION

Friction in a fluid creates a deceleration given by:

$$-\frac{g}{C^2(H+Z)}\ v|v| \qquad (1)$$

where:

- g = acceleration due to gravity
- H = the depth of the fluid
- Z = the surface displacement due to the disturbance. (In what follows we take $H+Z \sim H$)
- v = the velocity of the fluid at the point of measurement
- C = the Chézy coefficient of friction (In what follows we replace it by the dimensionless parameter $r = g/C^2$).

The constant r is determined empirically; in the case of numerical modelling, various values of r are tried to find the one giving the best agreement. Chézy's law of friction (1) has been verified to hold in rivers; the quadratic form of the law explains the presence of compound tides in rivers and the transfer of energy to other spectral frequencies.

Stock (1976) in his two dimensional model of the tide in the Gulf of California discovered empirically that for a single sinusoidal oscillation, a linear law of friction where (1) takes the form $-r'v_0v/H$ (v_0, the amplitude of the current, v, its instantaneous value, $r' = 8r/3\pi$) gave a better fit than the quadratic law (1). The best fit to the M_2 data collected by Filloux (1973) was obtained using $r' = .0115$ which is much larger than the commonly encountered value of .002 - .003. A fit, using the quadratic law needed values of r = .141 between the mouth of the gulf and its middle portion, r = .024 for the next quarter and r = .0047 for its northern extremity. These facts are at variance with the knowledge that a quadratic law of friction is the correct one and that a single value of r should be adequate to model the tide over the entire basin.

THE QUADRATIC LAW OF FRICTION APPLIED TO A TIDAL SIGNAL

The equations of hydrodynamics for a one dimensional channel of variable width and depth are (Defant, 1961):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial Z}{\partial x} - \frac{r}{H}\ u|u| \qquad (2)$$

$$\frac{\partial(BHu)}{\partial x} = -\frac{\partial(BZ)}{\partial t} \qquad (3)$$

where:

u = the current component in the x direction

B = the width of the channel at the point x

t = the time

The tidal oscillations of the Pacific Ocean at the mouth of the gulf are little affected by non linear effects and may be represented by a superposition of harmonics:

$$Z(x = O, t) = \sum_j Z_j \cos(s_j' t - a_j) \qquad (4)$$

$$u(x = O, t) = \sum_j u_j' \cos(s_j' t - b_j) \qquad (5)$$

where:

$\{s'_j\}$ = the frequencies present in the tidal signal, mainly diurnal and semidiurnal

$\{z_j, a_j\}$ = amplitudes and phase lags of the vertical oscillations

$\{u_j', b_j\}$ = amplitudes and phase lags of the tidal currents.

The convective term $u\partial u/\partial x$ and the frictional deceleration $-r\,u|u|/H$ should become sensible within the gulf; they cause an interaction between the components of the tide (4), (5), injected at the mouth of the basin. Strictly speaking a representation of the tide inside the gulf by harmonics is precluded because of this fact. In practice, if we can demonstrate that the convective and frictional terms are smaller than the linear ones in (2) and (3) over most of the gulf area, we may use a perturbation approach representing the signal inside the gulf by a superposition of the original harmonics in (4) and (5) and a first order correction term made up of the additional harmonics created by their interaction (Kravtchenko and Le Provost, 1970). Our first task is to investigate the relative magnitude of the terms in (2) and (3) over the body of the gulf; for this we need estimates of the current field u. These may be obtained with the help of the equation of continuity in conjunction with observations on the vertical tide (Forrester, 1972; Filloux, 1973). Such estimates of the barotropic current were made for the

Saint Lawrence river and were found to correspond very well with actual current measurements, which are integrated horizontally and vertically across a given section of the river. The current estimates are obtained by digitizing equation (3) into:

$$(BHu)_{n+2} = (BHu)_n + is(BZD)_{n+1} \tag{6}$$

D = the length of the segment of the channel over which the difference is calculated.

Z = the vertical tide (amplitude and phase) at the point n, interpolated from tide gauge measurements.

u = the unknown current to be determined.

We wrote $\partial Z/\partial t = -isZ$, s being the frequency of the component being investigated. The index in (6) denotes the grid point at which the specific variable is evaluated. The scheme of integration is started by assuming:

$$u = 0 \quad \text{at} \quad n = 0 \tag{7}$$

and is thereafter self sustaining once values of Z, B and H are available. We have used the schematization of Stock (1976) (Fig. 1) in order to deduce the current u implied from the equation of continuity; the length is discretized over units of 20 km and the depth as calculated by Stock, is averaged across the section. The current field deduced from Stock's cotidal charts and his schematization of the gulf are shown in Fig. 2 for the components M_2 and K_1; u was also calculated from the additional components N_2, S_2 and O_1. N_2 and S_2 have profiles similar to the one shown for M_2 and O_1 has a profile similar to K_1. Such currents help evaluate the magnitude of the terms in (2) and (3) and assess the relative importance of the non linear terms.

