A statistical model describing the sunshine in the southern metropolitan area of the Mexico Valley

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RESUMEN

Se propone un modelo estadístico que simule la densidad de probabilidades para la duración de la radiación solar basado en la suposición de que los intervalos de luz solar y de penumbra se distribuyen exponencialmente. Las medias respectivas t_1 y t_2 son calculadas para los meses del año y horas del día.A partir de estos parámetros se comparan las simulaciones con las medias. El modelo puede ser descrito como dos procesos alternativos Poisson. Se reportan histogramas para la duración de la radiación observada.

PALABRAS CLAVE: Radiación solar, probabilidad, Ciudad de México, clima.

ABSTRACT

A statistical model is proposed to simulate the probability density functions for sunshine duration under the assumption that the sunshine and shade time intervals are distributed exponentially. The respective means t_1 and t_2 are calculated for the months of the year and hours of day. With these parameters simulations are made and compared to measured data. The model can be described as two alternating Poisson processes. Histograms for the observed sunshine duration data are reported.

KEY WORDS: Sunshine, probability, Mexico City, climate.

INTRODUCTION

México is located in a transition zone between tropical and temperate regions; in 3/4 of its territory (about 1.5 millons km^2) long sunshine durations are observed due to the reduced average cluod cover. At present there is no reliable network of solar radiation stations that permite planning the optimun use of this resource. Nevertheless, sunshine duration is usually recorded by the meteorological network. The simplicity and low cost of such heliographic records justify their use for an approximated evaluation of available solar energy.

A. Ångström, 1924, 1956, first proposed the use of sunshine duration to estimate daily solar energy by using an empirical relation (Nagaraja-Rao *et al.*, 1985; Page, 1964; Ogelman *et al.*, 1984.). The same relation was used by Galindo and Chávez, 1977 to estimate the solar energy over Mexico on a monthly basis.

In the present work an analysis of the sunshine duration is made together with a statistical model that permits simulation of sunshine by using only two parameters. This work completes an earlier one (Galindo *et al.*, 1972-1973).

OBSERVED SUNSHINE DURATION FREQUENCY DISTRIBUTIONS

The analysis and the model developed in this work were made using hourly sunshine duration in tenths of hour (I) between 8 and 17 hrs, true solar time (tst). Records were obtained between 1968 and 1983 at the solar radiation observatory of the Instituto de Geofísica (19° 20' N and 99° 11' W), from a Campbell-Stokes heliograph, as reported (Bravo *et al.* , 1984).

From these data 108 histograms (12 months x 9 hours) were constructed. shown in Table 1. A distinct feature is that the higher frequencies appear at I=0 and I=1.0 hr and the lower frequencies at I=0.1 to 0.9 hr intervals ("U-shape" histograms). Fig. 1 and 2 show the percentage of hours with I=0 and I=1.0 hr of sunshine. On the abscissa the months of year are plotted while the hours of day are the ordinate. These figures show that the sunshine duration varies with the hour of day and with the seasons.

THE STATISTICAL MODEL

The proposed model assumes that the sunshin time intervals obey an exponential distribution with the parameter $1/t_1$; the same is expected for the following shading period with parameter $1/t_2$. Since sunshine is a function of hour of day and season, the parameters $1/t_1$ and $1/t_2$ must be also functions of these. The model can be considered as two alternating Poisson processes with t_1 and t_1 being intensity parameters (Kannan, 1979).

MODEL FITTING AND HISTOGRAM SIMULATION

Before the fitting of the model is made, it is assumed as a first approach that the mean geometry of the parameters of the exponential distributions may be considered constant during the hour to wich the model will be fitted. Initially it must be decided whether the hour begins with sunshine or with shade. This decision is made at random using the probability to obtain sunshine or shade for a given moment.

To evaluate this probability the proposed model is used. If n is the number of changes between sunshine to

TABLE I. Histograms in percentual values for the sunshine duration in Mexico City.

