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STATISTICAL ANALYSIS OF STRESS DROPS IN MEXICO WITH AN APPLICATION TO ACCELERATIONS OF THE GROUND

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RESUMEN

El análisis estadístico de caídas de esfuerzo, p , de 5 grupos de temblores en California y México confirma que la distribución de densidad de p es proporcional a $p^{-1+\alpha}$ ($\alpha < 0$). El resultado había sido predicho por Caputo (1976). La distribución de densidad de p nos permite computar el período de retorno de aceleraciones del suelo (Caputo, 1981a) y encontrar signos precursoros de temblores (Caputo, 1981b).

ABSTRACT

The statistical analysis of the stress drops, p , of 5 sets of earthquakes in California and Mexico confirms that the density distribution of p is proportional to $p^{-1+\alpha}$ ($\alpha < 0$). The result had been anticipated by Caputo (1976). The density distribution of p allows us to compute the return period of accelerations of the ground (Caputo, 1981a) and to find precursors of earthquakes (Caputo, 1981b).

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INTRODUCTION

It is known that b_0 and b , coefficients of the empirical laws of earthquakes frequency distributions versus magnitude and moment (e.g., Caputo, 1976)

$$\log n_0 = a_0 - b_0 \log M_0 \quad (1)$$

$$\log n = a - bM$$

are related to the density distributions, $\bar{n}_\ell(\ell)$, of linear dimensions of faults, ℓ , by

$$\bar{n}_\ell(\ell) = \ell^{-\nu} \quad , \quad \frac{\nu-1}{3} \gamma = b \quad , \quad \frac{\nu-1}{3} = b_0 \quad (2)$$

where γ is the parameter of the formula

$$\eta W = E = 10^{\beta+\gamma M} \quad (3)$$

with E the elastic energy radiated, W the energy released and η the seismic efficiency. It was anticipated by Caputo (1976) that the density distribution $\bar{n}_p(p)$ of the stress drop p associated with earthquakes, in a wide range of p , has the approximate density distribution

$$\bar{n}_p(p) = p^{-1+\alpha} \quad , \quad \alpha < 0 \quad (4)$$

A statistical analysis of the stress drops of 11 sets of earthquakes in Central and Southern California, Nevada, Italy, Japan, and Alaska-Aleutian Arc (Caputo, 1981b), confirms the validity of (4).

STATISTICAL ANALYSIS OF THE STRESS DROPS IN MEXICO AND BAJA CALIFORNIA

The statistical analysis of the stress drops for 5 sets of earthquakes in Baja California and Mexico proves the validity of (4) also for these regions. The results of the analysis are shown in Figure 1. The numerical values of $-1 + \alpha$ are in agreement with those observed in Southern and Central California, in Nevada, Japan, Italy and the Alaska-Aleutian regions.

