

The semi-cold approximation in magnetospheric and solar wind physics

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RESUMEN

Durante los últimos años se ha desarrollado un método para estudiar las ondas electromagnéticas ion-ciclotrón en plasmas multicomponentes. En este trabajo se revisa este método y se aplica a las pérdidas de protones en corrientes anulares y a la generación de ondas electromagnéticas ion-ciclotrón a la altura geostacionaria. Se aplica también este método a la aceleración preferencial de partículas alfa en condiciones de viento solar rápido. Se muestra que una pequeña velocidad positiva de arrastre entre partículas alfa y protones puede dar como resultado la aceleración de las partículas alfa hasta velocidades mucho mayores que la velocidad de conjunto de los protones. Durante el proceso de aceleración, el cual se supone que tiene lugar a distancias heliocéntricas menores que 0.3 UA, la velocidad de las partículas alfa podría exceder la velocidad de los protones y entonces desaparecería el gap que existe alrededor de la giro-frecuencia de las partículas alfa. También se muestra que para anisotropías térmicas de protones del orden de las observadas en el viento solar rápido, las ondas o crecen o no son excesivamente amortiguadas, de manera que éstas pueden existir y conducir así a las velocidades diferenciales observadas. La forma en que las partículas alfa exceden la velocidad de los protones no puede ser explicada por este mecanismo, pero se discuten algunas ideas concernientes al proceso inicial.

PALABRAS CLAVE: física de plasmas, magnetosfera, viento solar, ondas ion-ciclotrón.

ABSTRACT

During the past years, a method has been developed in order to study electromagnetic ion-cyclotron waves in multicomponent plasmas. The method is reviewed and applied to ring current proton losses, and to the generation of electromagnetic ion-cyclotron waves at the geostationary altitude. This method is also applied to the preferential acceleration of alpha particles in fast solar wind conditions. It is shown that a small positive drift velocity between alpha particles and protons can result in an alpha particle acceleration to velocities well in excess of the proton bulk velocity. During the acceleration process, which is assumed to take place at heliocentric distances less than 0.3 AU, the alpha particle velocity should exceed the proton velocity; then the gap which exists around the alpha particle gyrofrequency disappears. For proton thermal anisotropies of the order of those observed in fast solar wind, the waves either grow or are not damped excessively, so the waves can exist and might lead to the observed differential speeds. The way in which the alpha particles exceed the proton velocity is not accounted for by this mechanism, but some ideas concerning this initial process are discussed.

KEY WORDS: plasma physics, magnetosphere, solar wind, ion-cyclotron waves.

I. INTRODUCTION

The subject of this paper is the acceleration of alpha particles in fast solar wind flow.

The solar wind is composed of electrons and protons and a minority of heavier ions, mainly alpha particles. It is well known that quasilinear resonant interactions of alpha particles with left-hand polarized ion-cyclotron waves propagating parallel to the interplanetary magnetic field can preferentially accelerate alpha particles moving with proton speed (Dusenbery and Hollweg, 1981; Isenberg, 1983; Isenberg and Hollweg, 1983).

The preferential acceleration was shown to occur without taking into account the effect of the alpha particles in the dispersion relation of the electromagnetic ion-cyclotron waves. However, as pointed out by Isenberg (1984), there is no reason to neglect the dispersive effects of the alpha particles in the dispersion relation. When the dispersive properties of the alpha particles are included in the dispersion relation, the resonant acceleration of Dusenbery and Hollweg (1981) is no longer possible (Isenberg, 1984).

We show that the aforementioned acceleration can be recovered if a relative drift velocity between the protons and the alpha particles is included in the dispersion relation. Also, for large proton thermal anisotropies, $T_{\perp} / T_{\parallel} \approx 3 - 4$, the waves either grow or are not excessively damped in the frequency region of interest, so that they can efficiently accelerate alpha particles.

Our method has been developed in order to study ion-cyclotron waves in multicomponent plasmas. This method is called the semi-cold approximation for reasons which will become clear later on, and it has been extensively used in magnetospheric plasmas.

The initial drift velocity between alpha particles and protons is not explained by the aforementioned mechanism. By using a simple model of the source of the fast solar wind, it is shown that an initial proton thermal anisotropy can lead to the required alpha-proton drift velocity.

The paper is organized as follows. In Section II, the semi cold approximation for electromagnetic waves in multicomponent plasmas is derived. In Section III-1 the semi cold approximation is applied to ring current proton losses, as an illustration of the method. In Section III.2 it is applied to ion-cyclotron waves propagating at the geostationary altitude. In Section IV, the method is applied to the preferential acceleration of alpha particles in fast solar wind flow, and it is also shown that a proton thermal anisotropy can lead to a small relative drift velocity between alpha particles and protons. In Section V, the results are summarized and discussed.

II. THE SEMI-COLD APPROXIMATION

Consider a plasma composed by warm electrons and any number of ion species each having a mixed population of warm and cold components. The dispersion relation for electromagnetic ion-cyclotron waves (EICW) propagating parallel to an external magnetic field in a homogeneous plasma is given by (see, e.g., Gomberoff and Cuperman, 1982):

$$c^2 k^2 = \omega^2 + \pi \omega \sum_l \omega_{pl}^2 \int dv_{\parallel} \int dv_{\perp} v_{\perp}^2 \frac{(1 - kv_{\parallel}/\omega) \frac{\partial f_{0l}}{\partial v_{\perp}} + (kv_{\perp}/\omega) \frac{\partial f_{0l}}{\partial v_{\parallel}}}{\omega - kv_{\parallel} \pm \Omega_l} \quad (1)$$

where k is the wave vector parallel to the external magnetic field, ω_{pl} is the plasma frequency of the l component, f_{0l} is the distribution function of the l component, v_{\parallel} (v_{\perp}) is the particle velocity parallel (perpendicular) to the external field, and Ω_l is the gyrofrequency of the l component.

Assuming that the warm ion components are described by biMaxwellian distribution functions

$$f_{0l} = \frac{1}{\pi^{3/2} \alpha_{\perp}^2 \alpha_{\parallel}} \exp\left[-\left(\frac{v_{\perp}}{\alpha_{\perp}}\right)^2 - \left(\frac{v_{\parallel}}{\alpha_{\parallel}}\right)^2\right], \quad (2)$$

and the electrons by a single Maxwellian

$$f_{0e} = \frac{1}{\pi^{3/2} \alpha^3} \exp\left[-\left(\frac{v}{\alpha}\right)^2\right], \quad (3)$$

the dispersion relation, eq.(1), becomes

$$y^2 = \sum_l \left\{ \frac{z_l}{M_l} \eta_{lw} A_l - z_l \eta_{lw} x - \frac{z_l \eta_{lw}}{M_l^2 y \beta_{\parallel l}^{1/2}} Z\left(\frac{M_l x - 1}{M_l y \beta_{\parallel l}^{1/2}}\right) \right. \\ \left. [A_l (1 - M_l x) - M_l x] \right\} + \sum_{lc} \frac{z_l \eta_{lc} M_l x^2}{1 - M_l x}, \quad (4)$$

where Z is the plasma dispersion function (Fried and Conte, 1961), $y = kV_{Ap}/\Omega_p$, $V_{Ap} = B_0/(4\pi m_p n_e)^{1/2}$ is the Alfvén velocity, $x = \omega/\Omega_p$, $A_l = (T_{\perp}/T_{\parallel} - 1)_l$, $\beta_{\parallel l} = 8\pi m_p n_{wl} kT_{\parallel l}/m_l B_0^2$, $\eta_{lw} = \eta_{lw}/\eta_{pw}$, $\eta_{lc} = \eta_{lc}/\eta_{pw}$, z_l is the ionic charge, and $M_l = m_l/z_l m_p$.