TENTHS OF HOUR

Hours (TST)	0	1	2	3	4	5	6	7	8	9	10
JANUARY											
8-9	8.6	2.0	1.4	1.4	1.2	1.2	3.1	0.4	2.7	2.9	75.1
9-10	5.9	1.4	1.4	1.8	0.8	0.6	1.0	2.0	1.6	2.7	80.6
10-11	4.9	1.6	1.2	1.6	0.4	0.4	1.6	2.4	2.0	2.7	81.0
11-12	6.4	0.5	3.8	1.3	1.3	1.0	2.6	2.1	4.1	2.6	74.4
12-13	6.2	2.6	1.9	1.9	1.0	2.1	2.6	3.6	4.0	5.0	69.0
13-14	8.8	1.0	2.9	2.7	2.2	2.2	2.2	4.1	3.5	5.3	65.1
14-15	11.0	3.9	2.9	3.5	4.3	1.4	1.4	3.3	4.7	3.7	60.0
15-16	16.3	2.0	3.7	5.5	2.9	3.5	5.7	3.7	8.0	4.7	44.1
16-17	37.8	4.5	5.9	5.5	5.3	3.1	4.3	5.9	4.9	5.3	17.5
FEBRUARY											
8-9	6.7	1.1	1.3	1.6	1.3	0.2	2.2	2.0	1.8	2.5	79.1
9-10	4.5	0.9	0.9	0.4	0.4	0.9	2.0	2.2	2.0	1.6	84.0
10-11	3.1	0.2	1.3	0.7	0.9	1.1	1.1	1.8	2.2	1.1	86.3
11-12	3.1	0.7	0.7	1.3	0.9	1.1	1.1	2.9	2.9	2.0	83.1
12-13	2.5	2.0	2.0	2.2	2.0	1.3	4.0	3.1	2.5	4.0	74.2
13-14	7.0	2.0	2.0	2.9	3.4	2.9	3.1	4.9	4.9	3.8	62.9
14-15	8.8	2.3	2.9	3.8	3.6	2.9	6.1	5.0	4.1	7.7	52.9
15-16	15.7	3.8	5.6	3.4	3.8	2.7	3.1	4.3	6.3	4.9	46.3
16-17	26.5	3.8	6.5	4.3	4.0	2.7	4.3	6.1	3.6	5.6	32.6
MARCH											
8-9	5.7	0.2	0.6	0.6	0.6	1.3	0.8	1.7	1.5	3.4	83.4
9-10	3.6	0.2	0.2	0.8	0.6	0.8	1.3	1.5	2.3	2.5	86.1
10-11	2.5	0.4	0.6	0.6	0.4	0.6	1.5	2.3	2.5	2.3	86.2
11-12	4.0	0.0	0.6	0.4	1.3	0.6	1.5	1.3	2.7	2.5	85.2
12-13	5.0	1.0	1.3	1.0	1.5	1.3	1.3	3.8	3.3	5.0	75.6
13-14	6.5	3.3	2.1	2.3	2.3	1.5	2.3	4.2	6.9	6.9	61.8
14-15	10.4	4.2	2.5	5.2	4.2	4.2	4.2	5.6	6.7	4.2	48.6
15-16	18.6	3.8	5.4	4.0	4.4	3.1	3.8	5.6	5.2	4.0	42.2
16-17	29.2	5.6	4.5	3.5	4.2	2.9	3.5	4.8	5.0	5.0	31.5
APRIL											
8-9	5.0	0.2	0.7	0.7	1.3	0.9	0.7	0.4	2.2	1.1	86.9
9-10	3.1	0.9	0.7	0.4	0.9	0.4	0.2	0.0	1.7	2.0	89.8
10-11	1.5	0.0	0.9	1.1	0.7	0.7	1.3	2.4	2.4	2.8	86.3
11-12	3.1	0.4	1.1	1.3	2.2	1.5	2.6	3.1	2.8	5.3	76.6
12-13	6.2	2.1	3.0	2.8	1.7	2.1	4.5	3.4	6.2	7.7	60.3
13-14	14.1	4.1	3.9	2.8	4.8	3.0	4.3	4.1	5.9	3.7	49.1
14-15	20.7	5.2	4.3	4.6	3.0	2.6	3.9	5.2	6.3	5.7	38.5
15-16	29.2	4.5	4.7	6.2	5.8	4.9	4.1	4.5	4.1	4.9	27.3
16-17	43.1	5.4	4.8	4.6	4.4	3.3	3.9	3.3	5.9	3.3	18.1

TABLE I. Continue

TENTHS OF HOUR

Hours (TST)	0	1	2	3	4	5	6	7	8	9	10
M A Y											
8-9	6.8	1.7	2.5	0.4	1.4	1.9	2.3	2.5	2.1	3.5	74.9
9-10	5.2	2.9	1.0	1.7	1.0	0.4	2.1	3.1	2.5	2.9	77.2
10-11	4.1	1.4	2.3	1.6	1.6	0.8	1.9	2.1	1.9	2.7	79.6
11-12	6.6	2.5	2.5	1.4	3.1	1.2	2.3	1.9	4.3	6.4	67.8
12-13	8.5	2.3	3.1	4.5	2.5	1.9	4.9	4.7	6.2	8.0	53.4
13-14	16.7	5.2	4.5	4.3	6.0	2.7	4.7	5.6	5.4	8.7	36.3
14-15	31.1	4.7	5.2	5.4	3.3	3.9	5.6	5.4	5.4	7.8	22.3
15-16	45.2	5.4	5.4	2.9	2.5	4.1	4.5	4.8	3.3	3.7	18.2
16-17	53.4	6.6	5.4	3.7	3.3	2.7	3.7	2.9	3.3	1.9	13.2
J U N E											
8-9	25.5	3.0	3.6	4.2	2.3	3.2	5.5	1.9	4.2	3.6	42.9
9-10	18.7	3.4	4.0	3.2	2.5	2.8	2.5	3.4	4.2	4.7	50.5
10-11	12.7	3.2	3.6	3.0	2.8	3.2	5.5	4.4	4.2	5.7	51.7
11-12	12.7	1.9	3.0	1.9	2.5	2.3	4.2	4.0	7.4	5.5	54.4
12-13	15.5	2.5	3.8	2.1	1.3	3.2	5.1	7.0	5.1	9.1	45.3
13-14	23.0	5.1	4.0	3.8	3.2	4.2	5.1	5.1	4.9	7.2	34.6
14-15	30.0	4.4	5.3	2.1	3.6	3.0	3.8	3.0	5.1	10.1	29.7
15-16	35.7	8.0	4.4	4.0	4.4	2.5	4.6	4.6	5.7	6.5	19.4
16-17	53.8	5.3	6.5	2.5	3.0	3.0	3.8	3.0	2.7	2.3	14.1
J U L Y											
8-9	20.3	2.9	2.7	4.2	4.2	2.5	6.7	6.1	6.7	7.9	35.9
9-10	11.7	2.9	3.3	3.8	3.5	3.3	4.0	5.2	6.5	8.6	47.2
10-11	7.7	3.3	4.4	1.9	2.9	2.9	5.6	5.6	8.6	9.6	47.4
11-12	8.6	3.8	3.8	2.5	4.6	4.2	4.4	6.3	6.9	11.9	43.2
12-13	15.7	3.3	4.0	4.2	2.3	3.1	5.6	7.7	9.8	12.5	31.7
13-14	23.3	5.2	6.0	4.6	2.9	3.3	6.0	7.7	7.9	9.6	23.5
14-15	34.5	6.4	5.4	6.4	5.0	4.8	4.2	6,9	6.9	6.9	12.7
15-16	48.9	6.0	7.1	6.9	4.6	4.0	4.0	4.0	3.7	3.7	7.3
16-17	66.0	6.2	6.8	4.6	2.9	2.7	2.7	3.1	2.1	1.0	1.9
A U G U S T											
8-9	18.2	4.7	3.4	4.2	4.2	4.7	4.2	6.1	5.9	5.7	38.6
9-10	9.7	2.8	3.6	3.8	3.4	2.8	4.0	6.4	4.4	7.6	51.5
10-11	7.8	2.7	1.2	4.3	2.1	2.5	3.5	5.6	9.1	9.5	51.9
11-12	8.6	2.5	3.3	2.9	3.9	2.3	5.1	6.8	9.1	10.7	44.9
12-13	11.7	3.3	3.9	3.1	3.9	4.9	7.4	11.1	11.5	9.4	29.8
13-14	18.1	4.7	6.6	6.2	4.7	6.8	5.7	9.4	8.0	7.2	22.6
14-15	28.0	4.7	10.5	6.2	4.9	5.6	6.2	6.6	7.4	6.8	13.0
15-16	44.8	8.0	6.6	5.5	5.7	3.5	5.7	6.2	4.3	2.9	6.8
16-17	66.3	4.1	6.0	4.5	4.3	3.3	3.7	3.3	1.2	1.4	1.8
S E P T E M B E R											
8-9	27.1	5.5	5.7	4.1	4.6	3.7	4.4	5.5	5.7	3.9	29.9
9-10	14.0	3.9	4.8	3.9	4.6	5.0	4.6	6.1	6.1	7.9	39.1
10-11	12.4	2.2	3.7	3.1	2.8	3.3	6.6	8.3	8.1	10.5	39.1
11-12	14.4	3.3	4.4	3.7	4.4	2.4	3.9	5.9	6.1	11.1	40.4
12-13	13.9	5.0	6.1	3.0	4.3	4.1	7.2	8.0	6.7	10.6	31.0
13-14	20.3	3.9	7.9	4.6	5.0	4.1	5.9	7.4	10.0	7.4	23.4
14-15	27.5	7.6	8.1	4.8	6.1	4.4	4.8	6.8	7.2	5.0	17.7
15-16	42.6	6.8	5.9	5.7	4.4	5.0	3.3	4.4	5.5	4.1	12.4
16-17	61.8	6.3	6.6	4.6	3.9	3.1	1.7	3.3	0.9	1.5	6.3

