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# *A PARAMETER FOR ESTIMATING THE DEGREE OF MAGNETIC SUSCEPTIBILITY ANISOTROPY*

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## RESUMEN

Hay cierta confusión en la literatura acerca del empleo del término "grado de susceptibilidad magnética anisotrópica". Los parámetros propuestos previamente como índices de evaluación del grado de anisotropía carecen de base física y su motivación ha sido esencialmente instrumental. La susceptibilidad de bajo campo puede ser estimada por un tensor simétrico de segundo orden. El tensor adecuado para la susceptibilidad isotrópica, está representado por una esfera, en tanto que el de susceptibilidad anisotrópica configura un elipsoide. El grado de anisotropía en consecuencia, aumenta a medida que el elipsoide se aparta de la esfericidad. Parece entonces lógico que el grado de anisotropía pueda ser estimado a partir del grado de separación de la esfericidad y medido por comparación de los elipsoides (anisotropía) con la esfera (isotropía).

Para este propósito definimos aquí un parámetro basado en el argumento que, sólidos de igual área de superficie, pero de volúmenes diferentes, tienen distintas formas y se expresa como la relación del radio de volumen de una esfera –de igual área de superficie que el elipsoide– respecto al volumen del mismo elipsoide. De aquí se derivan expresiones simples para estimar el grado de anisotropía para casos de esferoides prolatos y oblatos. Finalmente se proponen expresiones aproximadas para estimar el grado de susceptibilidad magnética anisotrópica de las rocas.


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## ABSTRACT

Confusion has existed in the literature over the use of the term "degree of magnetic susceptibility anisotropy". Previous parameters proposed as estimators of the anisotropy degree have no physical basis, and their motivation has been mainly operational. The low-field susceptibility can be approximated by a symmetric tensor of second rank. The tensor for isotropic susceptibility is represented as a sphere and for anisotropic susceptibility as an ellipsoid. The anisotropy degree therefore increases as the ellipsoid departes from sphericity. Thus, it seems logical that the degree of anisotropy can be estimated from the degree of departure from sphericity and measured by comparing the ellipsoids (anisotropy) with the sphere (isotropy). For this purpose we define a parameter based on the argument that solids of equal surface areas but of different volumes have different shapes and express it as the ratio of the volume of a sphere, of the same surface areas as the ellipsoid, to the volume of the ellipsoid. Simple expressions for estimating the anisotropy degree for the cases of prolate and oblate spheroids are derived. Finally, approximate expressions for estimating the degree of magnetic susceptibility anisotropy of rocks are proposed.

## INTRODUCTION

Anisotropy susceptibility studies are now routinely used in palacomagnetism. They permit to study the effects of anisotropy in the ability of the rocks to accurately record the Earth's magnetic field. In addition, these studies have application in other fields, e.g., in petrofabric studies and in interpretation of magnetic anomalies. One of the properties required is the degree of anisotropy (Gough *et al.*, 1977). Confusion has existed in the literature over the use of the term "degree of magnetic susceptibility anisotropy". For example, depending on the effects of anisotropy in the direction of the magnetization, rocks have been often generally classified as isotropic and anisotropic, or as non-anisotropic, low-anisotropic and high-anisotropic. Attempts have been made to derive parameters for estimating the anisotropy degree which do not have physical basis, but merely quantify some of the factors affecting the magnetic anisotropy of rocks. As a result the degree of anisotropy has been loosely and ambiguously defined, meaning different things to different workers.

## DEGREE OF ANISOTROPY

Previous parameters (Nagata, 1953; Graham, 1966; Rees, 1966) proposed as estimators of the magnetic anisotropy degree have no physical basis and their motivation has been mainly operational. The purpose of this note is to develop an estimator based on a simple mathematical model. The susceptibility of rocks measured in weak magnetic fields is approximated by a symmetric tensor of second rank characterized by three principal susceptibilities, $k_1 \geqslant k_2 \geqslant k_3$. When $k_1 = k_2 = k_3$, the susceptibility is isotropic, otherwise the susceptibility is anisotropic. The tensor for isotropic susceptibility is represented as a sphere and for anisotropic susceptibility as an ellipsoid (Nye, 1957). The degree of anisotropy therefore increases as the ellipsoid departes from sphericity. Thus, it seems logical that the degree of anisotropy can be estimated from the degree of departure from sphericity and measured by comparing the ellipsoids (anisotropy) with the sphere (isotropy). There are several possible ways of doing this and in fact some have already been exploited in other fields, e.g., in petrography of sedimentary rocks to study the shape of grains (Wadell, 1932, 1933, 1934; Krumbein and Sloss, 1963; Krumbein, 1941).

## PREVIOUSLY DEFINED PARAMETERS

Nagata (1953) was the first to define a parameter for estimating the anisotropy degree. This is expresed as:

$$P = \frac{k_1}{k_3} \qquad (1)$$

The parameter implicitly assumes that anisotropies having the same degree are those of the same $k_1$, $k_3$, as it takes no account of the $k_2$ principal susceptibility and, in addition, the formula has no physical basis.