The maximum value of $u\partial u/\partial x$ is found between kms 400 and 440 (See Fig. 1): it has value:

$$u\ \partial u/\partial x \sim \frac{(27.5 - 14.5)(\text{cm/s}) \times 21\ \text{cm/s}}{40\ \text{km}} = 7 \times 10^{-5}\ \text{cm/s}^2$$

for the component M_2. Taking the value $C = 52\text{m}^{1/2}/\text{s}$ (or $r = g/C^2 = .0036$), a perfectly arbitrary but sensible value of the friction coefficient, the friction term at km 400 becomes for M_2:

$$ru|u|/H \sim \frac{.0036(31\ \text{cm/s})^2}{428\ \text{m}} = 8 \times 10^{-5}\ \text{cm/s}^2\ .$$

The friction term for M_2 reaches values of:

$$\frac{.0036\,(25\ \text{cm/s})^2}{52.9\ \text{m}} = 4 \times 10^{-4}\ \text{cm/s}^2$$

in the shallower sections near the head.

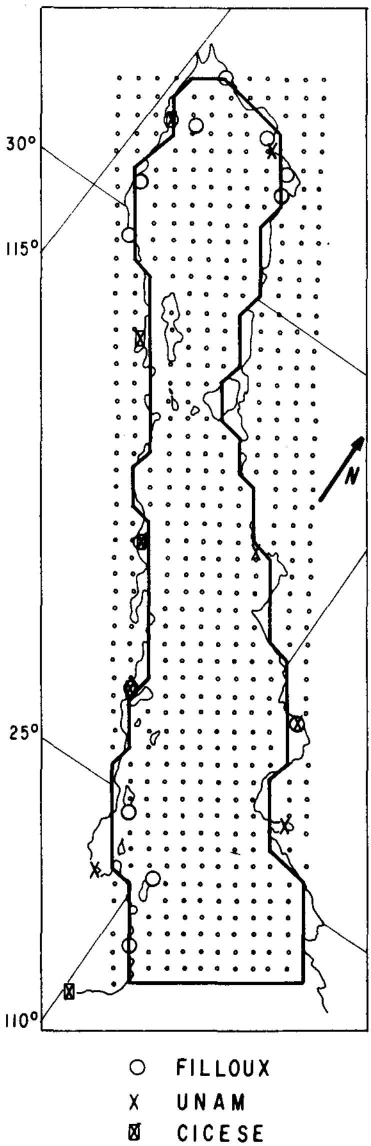

Fig. 1. Grid and schematization of the Gulf of California developed by Stock (1976) for his two dimensional numerical model of the tides. Distance between grid points = 20 km.

Gauges sites, clockwise from the bottom left:

O Filloux

Buena Vista
Isla Cerralvo
Isla San José
Loreto
Santa Rosalía
San Luis Gonzaga
Puertecitos
Roca Consag
San Felipe
Santa Clara
Puerto Peñasco North
Puerto Peñasco South
Rocas San Jorge
Yavaros