The first sum in eq.(4) is over all warm components, and the last sum is over the cold components.

Assuming $\omega = \omega_r + i\omega_i$ and $\beta_{\parallel l} < 1$, eq.(4) yields:

$$y^2 = \sum_l x^2 z_l M_l (\eta_{lw} + \eta_{lc}) / (1 - M_l x), \quad (5)$$

and

$$\gamma = \frac{\omega_i}{\Omega_p} = \sum_l \left(\frac{z_l^2 \pi}{M_l^4 \beta_{||l}} \right)^{1/2} \eta_{lw} \frac{[A_l(1 - M_l x) - M_l x]}{y F(x)} \exp\left[-\frac{(1 - M_l x)^2}{M_l^2 \beta_{||l} y^2}\right], \quad (6)$$

where

$$F(x) = \sum_l x(2 - M_l x)(\eta_{lw} + \eta_{lc}) / (1 - M_l x)^2. \quad (7)$$

Eq.(5) is the dispersion relation and eq.(6) is the growth or damping rate. Notice that the dispersion relation is the cold plasma dispersion relation and that thermal effects appear only in the expression for the growth rate. This is the reason why the approximation is called the semi-cold approximation (Cuperman *et al.*, 1975; Gomberoff and Cuperman, 1982).

III.1 RING CURRENT PROTON PRECIPITATION

As a first example of the semi cold approximation, consider a plasma consisting of a mixture of warm and cold protons.

In this case, eqs.(5) and (6) reduce to (Cornwall *et al.*, 1970; Cornwall and Shultz, 1971; Cornwall, 1972; Cuperman *et al.*, 1975):

$$y^2 = \frac{x^2(1 + \delta)}{1 - x}, \quad (8)$$

and

$$\gamma = \left(\frac{\pi}{\beta_{||p}} \right)^{1/2} \frac{[A_p(1 - x) - x]}{x^2(1 + \delta)^{3/2}/(1 - x)^{5/2}} \exp\left[-\frac{(1 - x)^2}{\beta_{||p} y^2}\right]. \quad (9)$$

From the last equation it follows that $\gamma > 0$ if $A_p > 0$. Moreover, γ is positive in the region $0 < x < x_m$ where x_m is the marginal mode and it is given by $x_m = A_p/(A_p + 1)$.

In Figure 1, the behaviour of the growth rate with varying concentration of cold protons is illustrated. We can see that the growth rate increases with increasing concentration of cold protons, until the cold proton concentration reaches a given value, decreasing thereafter. The maximum growth rate in the presence of a cold proton population can become one order of magnitude larger than the maximum growth rate in the absence of cold protons (Gomberoff and Rogan, 1987). In Figure 1 the dashed line corresponds to the results obtained by using the semi-cold approximation, eqs.(5) and (6), and the full line corresponds to numerical results obtained from the exact dispersion relation, eq. (4). We can see that the agreement is very good (Cuperman *et al.*, 1975).

In 1970, Cornwall, Coroniti and Thorne proposed a mechanism of ring current proton losses based on a possible interaction between the hot ring current protons and the rather cold magnetospheric plasma. According to these authors, the cold magnetospheric protons can trigger unstable EICW which in turn can interact with ring current protons leading to proton precipitation. The results illustrated in Figure 1 strongly support their proposal. Notice from the upper part of Figure 1 that the cold plasma dispersion relation extends from $\omega = 0$ up to $\omega = \Omega_p$, where there is a resonance.

III.2 ION-CYCLOTRON WAVES AT THE GEOSTATIONARY ALTITUDE

As a second example, let us consider a plasma composed by warm and cold protons and a minority heavier ion component consisting of cold He⁺ ions. In this case eqs.(5) and (6) reduce to (Gomberoff and Cuperman, 1982; Gomberoff and Neira, 1983; Gomberoff and Molina, 1985)

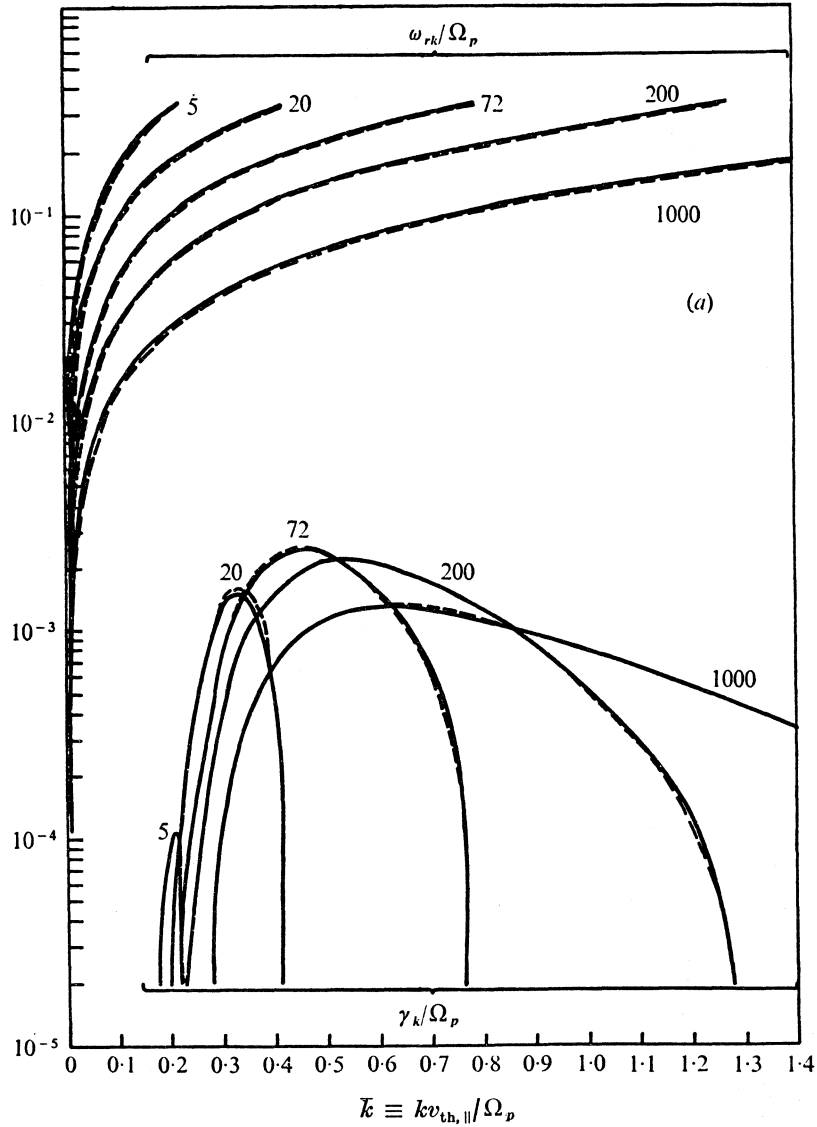


Fig. 1. Growth rates and real frequencies vs normalized wavenumbers $\bar{k} = kv_{th,||}/\Omega_p$, for several values of the ratio of cold to warm proton density.