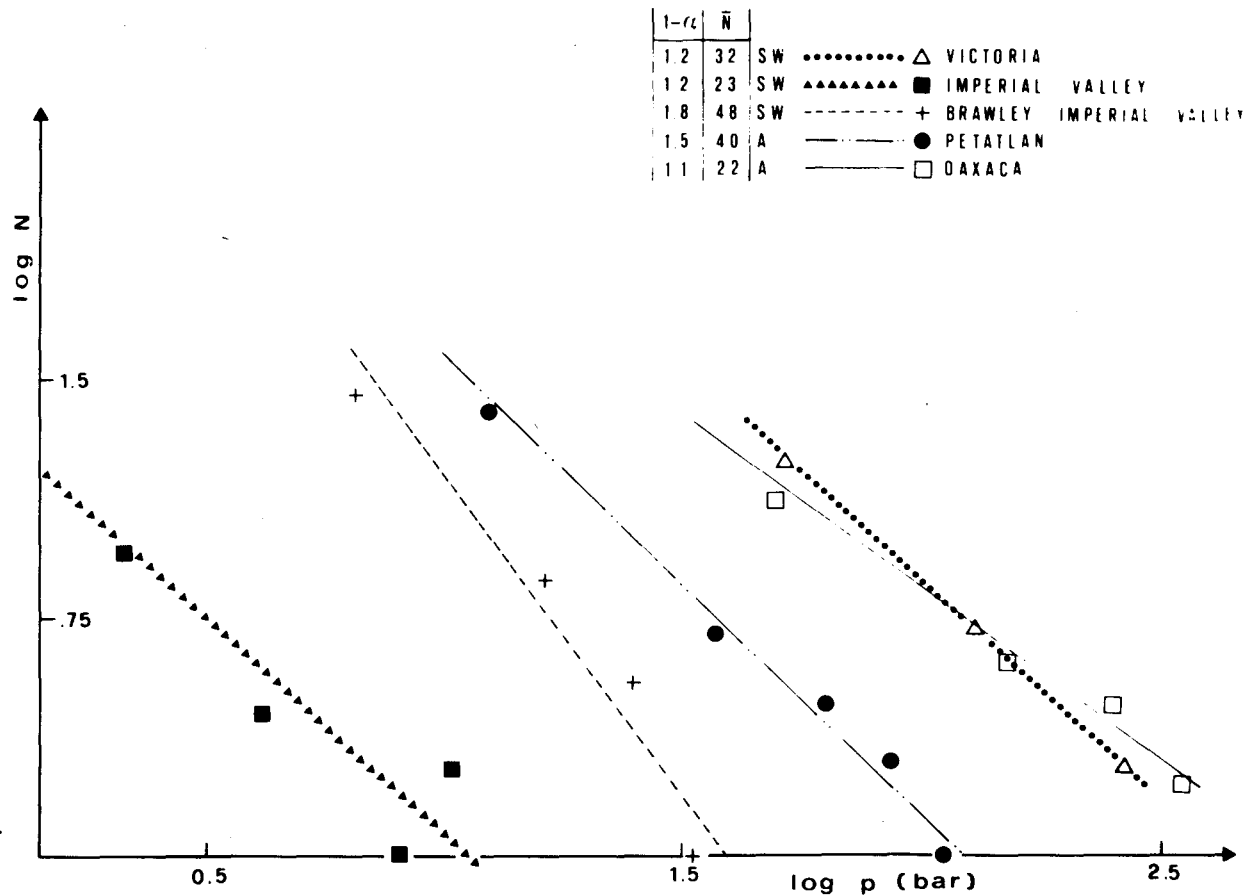


Fig. 1. Linear regression on frequency of the stress drops of aftershocks of the earthquakes in the Brawley-Imperial Valley 1975 swarm (Hartzell and Brune, 1977); the Imperial Valley 1977 swarm (Adair *et al.*, 1977); the 1978 Victoria (Mexico) swarm (Munguía *et al.*, 1978); the 1978 Oaxaca (Mexico) aftershock sequence (Munguía *et al.*, 1977-78); the 1979 Petatlan (Mexico) aftershocks (Zúñiga y Valdés, 1980). \bar{N} is the total number of events used in the analysis, SC means a set of scattered events, A means a set of aftershocks, SW means a set from a swarm. The regression lines have been computed with the method of maximum likelihood and $\sigma(\log N)/\sigma(\log p) = 1$.

PRECURSORS OF EARTHQUAKES

It has been suggested (Caputo, 1981b) that the value of $1 - \alpha$ may increase during the seismic cycle from aftershocks to foreshocks. This is now confirmed by the analysis of the stress drops of the foreshocks and aftershocks of the recent Mammoth earthquakes. The stress drops computed by Peppin (1981) for 316 events recorded at 4 stations have been analyzed (Caputo, 1981c) and the results of the analysis are shown in Figure 2, where one may see that the average value of $-1 + \alpha$ in the aftershock sequence is -1.2 and that of the foreshock sequence is -1.8 . This phenomenon had already been noted for Southern California (Caputo, 1981b).