x UNAM

La Paz
Puerto Peñasco
Guaymas
Yavaros
Topolobampo

☒ CICESE

Cabo San Lucas
Loreto
Santa Rosalía
Bahía Los Angeles
San Felipe

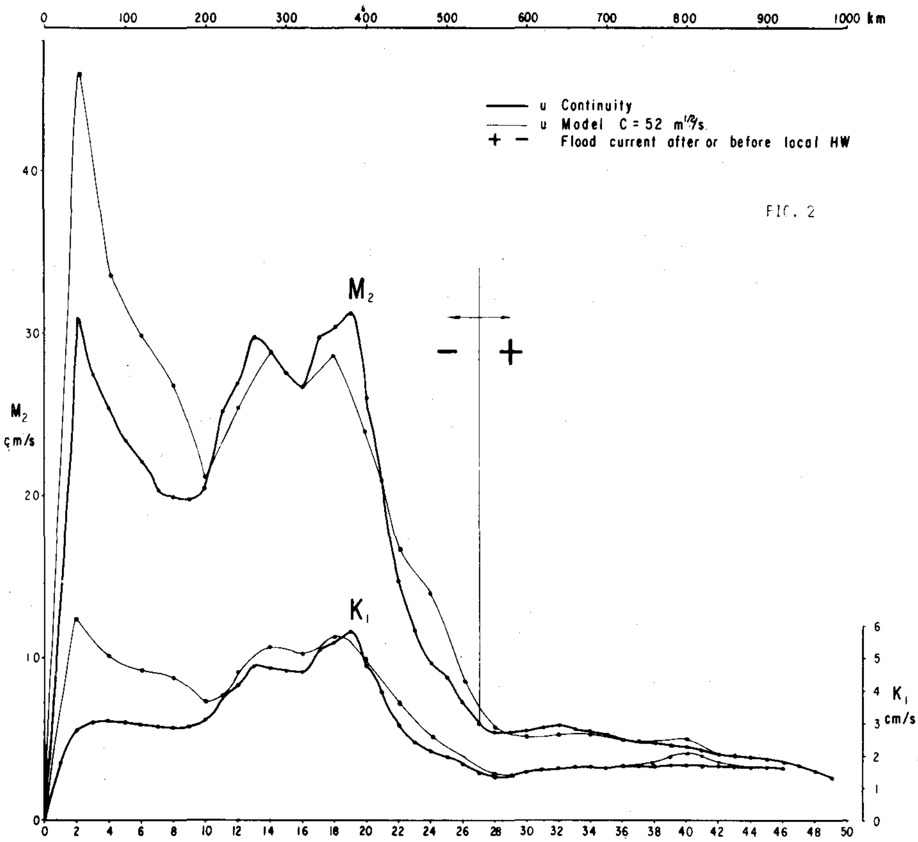

Fig. 2. Current speed inferred from the equation of continuity and the known distribution of the tidal amplitudes and phases (thick line) using Stock's schematization of the Gulf of California, one dimensionalized by averaging his depths across each section, and the current speed given by a one dimensional model of the Gulf of California (thin line) based on the same schematization and using the value $C = 52\ m^{1/2}/s$ for the Chézy coefficient of friction. The vertical line with the + and − delineates the zones where the M_2 flood current precedes or follows the local M_2 high water; the K_1 flood current always precedes the local K_1 high water.

The acceleration term $\partial u/\partial t$ has magnitude su, s being 1.405×10^{-4}/s for M_2 or $1.405 \times 10^{-4} u$ cms/s^2 everywhere in the basin where a harmonic representation is permissible. It amounts to:

$$2.8 \rightarrow 4.2 \times 10^{-3}\ \text{cm/s}^2 \qquad \text{between kms 0 and 400}$$

The pressure gradient $g\ \partial Z/\partial x$ for M_2 from the head to km 400 amounts to some:

$$\frac{981\ \text{cm/s}^2(180 - 30)\text{cm}}{400\ \text{km}} = 3.7 \times 10^{-3}\ \text{cm/s}^2$$

We therefore have the following orders of magnitude for the M_2 tide between the head of the Gulf and km 400:

$g\,\partial Z/\partial x \sim 4 \times 10^{-3}$, $\partial u/\partial t \sim 4 \times 10^{-3}$, $ru|u|/H \sim 4 \times 10^{-4}$, $u\partial u/\partial x \sim 7 \times 10^{-5}$ cm/s^2

The non linear terms amount to at most 10% of the linear ones except in the extreme northern extremity of the basin and in constricted channels where the depth is changing rapidly; they may therefore be neglected in a first approximation and then brought back in, for a better approximation.

Taking account of the non linear contribution

We write the solution of (2) and (3) as:

$$u = u_0 + u_1 \qquad\qquad Z = Z_0 + Z_1 \tag{8}$$

where u_0, z_0 satisfy the linear equations:

$$\frac{\partial u_0}{\partial t} + g\frac{\partial Z_0}{\partial x} = 0 \tag{9}$$

$$\frac{\partial (BHu_0)}{\partial x} + \frac{\partial (BZ_0)}{\partial t} = 0 \tag{10}$$

while the non linear corrections u_1, Z_1 satisfy to first order:

$$\frac{\partial u_1}{\partial t} + g\frac{\partial Z_1}{\partial x} = -u_0\frac{\partial u_0}{\partial x} - \frac{r}{H}u_0|u_0| \tag{11}$$

$$\frac{\partial (BHu_1)}{\partial x} = -\frac{\partial (BZ_1)}{\partial t} \tag{12}$$

The u_0, Z_0 solution is made up of the harmonic components of the tide injected in the mouth of the basin; among these we select M_2, S_2, N_2, K_1, and O_1 since they are the larger ones.

The non linear terms involve u_0 exclusively: we rewrite them in a more convenient form: we normalize the range of the current by noting:

$$u_0 \leqslant \sum_j u'_j \equiv U \tag{13}$$

at all times, U denoting the maximum possible instantaneous value of u. We write:

$$u_0 = U \sum_j u_j \cos s_j \equiv U u \tag{14}$$

where:

$$u_j = u'_j/U < 1 \qquad s_j = s'_j t - b_j \qquad \sum_j u_j = 1$$

The convective term can be reexpressed as:

$$u_0 \partial u_0/\partial x = (1/4)(\partial/\partial x) \sum_j [u_j^2(1 + \cos 2s_j) + 2 \sum_{j,k}^{j \neq k} u_j u_k [\cos(s_j + s_k) + \cos(s_j - s_k)]] U^2 \tag{15}$$

We see that in our approximation, the convective term contributes to the mean level and creates new frequencies $2s_j$ and $s_j \pm s_k$.

To rewrite the friction term, we need to deal with the absolute value $|u_0|$.