$$y^2 = \frac{x^2(1 + \delta)}{1 - x} + \frac{4\eta_{He^+}x^2}{1 - 4x}, \quad (10)$$

and

$$\gamma = \left(\frac{\pi}{\beta_{||p}}\right)^{1/2} \frac{1}{xy(1 + \delta)(2 - x)/(1 - x)^2 + 8\eta(1 - 2x)/(1 - 4x)^2} \cdot \exp\left[-\frac{(1 - x)^2}{\beta_{||p}y^2}\right]. \quad (11)$$

The dispersion relation, eq.(10), is illustrated in Figure 2. In the presence of heavier ions the dispersion relation contains a stop band starting at the heavier ion gyrofrequency, Ω_{He^+} , and ending at the normalized frequency cutoff $x_c = (1 + \delta + 4\eta) / 4(1 + \delta + \eta)$. The stop band divides the wave propagation spectrum into two regions: (1) a low frequency region, extending from $\omega = 0$ up to the resonance at $\omega = \Omega_{He^+}$, and (2) a high frequency region, extending from a cutoff frequency at $\omega = \omega_m$ up to a resonance at the proton gyrofrequency.

On the other hand, the growth rate given by eq.(11), is shown in Figure 3 as a function of the normalized frequency $x = \omega / \Omega_p$. The left side of the figure shows the growth rate in the absence of heavier ions, $\eta_{He^+} = 0$, and the right hand side the case when there is a small amount of heavier ions, $\eta_{He^+} = 0.07$.

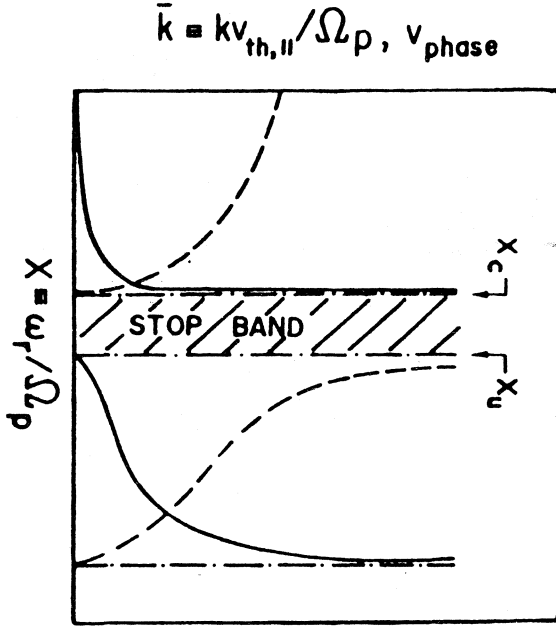


Fig. 2. Dispersion curves for electromagnetic proton-cyclotron waves in hydrogen-helium plasma.

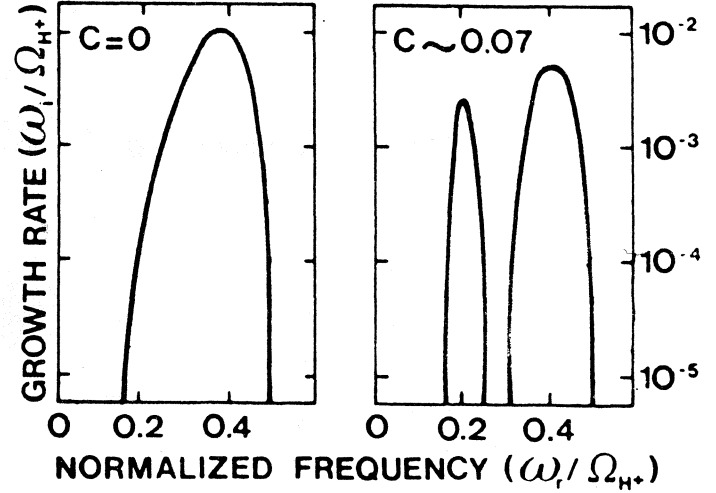


Fig. 3. Growth rate vs frequency for a proton distribution with $A_p = 1$ and $\beta_{||p} = 1$. Left: without He^+ . Right: with 7% of He^+ .

We see that the stop band also divides the unstable spectrum into a low frequency region and a high frequency region. The low frequency region extends from $x = 0$ up to the He^+ gyrofrequency, and the high frequency region from the cutoff frequency, x_c up to a marginal frequency given by $x_m = A_p / (A_p + 1)$, (see eq.11). Notice that when $x_m < x_c$, there is only a low frequency region (Gomberoff and Cuperman, 1982, and references therein).

Figure 4 shows a typical frequency time spectrogram of ultra low frequency waves (ULFW), as measured on board of GEOS 2 (Young *et al.*, 1979). The left hand ion-cyclotron wave spectrum organizes around the He^+ gyrofrequency, in good agreement with the previous results. The corresponding power spectrum is shown in Figure 5, where one can clearly see the two maxima around the He^+ gyrofrequency (Young *et al.*, 1979; Gendrin and Roux, 1980; Roux *et al.*, 1982; Gendrin, 1983a and 1983b).

The observations on board the GEOS satellites suggest that once the ion cyclotron waves are triggered, the He^+ ions are heated anisotropically (Young *et al.*, 1979; Roux *et al.*, 1982). Therefore, it becomes necessary to include in the model He^+ thermal and thermal anisotropy effects.

Including these effects in the expression for the growth rate yields (Gomberoff and Vega, 1987):

$$\gamma = \left(\frac{\pi}{\beta_{||p}}\right)^{1/2} \frac{[A_p(1-x) - x]}{yx F(x)} \exp\left[-\frac{(1-x)^2}{\beta_{||p} y^2}\right] + \left(\frac{\pi}{16^2 \beta_{||\text{He}^+}}\right)^{1/2} \eta_{\text{He}^+} \frac{[A_{\text{He}^+}(1-4x) - 4x]}{yx F(x)} \exp\left[-\frac{(1-4x)^2}{4\beta_{||\text{He}^+} y^2}\right], \quad (12)$$

where

$$F(x) = \frac{(2-x)(1+\delta)}{(1-x)^2} + \frac{4\eta(2-4x)}{(1-4x)^2}.$$

Equation (12) shows that if the He^+ thermal anisotropy is zero, the second term on the right hand side of the equation is always negative. Therefore, the He^+ thermal effects are stabilizing. On the other hand, if A_p is positive, the second term is positive between $x = 0$ and $x_{m\text{He}^+} = A_{\text{He}^+} / (A_{\text{He}^+} + 1)$. Since $x_{m\text{He}^+} \leq 1/4$, it follows that such a term can be positive only in the low frequency region. Thus, when the He^+ ions are anisotropic, they destabilize further the low frequency region and stabilize the high frequency region. The interpretation of this result is simple. When the He^+ ions become anisotropic, they can trigger their own ion-cyclotron waves. The mode $x_{m\text{He}^+}$ is the marginal mode of the He^+ ion-cyclotron instability