TABLE I. Continue

TENTHS OF HOUR

Hours (TST)	0	1	2	3	4	5	6	7	8	9	10
OCTOBER											
8-9	26.2	2.7	3.7	2.3	2.5	2.3	3.7	3.7	2.7	4.2	45.9
9-10	16.0	2.5	3.5	2.9	1.7	2.9	2.7	4.1	4.4	2.7	56.6
10-11	11.0	2.0	1.8	1.8	3.0	2.4	2.8	3.7	5.1	4.5	61.8
11-12	7.7	3.0	2.8	2.4	1.6	2.2	2.6	5.1	6.5	6.7	59.1
12-13	8.9	2.6	2.8	3.3	3.7	2.6	3.0	4.7	6.5	6.1	55.7
13-14	11.8	4.3	2.4	4.1	4.1	3.5	4.9	5.7	6.9	7.1	45.3
14-15	18.7	4.5	3.5	4.3	4.3	5.5	4.1	8.3	5.3	5.7	36.0
15-16	29.7	5.1	5.9	5.5	3.9	3.0	4.5	4.9	5.7	4.1	27.8
16-17	48.6	5.9	6.7	5.3	4.1	3.0	3.9	2.8	2.6	3.0	14.0
NOVEMBER											
8-9	14.0	3.4	1.8	1.8	1.1	0.5	2.5	2.9	2.0	3.6	66.3
9-10	8.8	1.6	1.4	1.1	1.6	0.7	2.3	1.8	3.4	2.9	74.5
10-11	6.8	1.1	0.7	0.9	0.5	1.1	1.8	2.3	0.7	3.6	80.6
11-12	5.2	2.0	1.6	0.5	1.8	0.7	2.7	2.3	2.0	3.4	77.9
12-13	4.7	2.3	2.0	1.1	2.5	2.0	2.5	2.9	3.6	5.4	70.9
13-14	8.6	1.6	2.3	4.3	1.8	2.0	5.2	5.2	7.4	4.7	56.9
14-15	14.1	4.3	3.0	3.2	2.3	3.2	5.0	5.5	4.8	4.5	50.2
15-16	23.3	4.1	4.5	2.7	4.7	5.0	5.2	3.4	4.3	4.7	38.1
16-17	40.6	5.2	5.0	5.2	3.4	4.7	5.6	5.6	6.5	3.8	14.2
DECEMBER											
8-9	15.5	3.0	0.9	0.9	1.8	1.1	1.4	2.0	4.3	3.2	65.9
9-10	9.5	0.9	1.1	1.1	0.9	1.1	2.0	3.4	2.3	3.4	74.2
10-11	6.3	3.4	1.4	1.4	1.1	1.8	1.1	1.4	3.8	1.8	76.5
11-12	4.5	0.5	1.6	0.7	2.0	1.4	2.5	3.6	2.0	3.4	77.9
12-13	4.9	0.7	2.0	2.0	1.8	2.5	2.5	3.4	5.2	4.9	70.2
13-14	8.7	3.8	2.7	2.9	2.5	1.1	3.6	4.3	4.7	4.0	61.7
14-15	13.0	4.0	3.8	2.5	3.4	4.9	3.8	4.9	4.0	4.9	50.7
15-16	22.4	3.8	7.0	4.9	3.6	3.1	6.5	4.5	4.9	2.7	36.5
16-17	46.2	4.9	4.9	4.0	3.6	3.6	2.7	5.8	4.9	4.5	14.8