It can be expressed as:

$$|u_0| = U|u| = U\sqrt{u^2|} \qquad 0 \leqslant |u| \leqslant 1$$

We approximate $u|u|$ by:

$$u|u| \sim (1/2)(mu + u^3/m) \tag{16}$$

where m is a suitable constant. The approximation results from the first iteration step of Heron's algorithm to calculate the square root of u_0^2 or $|u_0|$, (Korn and Korn, 1968). Using m = .7 approximates $u|u|$ to better than .02 over the range $.2 \leqslant u \leqslant .8$ and to better than .06 over the range $0 \leqslant u \leqslant 1$. (16) is used as a two term approximation to $u|u|$ and the iteration is not pursued. Le Provost (1974, 1976) expressed $u|u|$ in terms of an infinite series expressible in terms of elliptic integrals of the first and second kind, whenever a single component dominates the others. Unfortunately such a series expansion diverges for rectilinear currents and the assumption of a dominant component does not hold well in the Gulf of California. Approximation (16) suits rectilinear currents which are the usual type of currents encountered and does not depend on the dominance of a given component. A numerical experiment with (16) involving the known tidal currents in the body of the Bay of Fundy reproduced the exact $|u|u$ value with a RMS error of .023; (16) is therefore a fully adequate approximation to the friction term.

The substitution of (16) into the friction term gives, in term of the original tidal frequencies:

$$\frac{u_0|u_0|}{U^2} \sim \frac{1}{2} \left\{ \sum_j [(mu_j + (3/4m)u_j^3) + (3/2)u_j \sum_{k \neq j} u_k^2] \cos s_j \right. \tag{17}$$

$$+ (3/4) [\sum_j u_j^2 \sum_{k \neq j} u_k [\cos(2s_j + s_k) + \cos(2s_j - s_k)]] \tag{18}$$

$$+ (6/4m)[\sum_{j \neq k \neq m} u_j u_k u_m [\cos(s_j + s_k + s_m) + \cos(s_j + s_k - s_m)$$

$$+ \cos(s_j - s_k + s_m) + \cos(s_j - s_k - s_m)]] \tag{19}$$

$$\left. + (1/4m) \sum_j u_j^3 \cos 2s_j \right\} \tag{20}$$

The friction term has created new frequencies:

$$3s_j,\ s_j \pm s_k \pm s_m,\ 2s_j \pm s_k$$

while also contributing to the original frequencies s_j. The convective term has created the new frequencies $2s_j$ and $s_j \pm s_k$. Considering diurnal and semidiurnal inputs, the contribution of convection will overlap some of the original frequencies while also creating third and fourth diurnal frequencies. The friction term contributes directly to the damping of the original harmonics, creates sixth and third diurnal frequencies plus diurnal and semidiurnal frequencies which overlap the original frequencies and do not necessarily increase their damping. We note in (15) and in (17) - (20) that the u_j's reflect internal ratios within the tidal bands while the local value of the term is determined by U and H. It is known that away from points of amphidromy (nodal zones), these ratios are fairly constant over a given sea; it is therefore possible to estimate beforehand the relative importance of the terms in the friction term.

We note that the frictional damping of a given harmonic involves the effect of all those present in the original input; their damping is therefore mutual. This implies that if quadratic friction is to be applied, one must take into account all the components present in the input at the mouth and not consider only one; this explains Stock's findings about the relative value of linear and quadratic friction. Term (17) allows to evaluate beforehand the relative importance of the other components in the damping of a given component; it is found in practice that the damping of the minor components is controlled mainly by the major ones while the latter are less affected by their companions.

The development of the friction term into (17) to (20) allows the development of a practical scheme for the estimation of the tidal components inside the Gulf of California while taking account of quadratic friction without being obliged to have recourse to powerful computers and take into account all the components at once. We use (17) to write the friction term affecting the original frequency as:

$$rA_jU^2u_j/H$$

where A_ju_j stands for the expression in brackets. The interaction of the other components through the term $(3/2m)\ u_j \sum_{k\neq j} u_k^2$ and the interaction of the harmonic with itself are lumped into an amplifying factor A_j to u_j. The improved equation of motion is:

$$-is_ju_j' + g\partial Z_j'/\partial x = -(r/H)A_ju_jU^2$$

for the harmonic s_j. We recall the definition of U_j and note that:

$$u_jU = u_j' \ .$$

We rewrite the improved equation of motion as:

$$-is_ju_j' + g\frac{\partial Z_j'}{\partial x} = -\frac{r}{H}A_jUu_j' = -r_j(x)u_j' \qquad (21)$$

The term on the right of (21) is a damping term: it depends on the local depth H, on the local peak velocity U of the field of currents and on the interaction of the other harmonics with the harmonic s_j through the amplification factor A_j. This approximation should be adequate to represent the waves M_2, S_2, K_2, K_1 and O_1 in the Gulf of California if our hypothesis that convection is not too important overall. The term (18) contributes to N_2 due to the interaction of M_2 with L_2, but the latter harmonic seems minor, similarly M_2 and K_1 contribute to O_1

through (18) while M_2 and O_1 contribute to K_1 through the same term. Using the ratios in the gulf, term (18) amounts to less than 10% of (17) for K_1 and O_1. Its effect may therefore be neglected in a first approximation.