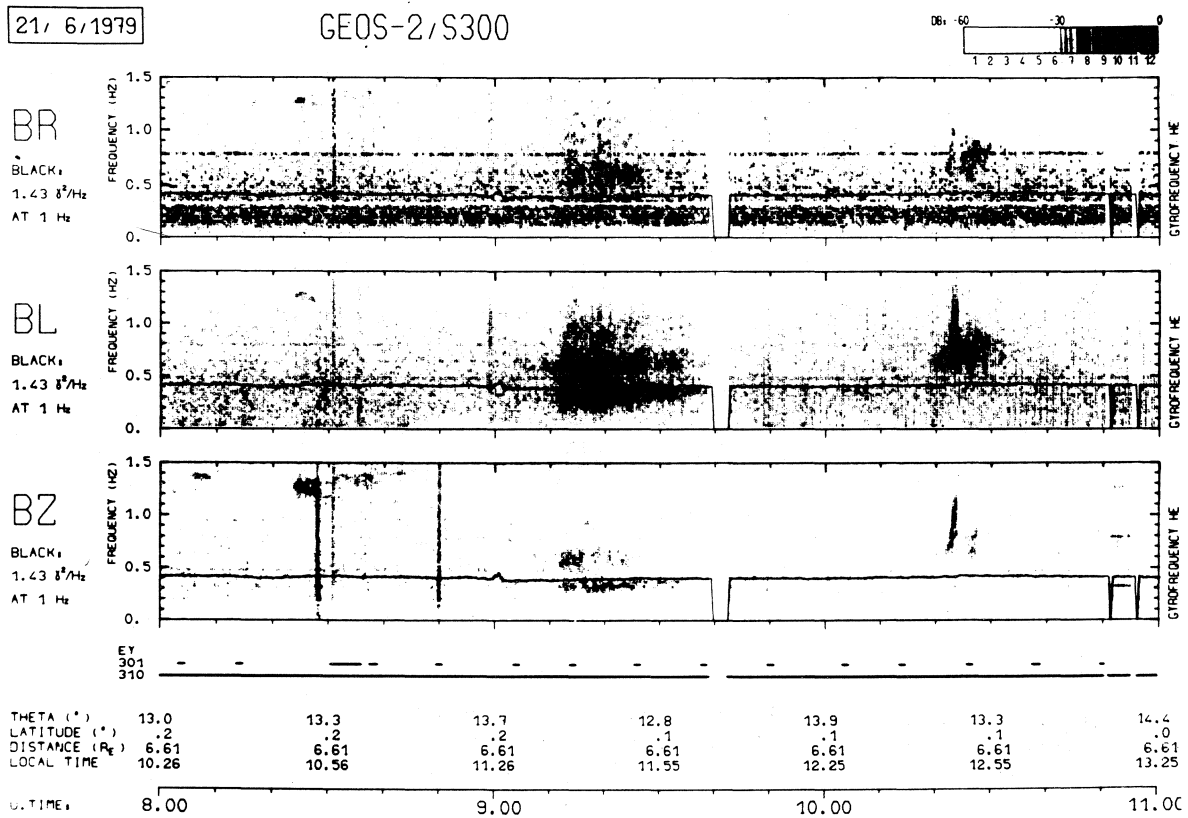


Fig. 4. Frequency time spectrogram of ULFW observed on June 21, 1979, on GEOS 2.

which is always less than the corresponding ion-cyclotron frequency and therefore it can only destabilize waves below this frequency.

These results are illustrated in Figure 6, where the growth rate, normalized to the proton gyrofrequency, is plotted against the normalized frequency for several values of the He⁺ thermal anisotropy. For He⁺ thermal anisotropies larger than zero, the maximum growth rate increases in the low frequency branch. In the high frequency branch, the maximum growth rate always decreases due to He⁺ thermal and anisotropy effects.

From the semi-cold approximation, it is possible to show that the effect of thermal He⁺ ions on the ion-cyclotron instability is negligible for He⁺ ion temperatures up to 1keV, with a similar behaviour for larger temperatures, but the approximation is no longer reliable.

These results are in very good agreement with exact numerical calculations (see Berchem and Gendrin, 1985; Gendrin *et al.*, 1984).

IV. ACCELERATION OF ALPHA PARTICLES IN THE SOLAR WIND

Ever since the discovery that alpha particles flow faster than the bulk protons in fast solar wind conditions (Asbridge *et al.*, 1976; Bosqued *et al.*, 1977; Neugebauer and Feldman, 1979), much effort has been devoted to explain the observations.

Coulomb collisions can at most account for the equalization of the bulk velocities (Geiss *et al.*, 1970), but the fact that their relative velocity can become of the order of and sometimes larger than the Alfvén velocity (Marsch *et al.*, 1982a) has remained unexplained.

On the other hand, it has been shown that quasilinear resonant interactions of alpha particles with parallel propagating left handed ion-cyclotron waves can preferentially accelerate alpha particles flowing with proton speed (Dusenbery and Hollweg, 1981; Marsch *et al.*, 1982b; Isenberg, 1983; Isenberg and Hollweg, 1983). Therefore, this wave particle interaction can at least be a first step mechanism leading to the observed preferential velocities.

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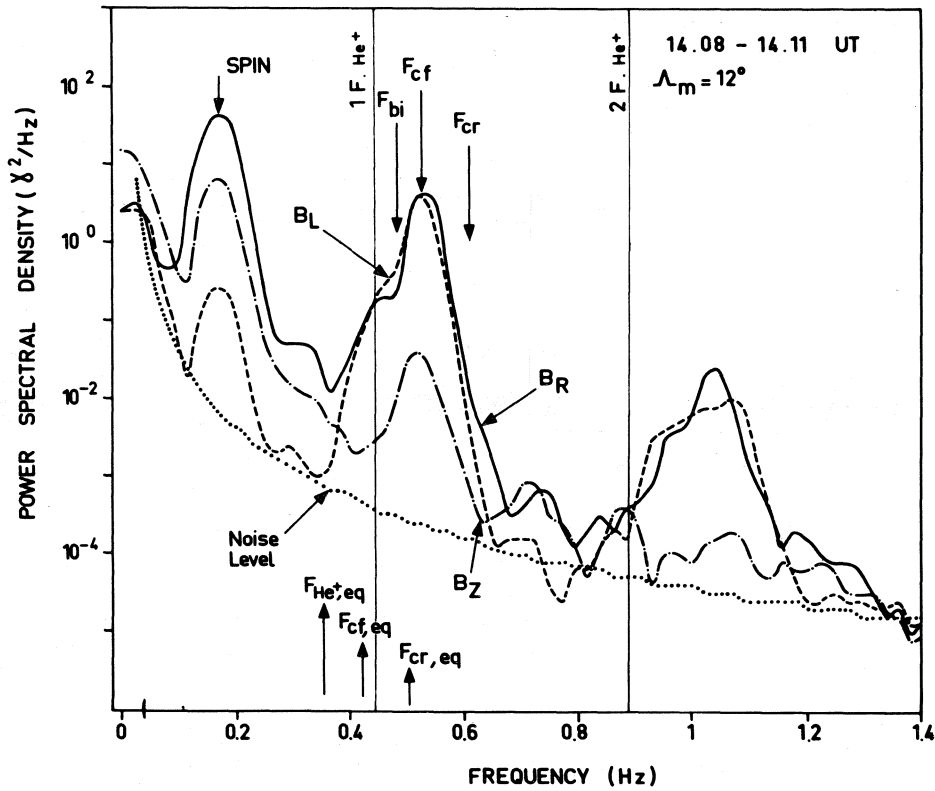


Fig. 5. Power spectrogram corresponding to Fig. 4.