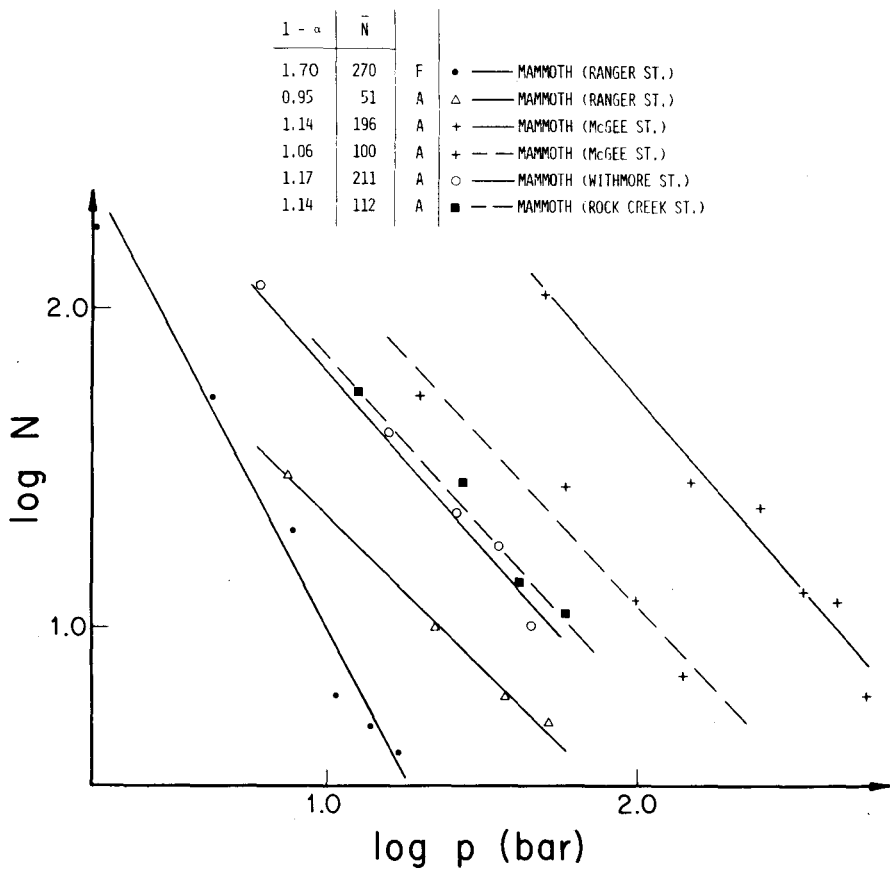


Fig. 2. Linear regression for the stress drops of the foreshocks (F) and aftershocks (A) of the Mammoth (California) May 1980 earthquake at different stations. The stress drops have been computed by Peppin (1981). The meaning of \bar{N} is as in Figure 1.

RETURN PERIOD OF THE ACCELERATIONS OF THE GROUND

In a recent paper (Caputo, 1981a), it has been shown that the return period of the r.m.s. accelerations of the ground caused by earthquakes may be computed from (2) and (3) and the known relation between a and ℓ , p (e.g., McGuire and Hanks, 1980) (ℓ is the linear dimension of the faults)

$$a = k p \ell^{1/2}$$

$$k = \frac{0.29}{\rho} Q^{1/2} R^{-3/2} \quad (5)$$

where Q is the specific dissipation, ρ the density and R the epicentral distance.

If p_1 , p_2 and ℓ_1, ℓ_2 are the minimum and maximum stress drops and linear dimensions of faults in the seismic region, the cumulative distribution of a is (Caputo 1981a)

$$n(a) = \frac{D}{1-\nu} \left\{ \frac{p_2^\alpha \ell_2^{1-\nu}}{\alpha} - \frac{2(\nu-1)\ell_2^{1-\nu-\frac{\alpha}{2}} a^\alpha}{\alpha(2\nu-2+\alpha)k^\alpha} - \frac{p_2^{2\nu-2+\alpha} a^{2-2\nu}}{(2\nu-2+\alpha)k^{2-2\nu}} \right\} \quad (6)$$

$$a > k p_1 \sqrt{\ell_2} \quad (\text{or } a > k p_2 \sqrt{\ell_1} \quad \text{whichever is larger})$$

$$n(a) = \frac{D p_2^\alpha}{\alpha} \frac{\ell_2^{1-\nu} - \ell_1^{1-\nu}}{1-\nu} - \frac{D}{\alpha} \frac{\ell_2^{1-\nu-\frac{\alpha}{2}} - \ell_1^{1-\nu-\frac{\alpha}{2}}}{1-\nu-\frac{\alpha}{2}} \left(\frac{a}{k} \right)^\alpha \quad (7)$$