We wish to use (21) along with the equation of continuity (3) to calculate values of the major harmonic components of the tide in the Gulf of California, using a single value of r (.0036 as said), over a one dimensional model of the area to show that we can derive values which fit the observations reasonably well. The integration is done numerically using the conventional leap frog integration scheme.

A fit to the observed values of the tide in the Gulf of California using a single value of the friction coefficient

We use the grid and the bathymetry developed by Stock (1976) in his two dimensional model of the area, but using as depth the average of the depths across each section. The scheme of integration is, dropping the component index:

$$(BHu)_{n+2} = isD(BZ)_{n+1} + (BHu)_n \qquad (22a)$$

$$Z_m = Z_{m-2} + (is - r_{m-1})(D/g)u_{m-1} \qquad (23)$$

n = even number

m = odd number

D = the finite integration step, 40 km.

However, in view of the steep depth gradients in the northern section, the large integration step (40 km) and the fact that most of the contribution to the change in phase originates in the last 200 km of the basin, the actual integration scheme for the velocity was:

$$(Bu)_{n+2}H_{n+1} = isD(BZ)_{n+1} + (Bu)_n H_{n-1} \qquad (22)$$

This amounted to selecting the depth half an integration step behind so that the influence of the shallow water would be taken into account more fully. The iteration is started by taking at

$$x = 0\ (\mathrm{km}\,0) \qquad Q = BHu = 0 \qquad (24)$$

and at

$$x = D/2\ (\mathrm{km} = 20) \qquad Z = 1 \qquad (25)$$

since equations (3) and (21) are linear in u and Z. The integration consist in the addition and multiplication of complex numbers over 24 steps and can be done in a few minutes for each tidal component on a suitably programmed hand held calculator. The preliminaries of the integration consist in:

a) Calculating U from the continuity values of u at each section for the individual components

b) Estimating u_j for each component

c) Calculating $\frac{1}{2}[mu_j + 3u_j^3/4m + (3/2m)u_j \sum_{k \neq j} u_k^2] \equiv A_j u_j$ for each component at each section

d) Calculating $r_j(x) = rA_jU/H(x)$ taking $r = g/C^2 = .0036$

which is a commonly encountered value of the friction coefficient (Dronkers, 1964). We shall soon see that it is not worthwhile yet to seek a more refined value of the coefficient of friction because of the actual state of the data. For the sake of interest the values of u_j and U are:

	M_2	S_2	N_2	K_1	O_1	U(cm/s)
km 80	.568	.245	.080	.069	.037	59.7
km 480	.544	.226	.081	.097	.052	24.8
km 1000	.430	.183	.065	.226	.114	9.3

the distances being measured from the head of the gulf.

The corresponding values of A_j are:

	M_2	S_2	N_2	K_1	O_1
km 80	.523	.382	.354	.353	.351
	.078	.359	.416	.419	.422
	.601	.741	.770	.772	.773
km 480	.509	.377	.354	.355	.352
	.074	.337	.385	.382	.389
	.583	.714	.739	.737	.741
km 1000	.449	.368	.353	.377	.357
	.109	.272	.302	.253	.293
	.558	.640	.655	.630	.650

The top value gives $(1/2)\ (m+(3/4m)u_j^2)$, the lower one is $(3/4m) \sum_{k \neq j} u_k^2$. The sum of the two is A_j; the second member gives an idea of the damping induced by the other components.

Once a table of $r_j(x)$ has been set up, one proceeds with the integration of (22) and (23). At the end of the integration, the calculated Z is adjusted to the Z imposed at the boundary condition at the open end, by multiplying it by the appropriate constant. The integration gives:

$$\text{a)} \qquad Z(x), \Delta\phi(x) \tag{24}$$

$Z(x)$ = the amplitude of the component at the point x

$\Delta\phi(x)$ = the phase difference between its phase at the open boundary and its phase at x

$$\text{b)} \qquad u(x), \Delta\phi'(x) \tag{25}$$

$u(x)$ = the velocity at x

$\Delta\phi'(x)$ = its phase change.