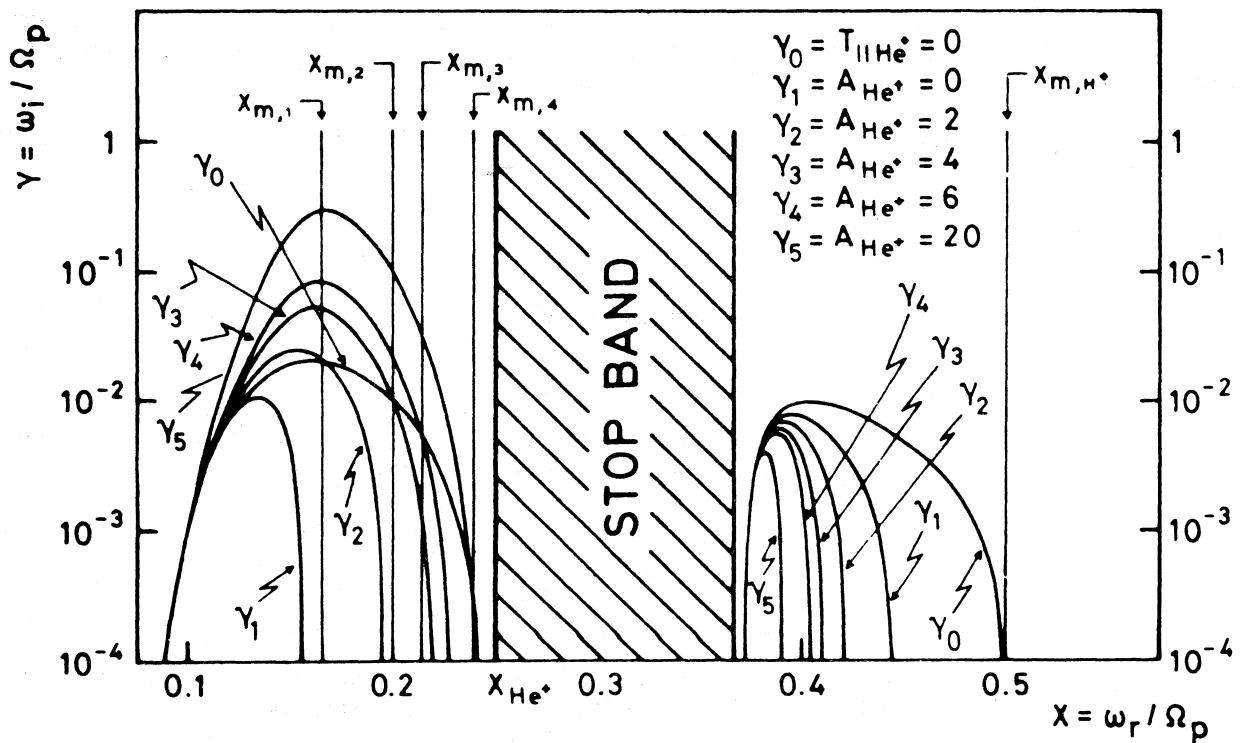


Fig. 6. Normalized growth rate vs. normalized frequency for $\beta_{\parallel p} = 0.5$, $B_0 = 130$ nT, $\beta_{\parallel He^+} = 0.25$, $A_p = 1$, $\eta_{pc} = 10$, $\eta_{He^+c} = 2$, $n_{wp} = 1 \text{ cm}^{-3}$ and several values of A_{He^+} .

In the aforementioned studies, alpha particles were either neglected or treated as test particles in the ion-cyclotron dispersion relation. More recently, Isenberg (1984) included alpha particle dispersion effects in the dispersion relation. By considering the cold plasma dispersion relation for parallel propagating ion-cyclotron waves, Isenberg (1984) showed that the stop band generated by heavy ions (see Gomberoff and Cuperman, 1982, and references therein) produces a gap in the resonant alpha particle velocities at the proton bulk speed. This gap implies that alpha particles can at most be accelerated up to the proton velocity by the resonant ion-cyclotron waves. Isenberg (1984) then showed that thermal effects can decrease the width of the stop band which appears in the cold plasma dispersion relation, and, for some critical values, the stop band can disappear altogether, thus recovering the preferential acceleration of Dusenbery and Hollweg (1981), who treated the heavy ions as test particles. However, as pointed out by Isenberg (1984) himself, the aforementioned effects are present under certain limited conditions which are likely to make this mechanism even less feasible to produce the observed differential velocities.

Isenberg (1974) assumed a plasma model consisting of protons and alpha particles with zero relative drift velocity. However, he pointed out that a complete study of the resonant interaction between the ion-cyclotron waves and the solar wind ions should include a relative drift between the ion species. The relative drift could substantially modify the dispersion relation.

We show now that a relative drift between the ion components adds a new branch to the dispersion relation which allows for wave propagation everywhere in the interval $0 < \omega < \Omega_p$. As a result, alpha particles can be accelerated to velocities well in excess of the proton bulk velocity.

Also, for large proton thermal anisotropies, $T_{\perp}/T_{\parallel} \approx 3-4$, the waves are strongly unstable around the He^{++} gyrofrequency and weakly damped around the proton gyrofrequency. Thus, the mechanism can efficiently accelerate alpha particles to large velocities for large thermal anisotropies.

Moreover, large thermal anisotropies of the order of 2 to 3 seems to be the prevailing condition in fast solar wind, in contrast to moderate and slow solar wind flow for which $T_{\perp}/T_{\parallel} < 1$, (Marsch *et al.*, 1982b).

IV.1 THE DISPERSION RELATION

We assume that the solar wind can be modeled by protons described by a biMaxwellian distribution function with thermal anisotropy A_p , and a minority He^{++} ion component (~4%) described by a drifting Maxwellian along the interplanetary magnetic field. The reference frame is selected as the proton rest frame.