Graham (1966) defined a parameter which expresses the degree in percentage by normalizing to the $k_1$ susceptibility. This is defined as:

$$A = \frac{k_1 - k_3}{k_1} \times 100 \qquad (2)$$

No account is taken of the $k_2$ susceptibility and A has no physical basis. This parameter can be expressed in term of P as:

$$A = (1 - \frac{1}{P}) \times 100 \qquad (3)$$

Finally, Rees (1966) defined a parameter which has the advantage that it takes account of the three principal susceptibilities, but it has no physical basis. The parameter is defined as follows:

$$h = \frac{k_1 - k_3}{k_2} \quad \text{or} \quad h = \frac{k_1 - k_3}{k_2} \times 100 \qquad (4)$$

In addition, these three parameters have the disadvantage that they do not take account of the difference between prolate and oblate susceptibilities (see section 5).

## A PARAMETER OF ANISOTROPY DEGREE

The parameter proposed here is based on the argument that solids of equal surface areas but of different volumes have different shapes. The parameter is defined as the ratio of the volume of a sphere ($V_S$), of the same surface area as the ellipsoid, to the volume of the ellipsoid ($V_e$). That is

$$\Phi = \frac{V_s}{V_e} \qquad (5)$$

Since the sphere is the solid that, for a given surface area, has the maximum volume, $\Phi$ is always greater than unity. When $\Phi = 1$ the susceptibility is isotropic and as $\Phi$ increases the degree of anisotropy increases.

To obtain a simple expression of this parameter we turn now to examine two cases of interest in magnetic anisotropy. (1) When $k_3 = k_2 < k_1$ and the ellipsoid is a prolate spheroid and (2) when $k_1 = k_2 > k_3$ and the ellipsoid is an oblate spheroid. These cases of prolate and oblate spheroids are of great importance in magnetic susceptibility and petrofabric studies (Rahman *et al.*, 1975). Primary depositional fabrics are represented by predominantly oblate spheroids (Graham, 1966; Hamilton and Rees, 1970; Rahman *et al.*, 1975), whereas secondary strain-controlled fabrics by predominantly prolate spheroids (Crimes and Oldershaw, 1967). Prolate spheroids are estimators of magnetic lineation and current direction, while oblate spheroids indicate magnetic foliation related to bedding plane (Rahman *et al.*, 1975; Gough *et al.*, 1977).

For prolate spheroids ($k_3 = k_2 < k_1$) we have

$$S_{e_p} = \text{surface area of prolate spheroid} = 2\pi k_3^2 + 2\pi \frac{k_1 k_3}{e} \sin^{-1} e \qquad (6)$$

where $e = \text{eccentricity} = \sqrt{1 - \left(\frac{k_3}{k_1}\right)^2}$

By expanding $\sin^{-1} e$ in a series:

$$\sin^{-1} e = e + \frac{1}{2\cdot 3} e^3 + \frac{1.3}{2.4.5} e^5 + \cdots\cdots \qquad (7)$$

and neglecting third and higher powers of e (e is < 1), the expression (6) is reduced to

$$S_{e_p} = 2\pi (k_3^2 + k_1 k_3) \qquad (8)$$

$$Ve_p = \text{volume of prolate spheroid} = \frac{4}{3}\pi k_1 k_3^2 \qquad (9)$$

$S_S$ = surface area of sphere = $4\pi\ r^2$ (10)

where r is defined as the normalizing radius = $\sqrt{\frac{k_1^2 + k_1\ k_3}{2}}$

$V_S$ = volume of sphere of the same surface area as the ellipsoid

$$V_S = \frac{4}{3}\pi\left(\frac{k_3^2 + k_1\ k_3}{2}\right)^{3/2} \tag{11}$$

and the degree of anisotropy is expressed as

$$\Phi_P = \frac{V_S}{V_{e_p}} = \frac{(k_1^2 + k_1\ k_3)^{3/2}}{\sqrt{8}\ k_1\ k_3^2} \tag{12}$$

For oblate spheroids ($k_1 = k_2 > k_3$) we have

$$S_{e_o} = \text{surface area of oblate spheroid} = 2\pi\ k_1^2 \left\{ 1 + (1-e)\ \frac{tgh^{-1} e}{e} \right\} \tag{13}$$

where e = eccentricity = $\sqrt{1-\left(\frac{k_3}{k_1}\right)^2}$

By expanding $tgh^{-1} e$ in a series

$$tgh^{-1} e = e + \frac{e^3}{3} + \frac{e^5}{5} + \cdots\cdots \tag{14}$$

and neglecting third and higher powers of e (e is < 1), the expression (13) is reduced to

$$S_{e_o} = 2\pi \, (k_1^2 + k_3^2) \tag{15}$$

$$V_{e_o} = \text{volume of oblate spheroid} = \frac{4}{3}\pi \, k_1^2 \, k_3 \tag{16}$$