We show in Fig. 3 a comparison between the Filloux amplitudes and

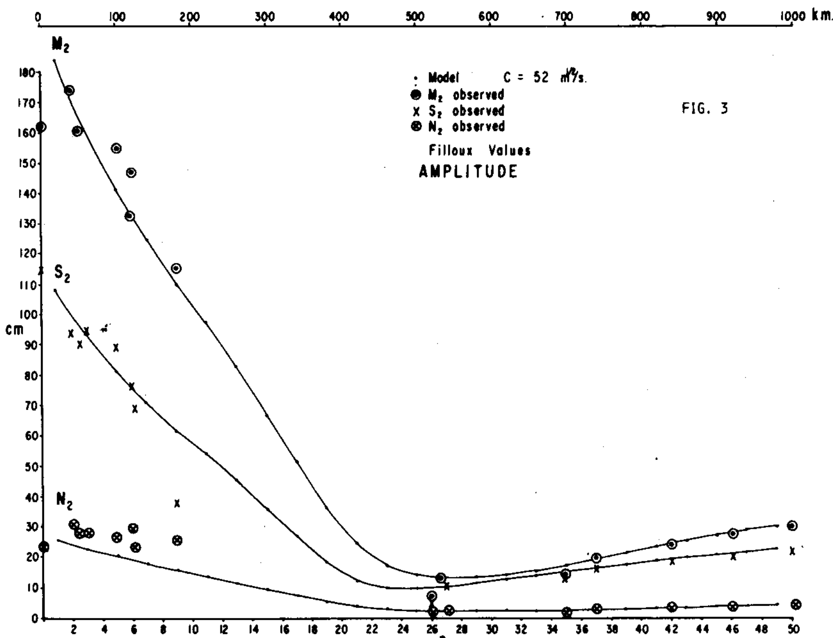

Fig. 3. Comparison between the Filloux constants and the values inferred from the one dimensional model of the Gulf of California, for the amplitude of the semidiurnal components M_2, S_2, and N_2.

those implied by the one dimensional model (22) and (23) for the semi-diurnal tides M_2, S_2 and N_2. Fig. 4 shows a similar comparison for $\Delta\phi$. We show in Fig. 5 a similar comparison for the diurnal components K_1 and O_1. The agreement for the semidiurnal components is good except

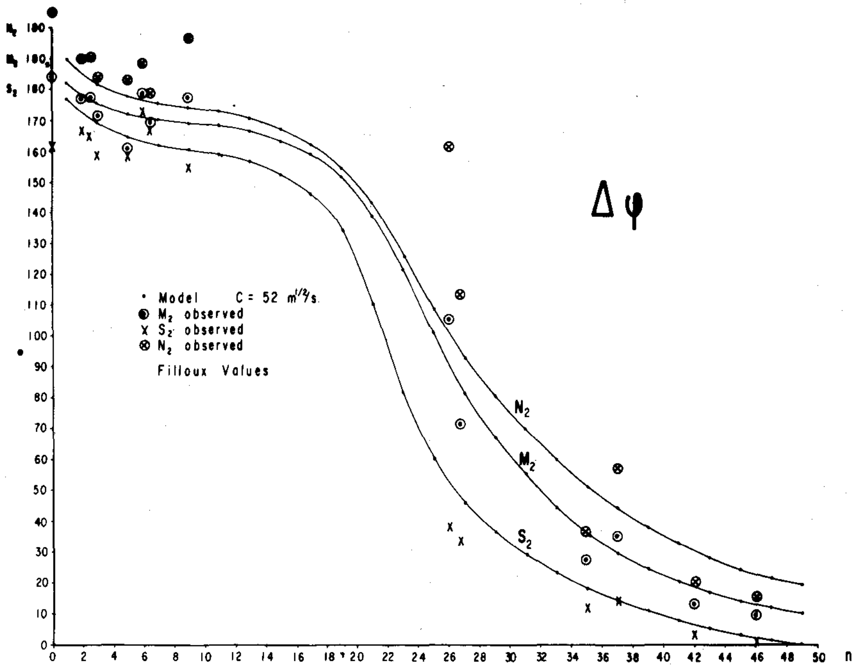

Fig. 4. Same comparison for the difference $\Delta\varphi$ between the phase of the component at the point x and its phase at the mouth of the Gulf, for N_2, M_2 and S_2. The scale was pushed upward by 10 and 20° for M_2 and N_2 in order to keep the curves separate.

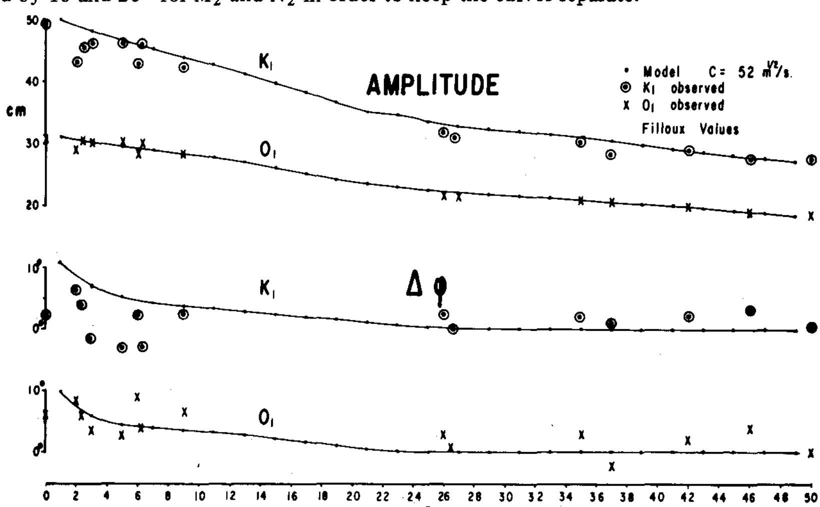

Fig. 5. Comparison between the amplitude and the phase difference $\Delta\phi$ implied by the Filloux constants and those given by the one dimensional model for K_1 and O_1.

for N_2; the model cannot adequately represent the $\Delta\phi$ of the diurnal components since it has a gradient across the gulf which is a two dimensional feature.