Under these conditions, the dispersion relation for left-hand polarized ion-cyclotron waves propagating in the direction of the magnetic field is (see eq.4):

$$y^2 = A_p - x - \frac{1}{y\beta_{\parallel p}^{1/2}} Z\left(\frac{x-1}{y\beta_{\parallel p}^{1/2}}\right) [A_p(1-x) - x] - 2\eta x + \frac{\eta(x-yU)}{y\beta_{\parallel \alpha}^{1/2}} Z\left(\frac{2x-2yU-1}{2y\beta_{\parallel \alpha}^{1/2}}\right), \quad (13)$$

where $y = k_{\parallel} V_{Ap}/\Omega_p$, $x = \omega/\Omega_p$, $\beta_{\parallel i} = 2kT_{\parallel i}/m_i V_{Ap}$, Z is the plasma dispersion function (Fried and Conte, 1961), $\eta = n_{\alpha}/n_p$ is the fractional alpha particle density, and U the drift velocity normalized to the Alfvén velocity V_{Ap} .

For β_i sufficiently small, we can use the asymptotic expansion of the Z function to obtain:

$$y^2 = \frac{x^2}{1-x} + \frac{4\eta(x-yU)^2}{1-2x+2yU}. \quad (14)$$

This is the cold plasma approximation. However, eq.(14) is also a good approximation to the warm plasma dispersion relation provided that $\beta_p \leq 1$ (see Gomberoff and Vega, 1989; Oscarsson and André, 1986; Ronnemark, 1983)

The dispersion relation for $U = 0$ is shown in Figure 7a, displaying the stop band starting at the He^{++} gyrofrequency and ending at the cutoff frequency $\omega_c/\Omega_p = (1/2)(1+4\eta)/(1+2\eta)$.

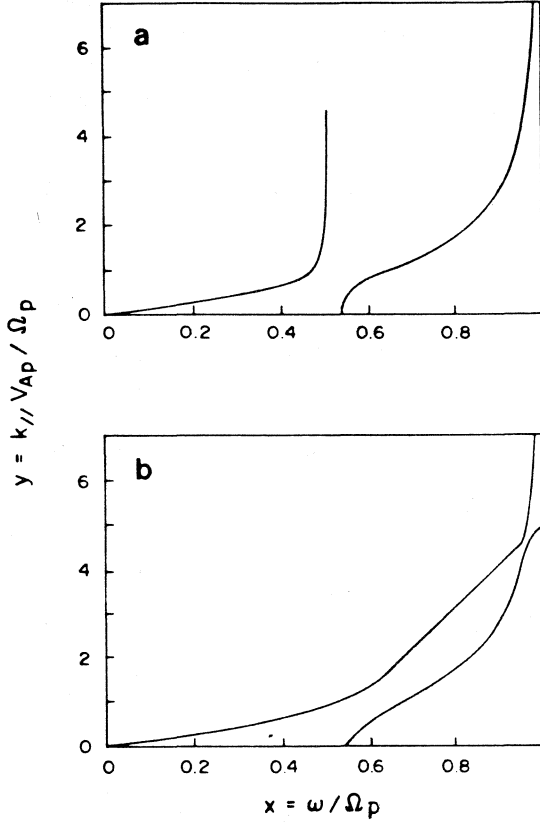


Fig. 7. Dispersion relation for ion-cyclotron waves with a minority alpha particle component, $\eta = 0.04$. Normalized wavenumber $y = k_{\parallel} V_{Ap} / \Omega_p$ vs. normalized frequency $x = \omega / \Omega_p$ for: (a) $U = 0$, and (b) $U = 0.1$.

Figure 7b shows eq.(14) for $U = 0.1$. We can see that there is no stop band at the alpha particle gyrofrequency. Instead, there is a continuous set of waves with frequencies in the range $0 < \omega < \Omega_p$. There is also another branch starting at the cutoff frequency ω_c .

We now determine the resonant velocities of an alpha particle moving in the wave field given by eq. (14). Following Dusenbery and Hollweg (1981), an ion moving with velocity V_{α}^r along the magnetic field will be in resonance with the waves when:

$$\omega - k_{\parallel} V_{\alpha}^r = \Omega_{\alpha} \quad (15)$$

Solving for V_{α}^r gives:

$$V_{\alpha}^r = (\omega / k_{\parallel})(1 - \Omega_{\alpha} / \omega), \quad (16)$$

which in terms of the variables x and y becomes

$$U_{\alpha}^r = (x/y)(1 - \Omega_{\alpha} / \Omega_p x), \quad (17)$$

where $U_{\alpha}^r = V_{\alpha}^r / V_{Ap}$.

Thus, an ion moving in the wave field of eq.(14) will resonate with the waves when its velocity is given by eq.(17).

Figure 8a shows the resonant velocity of an alpha particle when $U = 0$, and Figure 8b when $U = 0.1$. In the former case, there is a gap starting at $U_{\alpha}^r = 0$, implying that there are no waves which can resonate with alpha particles comoving with the protons. In contrast, from Figure 8b it follows that when $U = 0$ there is a continuous transition from $U_{\alpha}^r < 0$ to $U_{\alpha}^r > 0$.

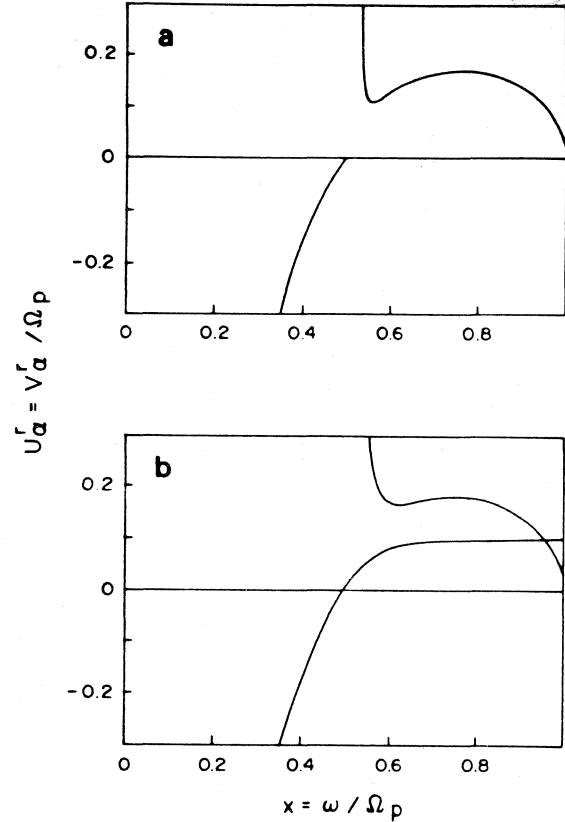


Fig. 8. Normalized alpha particle resonant velocity $U_{\alpha}^r = V_{\alpha}^r / \Omega_p$ vs. normalized frequency $x = \omega / \Omega_p$ for: (a) $U = 0$, and (b) $U = 0.1$.

Particles which resonate with the upper branch of the dispersion relation (see Fig. 7a) can be accelerated up to velocities equal to U . The other branch starting at $\omega = \omega_c$ (see Fig. 7a) gives rise to the upper curve in Figure 2b. The two curves almost touch each other at $\omega = 0.95 \Omega_p$. The gap between the curves is very small, $\Delta U_\alpha^T \approx 10^{-2}$, and becomes smaller as U decreases. If alpha particles can bridge this small gap, then a particle having $U_\alpha^T < 0$ initially can be continuously accelerated to large positive speeds.