shade, then the number of time intervals with duration t is n $f_1(t)dt$ for sunshine and n $f_2(t)dt$ for shade, where f_1 and f_2 are the probability density functions for sunshine and shade. The sum of those time intervals must be t n $f_1(t)dt$ and t n $f_2(t)dt$: then the ratio of total sunshine to total time must be:

$$\pi_1 = \lim_{n \to \infty} \frac{n \int_0^{\infty} t\, f_1(t)\, dt}{n \left[\int_0^{\infty} t\, f_1(t)\, dt + \int_0^{\infty} t\, f_2(t)\, dt \right]} =$$

$$\frac{E_1(t)}{E_1(t) + E_2(t)} = \frac{t_1}{t_2 + t_1} \qquad (1)$$

where $E_1(t)$ and $E_2(t)$ are expected values of t for sun and for shade. Here f_1 and f_2 are exponential density functions with parameters $1/t_1$ and $1/t_2$ respectively. Similarly, the ratio of shade to total time is:

$$\pi_2 = \frac{t_2}{t_1 + t_2} \qquad (2)$$

The simulation begins by assigning randomly, with probability π_1 or π_2 sunshine or shading, respectively, to

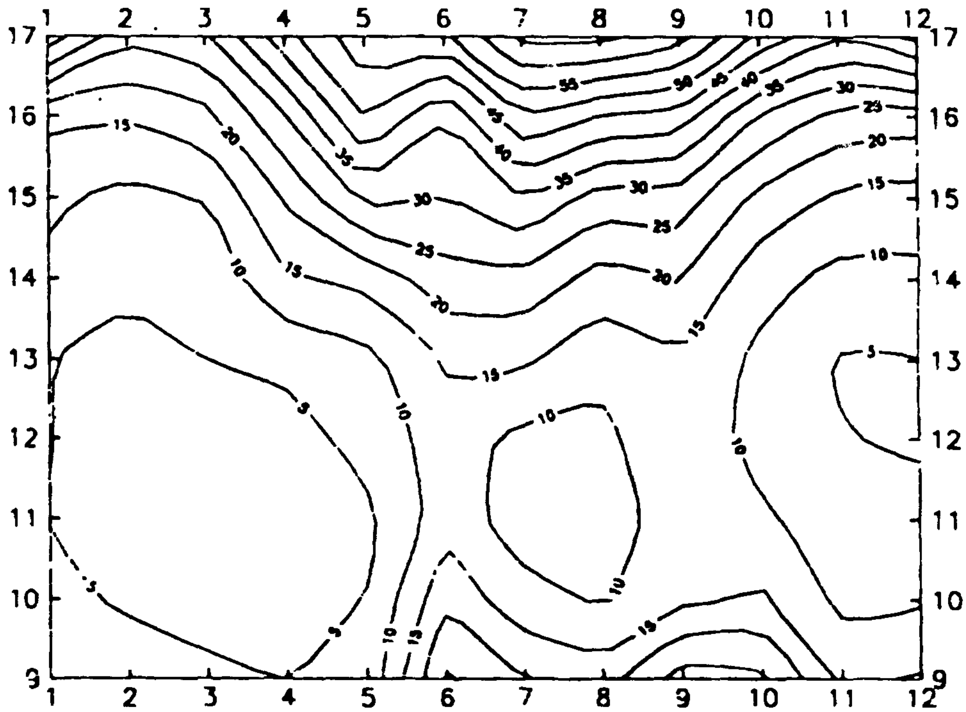

Fig. 1. Percentual values for I=0 hr sunshine duration

the beginning of the hour. Next an exponentially distributed time interval is selected, with $1/t_1$ or $1/t_2$ as parameter, depending on whether the hour starts with sunshine or shade. If the time interval is greater than or equal to one hour it will be assigned to the hour I=1.0 or I=0 hr of sunshine duration.

When the time interval is less than one hour, another exponentially distributed time interval is selected, simulating shade or sunshine, as needed, until the hour is completed. The total times of sunshine and shade are counted separately. Thus sunshine duration will be the sum of time intervals simulated with the parameter $1/t_1$ expressed in tenths of hour. Simulation of one histrogram is made by repeating this process N times, where N is the number of observed hours in each case.

The fitting of the simulated to the observed histograms is made by finding the asymptotic perfomance of I=1.0 and I=0 hr sunshine classes and selecting the values of t_1 and t_2 such that asymptotic values fit with observed ones. Asymptotic simulated values for I=1.0 hr of sunshine are found by considering that the probability of starting a simulated hour with sunshine is $\pi_1=t_1/(t_1 + t_2)$. When sunshine is selected at the beginning, the probability of no ocurrence of an event during one hour will be $\exp(-1/t_1)$.

Then, the probability of I=1.0 hr simulated sunshine is:

$$P_{10} = \frac{t_1}{t_1 + t_2} e^{-\frac{1}{t_1}} \qquad (3)$$

Similarly, for I=0 sunshine:

$$P_0 = \frac{t_2}{t_1 + t_2} e^{-\frac{1}{t_2}} \qquad (4)$$

By equating P_{10} and P_0 with the observed relative frequencies we obtain two simultaneous equations, with t_1 and t_2 as unknowns. These equations are solved using Newton's method, and the parameters t_1 and t_2 are found.