$$k p_2 \sqrt{\ell_1} < a < k p_1 \sqrt{\ell_2}$$

In order to be able to use formulas (6) and (7), one must know γ , α and the constant D , appearing in them. For the southwest part of Mexico, one may assume $b \sim 1$ (Ponce, 1981) which gives $\gamma = 3$, with $\gamma = 1.5$ as generally assumed. The value of $1 - \alpha$ for the same region can be approximately assumed -1.5 from the data on the Petatlan earthquake, 1979, aftershocks which are more numerous than those of the Oaxaca earthquake. This value is also closer to other values estimated in Central California with much more data. The density distribution of the acceleration of the ground at a distance R from a seismic area in Southern Mexico can be estimated from the following formulas according to the method of Caputo (1981a)

$$n(a) = D \left\{ \frac{p_2^{-1.5} \ell_2^{-2}}{3} - \frac{\ell_2^{-1.25}}{1.875} \left(\frac{a}{k} \right)^{-1.5} + \frac{p_2^{2.5}}{5} \left(\frac{a}{k} \right)^{-4} \right\}, a > k p_1 \sqrt{\ell_2} \quad (8)$$

$$n(a) = D \left\{ \frac{p_2^{-1.5}}{3} (\ell_2^{-2} - \ell_1^{-2}) + \frac{\ell_1^{-1.25} - \ell_2^{-1.25}}{1.875} \left(\frac{a}{k}\right)^{-1.5} \right\}, a < kp_1 \sqrt{\ell_2} \quad (9)$$

The constant D, can be obtained from the catalogue of earthquakes of the area with the method used by Caputo (1981a).

Figure 3 is a diagram with the isolines of magnitude computed for California by Caputo (1981a) from the formulas