The constants obtained by Filloux differ markedly from those inferred from longer sets of observations at the site of the permanent gauges maintained by UNAM and CICESE around the gulf, especially in the case of N_2 and K_1. The constants resolved from these stations should be considered as more reliable since they are repeatable from year to year (Godin *et al.*, 1981), but the stations themselves may exhibit local distortions due to the siting of the gauges: this may be specially true for La Paz, Topolobampo, Yavaros and possibly Guaymas. Loreto and Bahía Los Angeles are located near relatively narrow channels where the signal may be distorted by the nearby islands. We are therefore in the presence of two sets of constants which cannot be merged together till the original Filloux observations are reevaluated. Morales-Pérez (1983) reviewed the analyses of the data originating from the UNAM and CICESE sites and drew a set of cotidal charts based on what are essentially nine points of observations. In order to compare the one di-

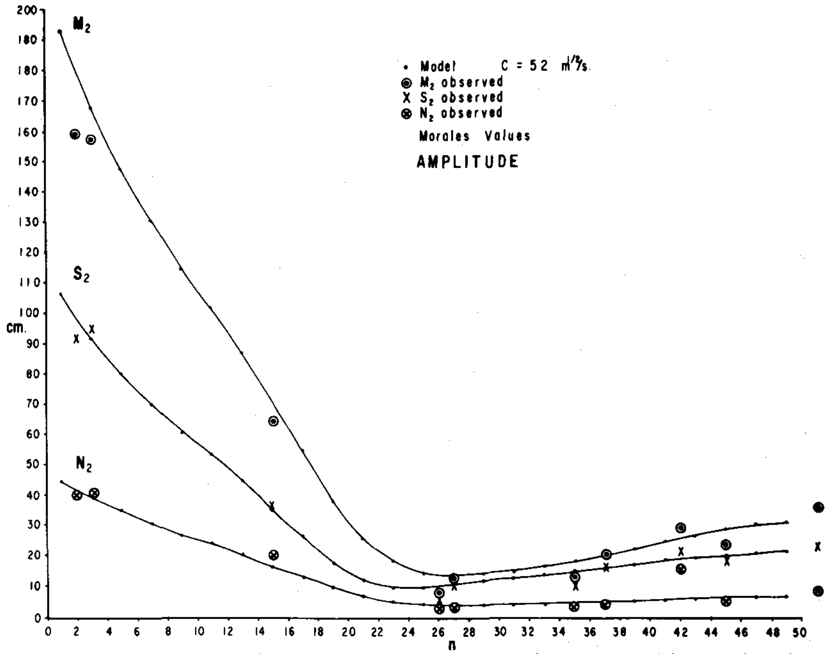

Fig. 6. Comparison between the values of the amplitude of M_2, S_2, and N_2 predicted by the model (rescaled so that amplitude at the mouth coincides with the one given in the Morales Maps) and the constants reviewed by Morales (1983). The M_2, S_2 and N_2 amplitudes at Cabo San Lucas have been entered at point 52.

mensional model with the set of constants derived from the UNAM and CICESE sites, the Z at the boundary was rescaled to conform with the value implied by the Morales charts while $\Delta\phi$ remained the same. Figs. 6, 7 and 8 show a comparison between the rescaled model and this set