To obtain an approximation to the asymptotic values for sunshine duration between 0.1 and 0.9 hr, 108 couples of parameters were fitted, and with these parameters the means of the 9 central classes for 100 simulated histograms were obtained. This procedure was repeated for each observed histogram.

Notice that the distribution of the number of events in each histogram class between 0.1 and 0.9 must be binominal with a small probability parameter, which can be approximated by a Poisson distribution (Hoog and Craig, 1965). It was considered that 100 simulations were sufficient. If we accept the Poisson distribution approximation, the variance can be approximated by the mean. Then the variance of the histogram means should be 1% of the variance because variance of the mean of a sample from a random variable is the variance of the random variable divided by the size of the sample (Hoog and Craig, 1965). Table II shows the means of the exponential distributions for the hours of day and the months.

GOODNESS OF FIT

The Chi-square test was used to verify the model goodness of fit. This test requires a number of not less than 5 expected observations for each class (Hoog and Craig, 1965). Hence the nine central classes were grouped in three: from 0.1 to 0.3, from 0.4 to 0.6 and from 0.7 to 0.9 hr of sunshine duration. The test was made only with these three grouped classes. Notice that data of intermediate classes are not used to obtain t_1 and t_2 values; hence, there are 3 classes and 2 degress of freedom.

The Chi-square test was applied to each of the 108 histograms using as expected frequency the asymptotic ones, obtained from the mean of 100 simulations. Values are shown in Table III. Rejection value at 0.05 confidence level is 5.99; 12 cases were rejected at this level. The reamining 89% are in the area where the goodness of fit is acceptable. Notice that the Chi-square test is performed at 0.05 confidence level several times. Hence 5% of the values must be in the rejection zone. In this case, we had 11%. Notice also that there are 4 outliers; August, 16 to 17 hrs and 12 to 13 hrs; July, 12 to 13 hrs and February,

TABLE II: Parameters t_1 and t_2 (expresed in hours) of the exponential distributions for sunshine and shade respectively.

Hour		JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
8-9	t1	8.73	10.38	14.19	19.74	7.67	3.09	2.02	2.06	1.83	3.44	6.71	7.08
	t2	1.64	1.54	1.66	1.86	1.30	2.13	1.41	1.30	1.72	2.28	2.01	2.25
9-10	t1	11.00	13.22	14.84	22.19	7.98	3.55	2.52	2.67	1.97	4.02	8.51	8.72
	t2	1.46	1.36	1.27	1.44	1.14	1.75	1.07	0.97	1.08	1.63	1.64	1.76
10-11	t1	10.31	14.33	13.12	11.60	8.70	3.09	2.26	2.67	1.91	4.53	11.86	8.25
	t2	1.26	1.16	0.98	0.73	1.05	1.24	0.81	0.87	0.99	1.35	1.67	1.30
11-12	t1	9.44	10.57	14.02	6.42	5.01	3.47	2.01	2.13	2.10	3.54	8.37	7.95
	t2	1.23	0.99	1.31	0.79	1.04	1.31	0.82	0.84	1.12	0.97	1.17	1.05
12-13	t1	6.34	5.46	7.09	3.50	2.89	2.59	1.58	1.36	1.49	3.20	5.31	5.18
	t2	1.03	0.67	1.05	0.86	0.94	1.30	1.06	0.84	0.96	1.01	0.90	0.91
13-14	t1	4.85	4.04	3.77	2.91	1.90	2.04	1.32	1.17	1.25	2.35	3.33	4.11
	t2	1.21	0.98	0.91	1.29	1.19	1.57	1.31	1.04	1.16	1.04	1.01	1.12
14-15	t1	4.18	2.86	2.57	2.26	1.42	1.93	0.95	0.88	1.10	1.96	3.04	2.99
	t2	1.30	0.95	1.01	1.51	1.75	1.94	1.64	1.31	1.40	1.31	1.31	1.24
15-16	t1	2.53	2.72	2.48	1.78	1.51	1.35	0.80	0.73	1.04	1.77	2.37	2.18
	t2	1.33	1.34	1.45	1.86	2.78	1.97	2.35	2.02	2.16	1.85	1.70	1.58
16-17	t1	1.28	2.03	2.06	1.44	1.32	1.42	0.52	0.52	0.90	1.27	1.12	1.27
	t2	2.05	1.76	1.96	2.55	3.37	3.54	3.57	3.62	3.75	2.83	2.11	2.63