$$S_S = \text{surface area of sphere} = 4\pi \, r^2 \tag{17}$$

where r is defined as the normalizing radius $= \sqrt{\dfrac{k_1^2 + k_3^2}{2}}$

$V_S$ = volume of sphere of the same surface area as the ellipsoid

$$V_S = \frac{4}{3}\pi \left( \frac{k_1^2 + k_3^2}{2} \right)^{3/2} \tag{18}$$

and the degree of anisotropy is expressed as

$$\Phi_o = \frac{V_S}{V_{e_o}} = \frac{(k_1^2 + k_3^2)^{3/2}}{\sqrt{8}\, k_1^2 \, k_3} \tag{19}$$

For $k_1 = k_2 = k_3$, the two expressions (12 and 19) give the same result, that is $\Phi_p = \Phi_o = 1$, and the susceptibility is isotropic.

## COMPARISION WITH PREVIOUS PARAMETERS

By using the expressions discussed in section 3 (1, 2 and 4) we can compare their results for the cases of prolate and oblate spheroids. Expression (2) is not, for this comparision, taken in percentage.

For prolate spheroids ($k_3 = k_2 < k_1$), the expressions are

$$P_P = \frac{k_1}{k_3} \tag{20}$$

$$A_P = \frac{k_1 - k_3}{k_1} \tag{21}$$

$$h_P = \frac{k_1 - K_3}{k_2} \tag{22}$$

and for oblate spheroids ($k_1 = k_2 > k_3$)

$$P_o = \frac{k_1}{k_3} \tag{23}$$

$$A_o = \frac{k_1 - k_3}{k_1} \tag{24}$$

$$h_o = \frac{k_1 - k_3}{k_1} \tag{25}$$

By comparing these expressions we have

$$P_P = P_o \tag{26}$$

$$A_P = A_o \tag{27}$$

$$h_P = \frac{k_1}{k_3} h_o \tag{28}$$

That is, the two first expressions give the same degree of anisotropy for prolate and oblate spheroids, no distinction is made, and the third expression indicates that the degree of anisotropy for prolate spheroids is greater $\left( \frac{k_1}{k_3} \text{ times} \right)$ than the degree for oblate spheroids. This is here interpreted as due to the formulation of h, and not as a true relation.

By comparing with the expressions obtained here, from (12) and (19) we have

$$\Phi_P = \frac{k_1}{k_3} \left\{ \frac{1 + k_1/k_3}{1 + k_1^2/k_3^2} \right\}^{3/2} \Phi_o \tag{29}$$

that is

$$\Phi_P = f\left( \frac{k_1}{k_3} \right) \Phi_o \tag{30}$$

By examining $f\left(\frac{k_1}{k_3}\right)$ we can observe that when $\frac{k_1}{k_3} <\sim 1.593$, $f\left(\frac{k_1}{k_3}\right) > 1$ and for values of $\frac{k_1}{k_3} >\sim 1.593$, $f\left(\frac{k_1}{k_3}\right) < 1$. This suggests that the anisotropy degree for prolate spheroids is greater than for oblate spheroids when $\frac{k_1}{k_3} < \sim 1.593$ and less when $\frac{k_1}{k_3} > \sim 1.593$. These results are of interest in studies concerning with comparisions of the degree of anisotropy where prolate and oblate spheroids are observed.

## FINAL REMARKS

To use the parameters defined for rocks which are neither exclusively

prolate nor exclusively oblate we can modify the expressions (12) and (19) to obtain,
for predominatly prolate spheroids

$$\Phi_p = \frac{(k_1^2 + k_1 k_{23})^{3/2}}{\sqrt{8} k_1 k_{23}^2} \qquad (31)$$

where

$$k_{23} = \frac{k_2 + k_3}{2}$$

and for predominantly oblate spheroids

$$\Phi_0 = \frac{(k_{12}^2 + k_3^2)^{3/2}}{\sqrt{8} k_{12}^2 k_3} \qquad (32)$$

where $k_{12} = \dfrac{k_1 + k_2}{2}$

For $k_1 = k_2 = k_3$, the two expresions give the same result, that is $\Phi_p = \Phi_0 = 1$, and the susceptibility is isotropic.

Similar expressions for estimating degree of anisotropy can be derived by using different properties of sphere and ellipsoid. Also an expression for ellipsoids formed by rotation of an ellipse of variable axes can be derived by using the equation of surface area given by Frost (1875). The parameter here proposed has the advantage that, in addition to the mathematical and physical formulation which is, I think, a more rational approach, it posses both geological and physical significances as it can be directly related to the sedimentological and deformational processes (Graham, 1966; Rees, 1965; King and Rees, 1966). Studies of these expressions and their utility in studying fabrics of deformed and non-deformed rocks assuming volume and surface area changes of the susceptibility ellipsoid are in progress and the results will be reported elsewhere.

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