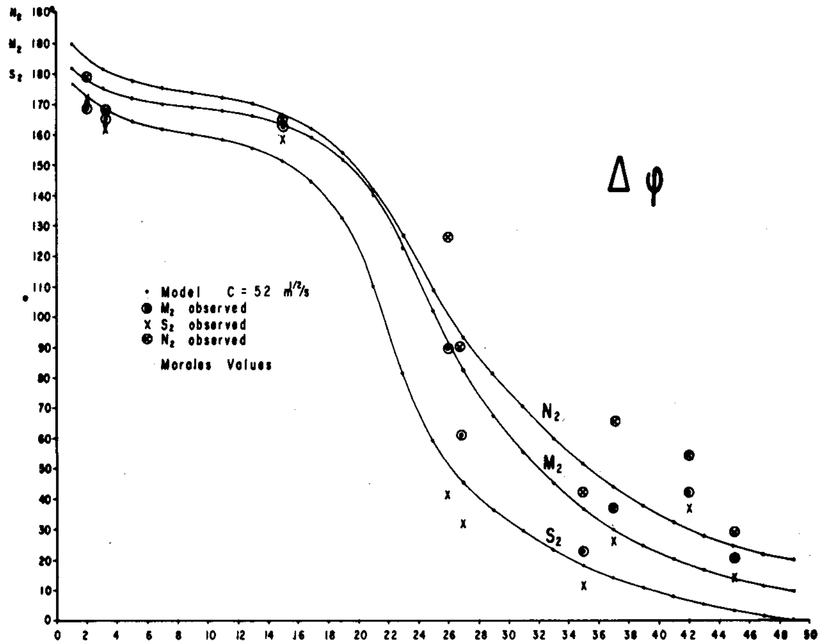

Fig. 7. Comparison between the phase differences predicted by the model and the constants reviewed by Morales for N_2, M_2, and S_2.

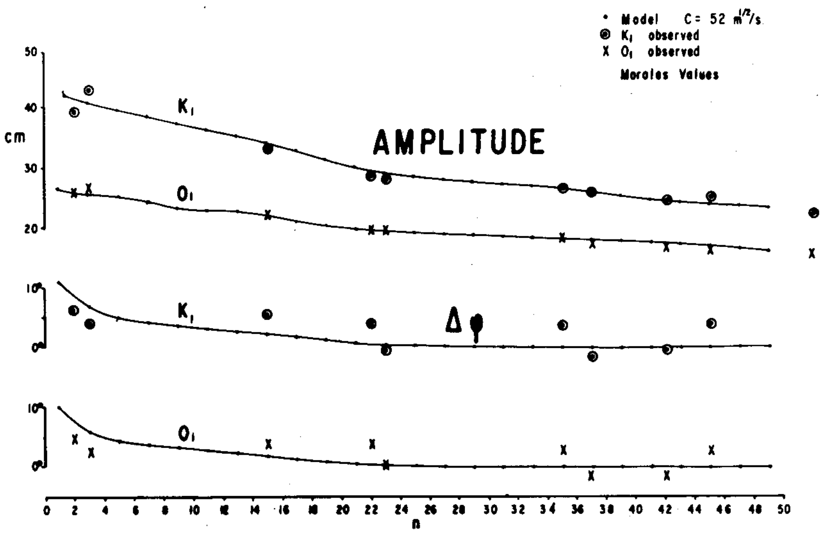

Fig. 8. Comparison between the amplitude and phase difference predicted by the model for K_1 and O_1 and the constants reviewed by Morales.

of observations. These are fewer in numbers but the model reproduces them adequately, the N_2 and K_1 fits being better.

A model which reproduces faithfully the elevations may still be most inadequate if it fails to reproduce the known velocity field. Systematic current observations are yet lacking in the gulf but we may return to the current estimates inferred from the equation of continuity and check if the model supplies the barotropic current implied by the observed elevations and the extant bathymetry. The u implied by the model to fit the Filloux data is shown as a fine line in Fig. 2. We see that the model gives unrealistic velocities in the northern extremity of the gulf but that it gives speeds which conform closely with the continuity values past km 200. Since friction is small over most of the gulf, we have also shown by a + and –, the phase relation between the semidiurnal currents and the corresponding vertical tide in the same section. South of km 550 the semidiurnal flood current (directed northward) will be some 90° out of phase behind the local semidiurnal high water while north of km 550 the same flood current will be 90° of phase or less ahead of the local high water. On the other hand the phase of the semidiurnal currents will be rather uniform over the gulf, the extreme northern currents being half an hour or more later than those at the entrance. The flood diurnal currents will run 90° or less ahead of the local diurnal high water all over the gulf; they will be nearly simultaneous with some delay in the north. South of km 550 the semidiurnal currents should have an average velocity of 5 cm/s while the diurnal ones should run at some 2 cm/s.

CONCLUSIONS

The tidal observations available in the Gulf of California were used in conjunction with the equation of continuity to estimate the relative magnitude of the frictional and convective terms with respect to the linear ones in the equations of hydrodynamics. Since the non linear terms are rather small, we may study the frictional damping of the tide inside the gulf using a perturbation approach. It was understood first that quadratic friction makes it necessary to take into consideration all components of the tide simultaneously; on the other hand, it is possible, within a perturbation approach, to evaluate approximately the mutual damping of the components and to rewrite the equation of motion into a linear form. The numerical integration of the equations of hydrodynamics over a one dimensional schematization of the gulf based on the two dimensional model of Stock (1976), using a single value of the friction coefficient, gave values of the harmonic components which compared adequately with the observations. The present exercise has exclusively pedagogical intentions. Because of the easy accessibility of computers, a perturbation technique is not necessary to integrate the equations although it may help as a first approximation. The major components can be included all at once and the non linear effects can be evaluated from the analysis of the levels generated by the model at various grid points. Filloux' data needs to be revised and reanalyzed to make it compatible with the long term observations available in the area before a more refined numerical model of the tide in the Gulf of California should be comtemplated.

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