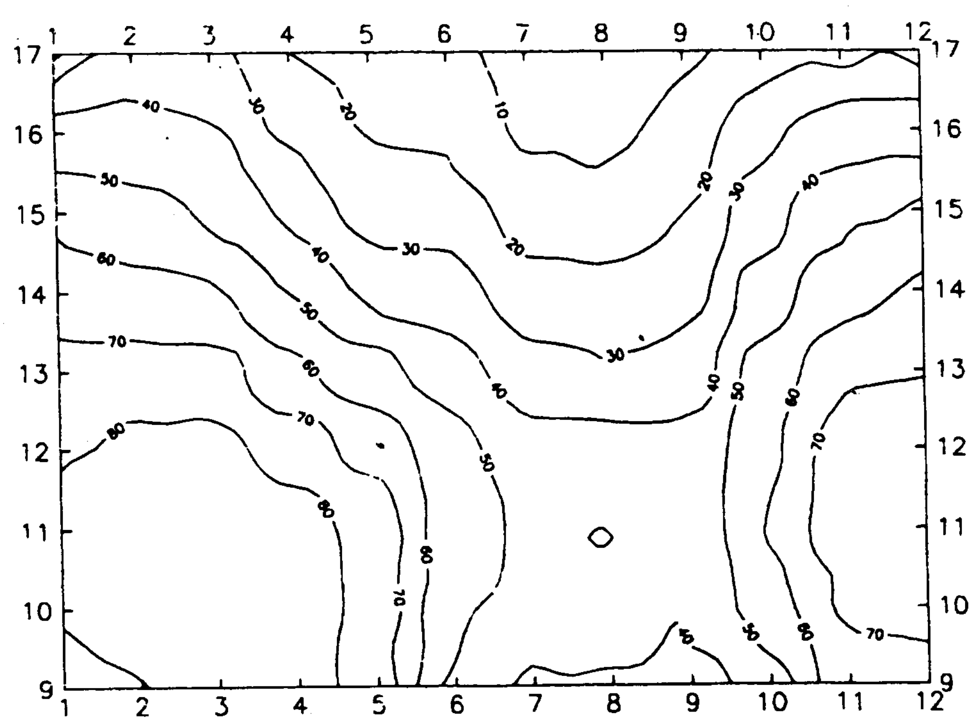

Fig. 2. Percentual values for I=1.0 hr sunshine duration.

12 to 13 hrs. This suggests that there might be a systematic meteorological phenomenon during these periods that perturbs the normal behavior of the sunshine duration.

RESULTS AND DISCUSSION

Figs. 1 and 2 show the probability of ocurrence of I=0 and I=1.0 hr sunshine duration, respectively. On the abscissa the months of the year are plotted, and on the ordinate the hours of day (tst). Fig.1 shows a fringe of low probability for 0 sunshine; this fringe remains all year between 8 and 13 hrs. The rest of the isolines have a tendency to be parallel, and present an increase of probability in the afternoon. Fig. 2 shows a region of high probability (greater than 0.8) between 8 and 13 hrs during February, March and April. This probability is greater than 0.7 between 8 and 13 hrs from November to May, with a maximun in the dry season for Mexico Valley (October to May). Even so, notice that in the wet season (June to September) the probability of hours with I=1.0 hr sunshine duration diminishes considerably after 15 hrs. In fact, a very low probability region is noticeable from July to September after 15 hrs, with probability of ocurrence less than 0.1.

Figs. 3 and 4 show isolines of the parameters t_1 and t_2. The abscissa is the month of the year and the ordinate is the hour of day (tst). The behaviour of these parameters is similar to that exhibited by the probability of ocurrence of I=1.0 and I=0 hr sunshine duration, respectively.

Fig. 5 shows the probability of I=1.0 hr sunshine duration for each season of the year. Notice again the general tendency for a decrease in probability at late hours during all four seasons of year. However, in winter and spring the probability remains high between 8 and 12 hrs.

Fig. 6 shows the mean probability for I=1.0 hr sunshine duration between 8 and 13 hrs versus the month of the year. A slow transition is noted between the first maximun and minimun of probability, while the transition between the minimun and the second maximum is faster. This could be related to the variability of the beginning of the rainy season at Mexico Valley and with the low probability or rain in October and following months (Camarillo, 1984).

CONCLUSIONS

Using two alternating exponentially distributed time intervals, one for the period of sunshine and the other for the period of cloudinnes, it is possible to simulate the sunshine duration probability density function. This was performed on the basis of hourly sunshine duration data recorded at a site located in the southern area of Mexico Valley.

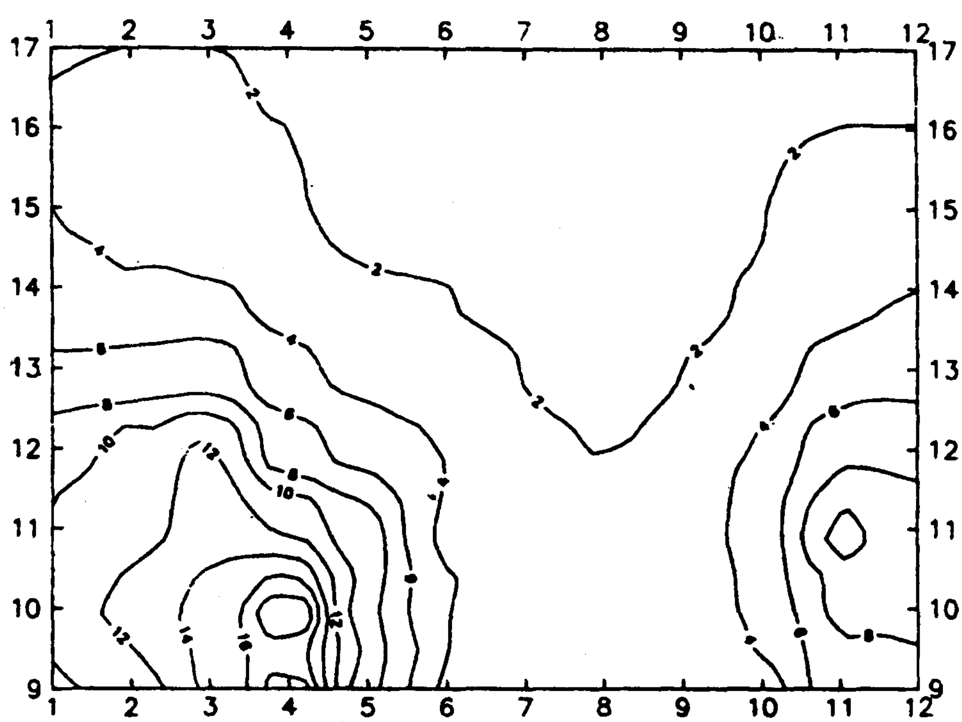

Fig. 3. t_1 Parameter for sunshine.