$$10^{\gamma M + \beta} = \eta \frac{k \ell^3 p^2}{2\mu} \quad (10)$$

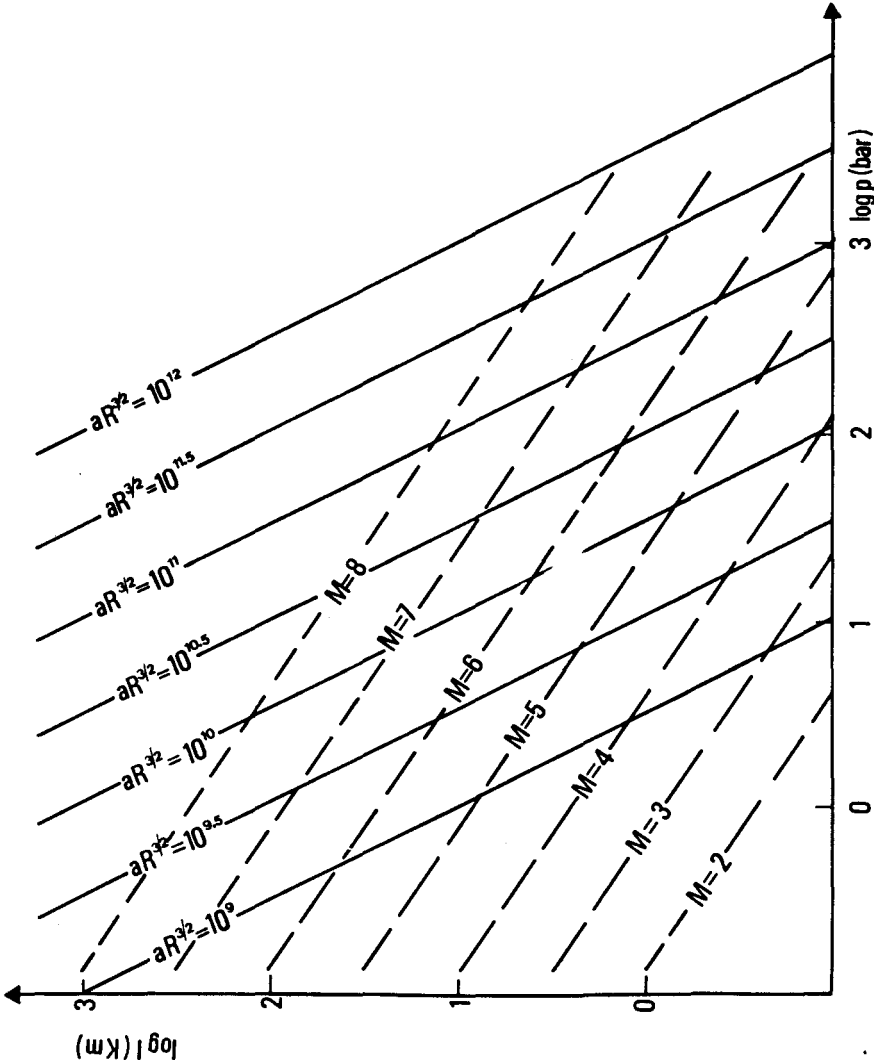


Fig. 3. Isolines of magnitude and acceleration of the ground in the plane of the linear dimensions of faults, ℓ , and stress drops, p . The lines $aR^{3/2} = \text{const}$. have been computed from equation (5). The lines $M = \text{const}$. have been computed from equation (10) with $\gamma = 1.46, \beta = 11.1$

and the isolines of the accelerations of the ground at the same distance for the same region are shown (Caputo, 1981a). Once the constants $\frac{10^{\beta}}{\eta}$ and γ of formulas (10) have been estimated for the seismic regions of Mexico, one may draw an exactly analogous diagram for these regions and find the various values of the accelerations of the ground associated with any value of the magnitude. One may anticipate that, as in California, the same magnitude, at the same distance, could cause accelerations of the ground which are different by an order of magnitude or more.

In general, the second and third terms of (8) are much smaller than the third, therefore we may write (8) in the very simple form

$$\begin{aligned} \log n(a) &= \log(D\rho^4 p_2^{2.5}/Q^2 3.536 \cdot 10^{-2}) - 4 \log a - 6 \log R \\ &= \text{const.} - 4 \log a - 6 \log R \end{aligned} \quad (11)$$

where the only unknown is the constant which could be determined empirically, from catalogs of earthquakes.

Formulas (11), (6), (7), (8), and (9) can be generalized to the case of extended seismic regions by integrating over the area of the region according to the method of Caputo (1981a).

Formulas (6) and (7) have been obtained by using the relation given by McGuire and Hanks in equation (5); however, the method given here to obtain $n(a)$ is valid for any formula relating a to ℓ and p .

APPENDIX

As a check, the value of $-1 + \alpha$ for the set of foreshocks of the Mammoth earthquake was computed also in the following manner.

Let us assume that the values of ℓ_1 , ℓ_2 , p_1 and p_2 are such that

$$\ell_2^3 p_1 < \ell_1^3 p_2 \quad (12)$$

In this case, the number of events with seismic moment larger than M_0 is obtained from the density distributions of the linear dimension of the faults, and of the stress drops, that is the number of faults with linear dimensions and stress drops in the ranges ℓ , $\ell + d\ell$ and p , $p + dp$:

$$\begin{aligned} \ell^{-\nu} d\ell \\ p^{-1+\alpha} dp \end{aligned} \quad (13)$$

In fact, following the method of Caputo (1976), we obtain for the cumulative distribution:

$$n_o(M_o) = D \int_{\varrho_1}^{\varrho_2} \varrho^{-\nu} d\varrho \int_{\frac{M_o c}{\varrho^3}}^p p^{-1+\alpha} dp = D \left\{ \frac{\varrho_2^{1-\nu} - \varrho_1^{1-\nu}}{\alpha(1-\nu)} P_2^\alpha - \frac{(M_o c)^\alpha}{\alpha} \frac{\varrho_2^{-\nu-3\alpha+1} - \varrho_1^{-\nu-3\alpha+1}}{-\nu+1-3\alpha} \right\} \quad (14)$$

and for the density distribution $\bar{n}_o(M_o)$:

$$\log \bar{n}_o(M) = \log \frac{D c^\alpha}{\gamma + 3\alpha - 1} \left\{ \varrho_1^{-\nu-3\alpha+1} - \varrho_2^{-\nu-3\alpha+1} \right\} + (-1 + \alpha) \log M_o \quad (15)$$

The value of $\alpha - 1$ was obtained by using formula (15) and with the statistical analysis of the values of M_o observed prior to the 1980 Mammoth earthquake; only data with corner frequency in the range $7H_z - 10$ Hz have been used in order to satisfy condition (12); a value $-1 + \alpha = 1.7$ was obtained, in good agreement with direct statistical analysis of p .

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