Studies of low frequency electromagnetic waves generated by drifting newborn cometary ions show that thermal effects can actually close the gap mentioned above (Brinca and Tsurutani, 1987).

IV 2. STABILITY ANALYSIS

In order that the acceleration mechanism be efficient, damping rates should either be small or the waves should grow in the frequency region $\Omega_\alpha < \omega < \Omega_p$.

To address this question, let us take in eq.(14) $\omega = \omega_r + i\omega_i$ and assume $\omega_r \gg |\omega_i|$. This assumption will be justified *a posteriori*.

Then, a Taylor expansion of the Z functions to first order in ω_i yields:

$$\gamma = \left(\frac{\pi}{\beta_{\parallel p}}\right)^{1/2} \frac{[A_p(1-x) - x]}{y \left[\frac{x(2-x)}{(1-x)^2} + \frac{8\eta(x-yU)(1-x+yU)}{(1-2x+2yU)^2} \right]} \exp\left[-\frac{(1-x)^2}{y^2 \beta_{\parallel p}}\right], \quad (18)$$

where $\gamma = \omega_i / \Omega_p$ and $\beta_{\parallel p} = 8\pi KT_{\parallel p} / |B_0|^2$.

Notice that the procedure leads to a complex algebraic equation which when resolved into real and imaginary parts, yields eqs.(14) and (18).

From the last equation it follows that the normalized growth rate γ is positive in the region $0 < x < A_p / (A_p + 1)$.

In Figure 9 the growth rate γ versus the normalized frequency x is shown for several values of the thermal anisotropy

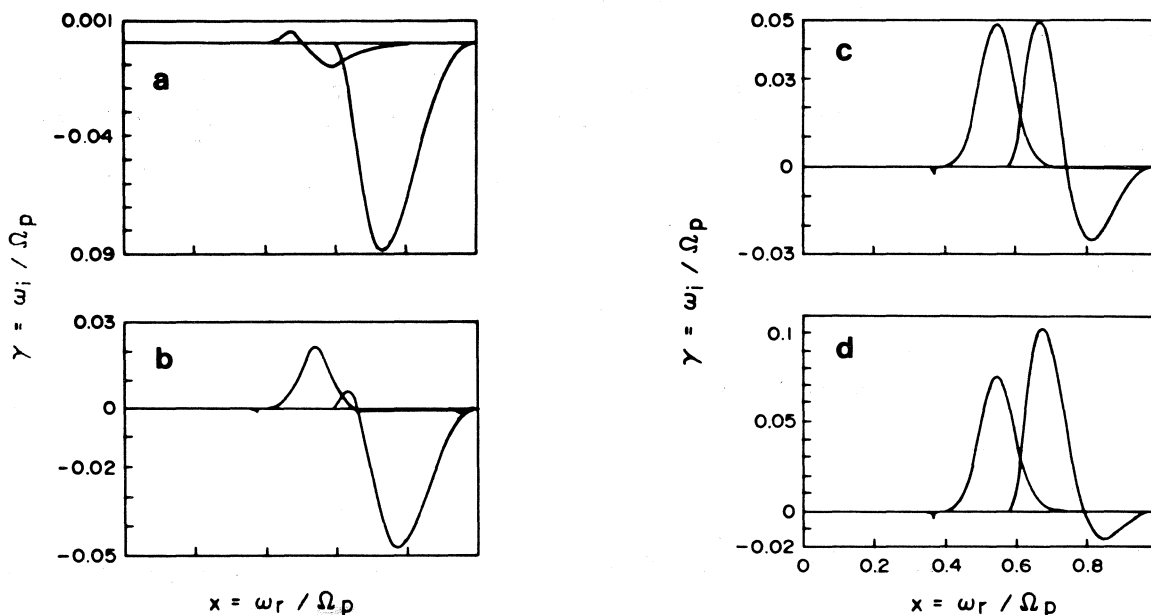


Fig. 9. Normalized growth rate $\gamma = \omega_i / \Omega_p$ vs. normalized frequency $x = \omega_r / \Omega_p$ in a plasma with $\beta_{\parallel p} = 0.1$, $\beta_{\perp p} = 0$, and $\eta_\alpha = 0.04$ for: (a) $A_p = 1$, (b) $A_p = 2$, (c) $A_p = 3$, and (d) $A_p = 4$.

A_p . In Figures (9b) - (9d), the first peak corresponds to the upper branch of the dispersion relation and the second peak to the branch beginning at $\omega = \omega_c$ (see Fig. 7b).

Since for $A_p = 1$ the marginal mode, $x_m = A_p/(A_p + 1)$, occurs at $x_m = 1/2$ the branch of the dispersion relation which starts at $\omega = \omega_c$ is completely damped, and there is only one peak which corresponds to the upper branch of the dispersion relation (see Fig. 9a).

As A_p increases, x_m becomes larger than x_c and the lower branch of the dispersion relation leads to unstable waves in the region $x_c < x < x_m$.

From Figures 9b-9d it follows that the effect of increasing thermal anisotropies is the following: (1) the instability region is enhanced, (2) maximum growth rates increase, and (3) damping rates beyond x_m decrease.

We conclude that the resonant mechanism of the previous section can effectively accelerate alpha particles if the proton thermal anisotropy is sufficiently large - of the order of $A_p \approx 2 - 3$.

Large thermal anisotropy in the core of the proton distribution function is persistently observed in fast solar wind everywhere between 0.3 and 1 AU, being more pronounced for distances closer to the sun. Typical values range from $A_p = 1$ to 3 in seemingly good agreement with the present conclusions (see Marsch *et al.*, 1982b and references therein).

From Figures 9b-9c it follows that maximum growth and damping rates are at least one order of magnitude smaller than the corresponding frequencies, in agreement with the assumption that $\omega_r \gg \omega_i$.

IV. 3 THE INITIAL ALPHA-PROTON DRIFT VELOCITY

We show now that a proton thermal anisotropy can lead to an alpha-proton drift velocity slightly larger than zero.

Let us write the equation of motion for particles of type l in the following form:

$$\frac{D^{(l)}\vec{U}_l}{Dt} + \frac{1}{m_l n_l} \vec{\nabla} \cdot \vec{P}_l - \frac{e_l}{m_l} \{ \vec{E} + \frac{1}{c} \vec{U}_l \times \vec{B} \} = \left\{ \frac{\partial \vec{U}_l}{\partial t} \right\}_{c,w}, \quad (19)$$

where D/Dt is the convective derivative, \vec{P} is the pressure tensor, \vec{U} is the fluid velocity, $\vec{E} = -T_e \vec{\nabla} p_e / c$, \vec{B} is the external magnetic field, and the last term contains all other interactions, including the interactions between the particles and wave-particle interactions.