The probability of obtaining sunshine at a given moment is $t_1/(t_1+t_2)$, where t_1 and t_2 are the means of the aforementioned exponential distributions. These parameters are functions of the hour of day and the season. The presence of some meteorological phenomenon that perturbs the sunshine duration is suggested by very high Chi-square values for 12 to 13 hrs (tst) during February, July and August, and between 16 to 17 hrs (tst) during August also. Maximum sunshine occurs between November and May from 9 to 13 hrs (tst).

The season of highest sunshine occurs between November and May from 9 to 13 hrs (tsv). It is the season when the probability of I=1.0 hr sunshine duration is higher than 0.70. The probability of hours with I=0 hr sunshine has a minimun between 9 and 13 hrs during all the year, and increases in the afternoon hours. In the morning of the rainy season the probability of getting hours in which sunshine duration differs from 0 remains above 0.85.

The minimun value for the probability of I=1.0 hr of sunshine is greater than or equal to 0.39 during morning hours of all months of the year. This means that the probability of getting hours with I=1.0 hr of sunshine during the morning, for all months of year, is greater than 0.39.

TABLE III: Chi-square test for goodnes of fit

Hour	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
8-9	2.1	0.3	7.2	1.7	0.1	3.7	5.0	0.7	0.1	0.3	3.7	1.2
9-10	5.6	0.7	5.7	1.3	4.8	1.4	1.1	0.1	0.1	1.4	0.3	1.8
10-11	5.2	0.2	3.2	1.6	5.5	0.2	1.8	6.5	8.2	1.8	0.8	3.5
11-12	1.3	1.5	6.6	2.1	1.1	2.9	1.7	5.6	3.5	5.1	0.2	2.3
12-13	2.0	10.5	8.7	1.6	2.4	5.7	12.4	12.2	1.4	0.9	0.1	0.3
13-14	1.3	0.3	7.9	1.1	0.6	0.2	6.6	1.4	2.8	0.3	0.8	4.1
14-15	4.3	1.1	1.1	4.3	6.0	2.7	4.3	1.6	0.7	1.1	0.1	0.8
15-16	0.04	2.9	1.1	0.6	0.6	1.8	0.2	2.0	0.4	2.0	4.7	2.5
16-17	3.5	1.2	1.2	0.4	1.9	0.7	0.1	9.9	1.7	2.3	2.4	1.9

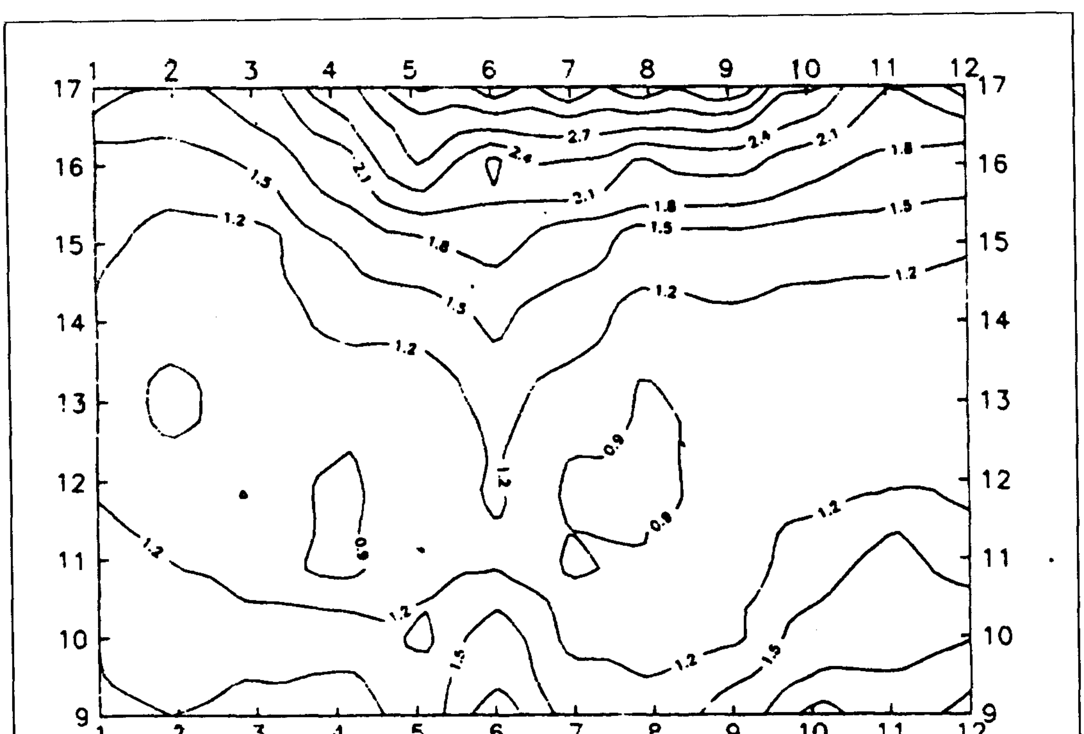

Fig. 4. t_2 parameter for shade.

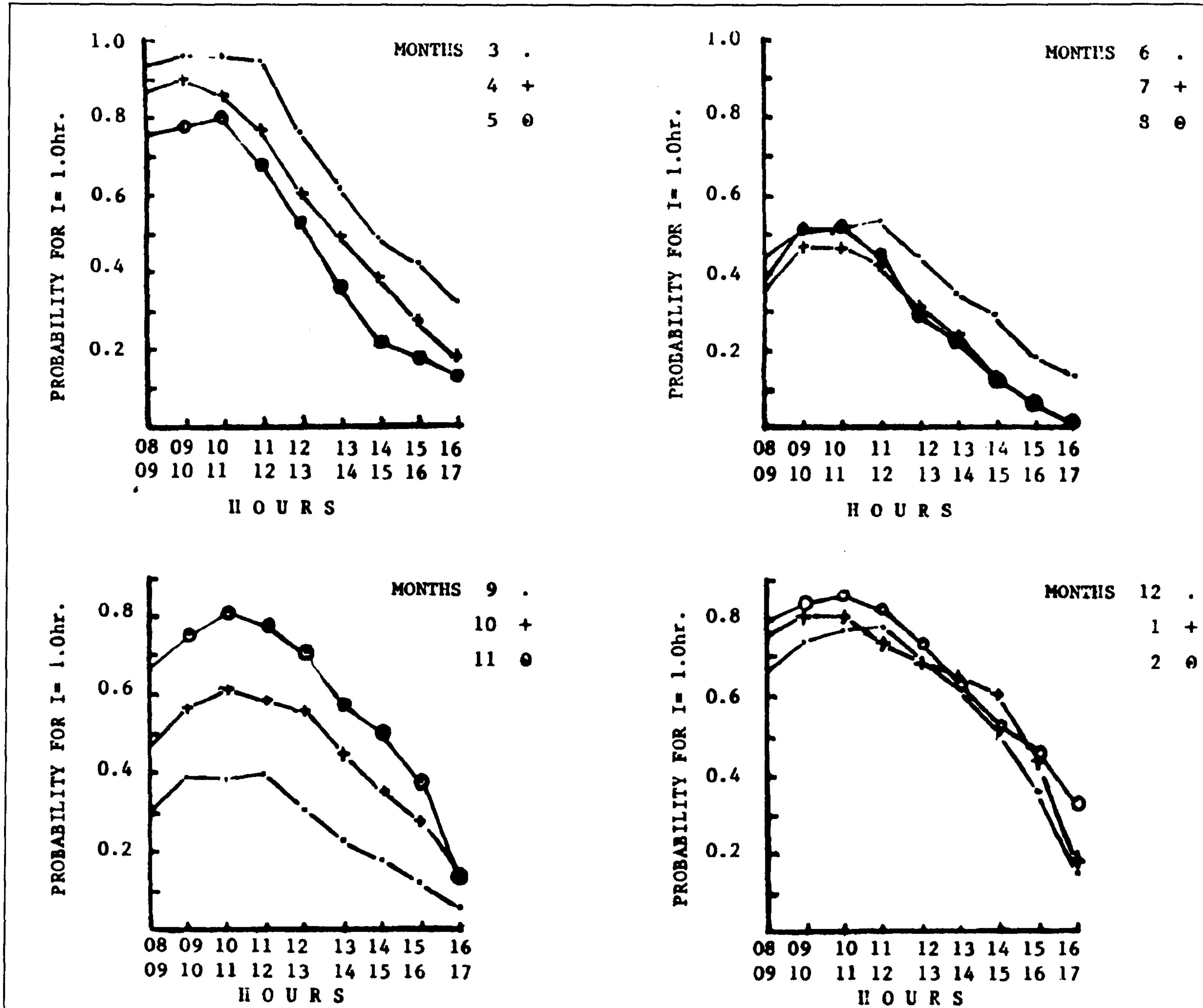

Fig. 5. Probability of I = 1.0 hr sunshine duration for each season of year.

TABLE III.: Chi-square test for goodnes of fit

Hour	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
8-9	2.1	0.3	7.2	1.2	0.1	3.7	5.0	0.7	0.1	0.3	3.7	1.2
9-10	5.6	0.7	5.7	1.3	4.8	1.4	1.1	0.1	0.1	1.4	0.3	1.8
10-11	5.2	0.2	3.2	1.6	5.5	0.2	1.8	6.5	8.2	1.8	0.8	3.5
11-12	1.3	1.5	6.6	2.1	1.1	2.9	1.7	5.6	3.5	5.1	0.2	2.3
12-13	2.0	10.5	8.7	1.6	2.4	5.7	12.4	12.2	1.4	0.9	0.1	0.3
13-14	1.3	0.3	7.8	1.1	0.6	0.2	6.6	1.4	2.8	0.3	0.8	4.1
14-15	4.3	1.1	1.1	4.3	6.0	2.7	4.3	1.6	0.7	1.1	0.1	0.8
15-16	0.04	2.9	1.1	0.6	0.6	1.8	0.2	2.0	0.4	2.0	4.7	2.5
16-17	305	1.2	1.2	0.4	1.9	0.7	0.1	9.9	1.7	2.3	2.4	1.9

The transition between high to low values of sunshine duration in spring is slower than the transition between the low to high values during fall.

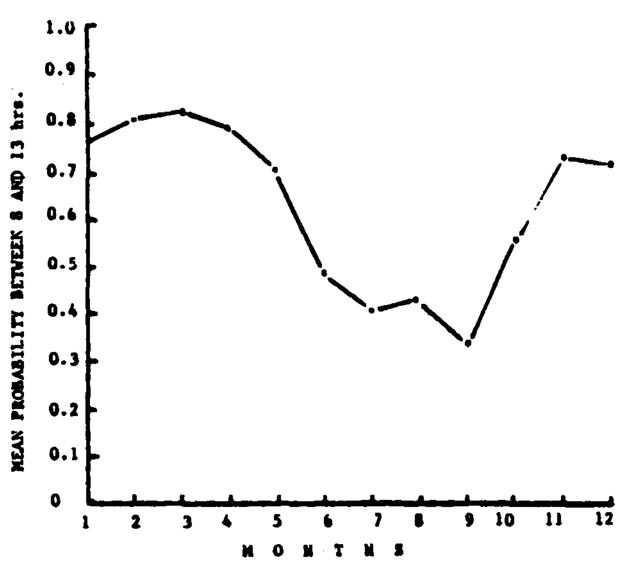

Fig. 6. Mean probability for I = 1.0 hr sunshine duration between 8 and 13 hrs (tst)

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