Assuming that there are protons and alpha particles, and introducing the following magnitudes:

$$\vec{U}_d = \vec{U}_\alpha - \vec{U}_p, \quad (20)$$

$$\vec{V}_{sw} = \hat{\rho}_\alpha \vec{U}_\alpha + \hat{\rho}_p \vec{U}_p, \quad (21)$$

where

$$\rho_{\alpha,p} = m_{\alpha,p} n_{\alpha,p}, \quad (22)$$

and

$$\hat{\rho}_{\alpha,p} = \frac{\rho_{\alpha,p}}{\rho_\alpha + \rho_p}, \quad (23)$$

a combination of eqs.(19), for protons and alpha particles, yields:

$$\frac{D\vec{U}_d}{Dt} = \left\{ \frac{\partial \vec{U}_d}{\partial t} \right\}_{c,w} + \left\{ \frac{\partial \vec{U}_d}{\partial t} \right\}_g, \quad (24)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V}_{sw} \cdot \vec{\nabla}. \quad (25)$$

Assuming that, when \vec{U}_d is zero or very small, the collision and the wave particle interaction term can be neglected and also that the distances involved are sufficiently short so that $\nabla U_d = \nabla \rho_{\alpha, p} = \nabla \ln P_e = 0$, the last term in eq.(24) can be written as follows:

$$\left\{ \frac{\partial \vec{U}_d}{\partial t} \right\}_g = - \left(\frac{1}{m_{\alpha} n_{\alpha}} \vec{\nabla} \cdot \bar{P}_{\alpha} - \frac{1}{m_p n_p} \vec{\nabla} \cdot \bar{P}_p \right). \quad (26)$$

Writing the pressure tensor in the following way:

$$\bar{P} = p_{\parallel} \hat{b}\hat{b} + p_{\perp} \{ \bar{1} - \hat{b}\hat{b} \}, \quad (27)$$

where $p_{\parallel, \perp}$ is the pressure parallel and perpendicular to the external magnetic field whose direction is \hat{b} , and introducing $p = 1/3(p_{\parallel} + 2 p_{\perp}) = nT$, where T is the kinetic temperature of the corresponding ion-species, eq.(24) reduces to

$$\frac{D\vec{U}_d}{Dt} = \left\{ \frac{T_{\alpha}}{m_{\alpha}} \frac{3A_{\alpha}}{2A_{\alpha} + 3} - \frac{T_p}{m_p} \frac{3A_p}{2A_p + 3} \right\} \{ \hat{b} \cdot \vec{\nabla} \hat{b} - (\hat{b} \cdot \vec{\nabla} \ln B) \hat{b} \}. \quad (28)$$

The last equation has been obtained by assuming that the gradients of p_{\parallel} and p_{\perp} can be neglected, and that the magnetic field is weakly inhomogeneous.

Introducing the thermal velocity, $\alpha_p^2 = 2T_p/m_p$, and the definitions. $T = T_{\alpha}/T_p$, and $M = m_{\alpha}/m_p$, eq.(28) reduces to:

$$\frac{D\vec{U}_d}{Dr} = - \frac{3}{4} \frac{\alpha_p^2}{V_{Ap} V_{sw}} \left\{ \frac{A_p}{A_p + 3/2} - \frac{T}{M} \frac{A_{\alpha}}{A_{\alpha} + 3/2} \right\} \Gamma, \quad (29)$$

where $\vec{U}_d = U_d/V_{Ap}$, $r = V_{sw} t$, and Γ is the last term in eq.(28).

In order to evaluate Γ in the last equation, consider a very simple model of the magnetic field which agrees with the observed topology in the vicinity of the solar corona (Tyan Yeh, 1977; Tyan Yeh and G. W. Pnewman, 1977).

Assuming cylindrical symmetry and letting r be the axis of symmetry,

$$B_r = B_0 \left\{ \frac{r_0}{r} \right\}^{\gamma}, \quad (30)$$

where B_r is the r -component of the magnetic field, and B_0 , r_0 , and γ are constants or free parameters.

The magnetic field is then given by:

$$\vec{B}(r, \rho) = B_r \hat{r} + B_{\rho} \hat{\rho}, \quad (31)$$

where $\hat{\rho}$ is in the radial direction.

Assuming that $\partial B_r / \partial r$ is approximately constant, from $\vec{\nabla} \cdot \vec{B} = 0$, it follows that:

$$\vec{B}(r, \rho) \simeq B_r \left\{ \hat{r} - \frac{1}{2} \gamma \frac{\rho}{r} \hat{\rho} \right\}. \quad (32)$$

and

$$\vec{\Gamma} = - \frac{\gamma}{r} \hat{r} - \frac{1}{2} \gamma \frac{\rho}{r^2} \hat{\rho}. \quad (33)$$

Moreover, if $\rho \ll r$, i.e., the particle flux is around a vicinity of the symmetry axis, then

$$\vec{\Gamma} \simeq -\frac{\gamma}{r} \hat{r} \quad (34)$$

From eq.(34) it follows that eq.(29) takes the form

$$\frac{D\tilde{U}_d}{Dr} = \frac{U_0}{r}, \quad (35)$$

so that

$$\tilde{U}_d = U_0 \ln(r/r_0). \quad (36)$$

As a first approximation, both \tilde{U} and \tilde{U}_0 are in the r-direction; thus we obtain:

$$\tilde{U}_0 = -\frac{3}{4} \gamma \frac{\alpha_p^2}{V_{Ap} V_{sw}} \left\{ \frac{A_p}{A_p + 3/2} - \frac{T}{M} \frac{A_\alpha}{A_\alpha + 3/2} \right\}. \quad (37)$$

Let us now evaluate the drift velocity given by eq.(36), for fast solar wind conditions. Thus, taking $V_{sw} = 700$ km/sec, $\alpha_p = 100$ km/sec, $V_{Ap} = 100$ km/sec, $M = 4$, $T = 1$, $A_\alpha = A_p = 2 - 3$, and for an unfavorable condition $T_p = T_\alpha$, eqs.(36) and (37) yield:

$$U_d \simeq -5\gamma \ln \frac{r}{r_0}, \quad (38)$$

where r_0 is some reference distance where the plasma is injected with zero drift velocity in a magnetic field weakly convergent and characterized by the index γ , which has to be negative if the field lines are convergent (Tyan Yeh, 1977).

Clearly, the drift velocity given by eq.(38) is a small quantity.

V. SUMMARY AND CONCLUSIONS

We present a semi cold approximation for EICW in multicomponent plasmas. We have applied the approximation to magnetospheric plasmas and to the solar wind.

In magnetospheric plasmas, the semi cold approximation describes successfully the amplification of ion cyclotron waves due to the presence of cold protons, a mechanism proposed by Cornwall *et al.* (1970) to account for ring current proton losses. We then study EICW propagating in a plasma composed by protons and He^+ ions, each having a cold and hot population. The unstable spectrum exhibits a low frequency region ending at the He^+ gyrofrequency and a high frequency region starting at a cutoff frequency which depends on the cold He^+ density. The results are in good agreement with the observations performed on board of the GEOS satellites.

Concerning the solar wind, we have shown that a relative drift velocity between alpha particles and protons can accelerate the alpha particles to velocities well in excess of the proton bulk velocity.

The relative drift allows the alpha particles to resonate with the waves continuously from negative to positive velocities. Waves belonging to the upper branch of the dispersion relation (see Fig. 7) will accelerate alpha particles to a velocity equal to the drift velocity U . On the other hand, waves belonging to the lower branch will accelerate alpha particles which have velocities slightly larger than U to larger velocities. Thermal effects on the real part of the dispersion relation might close the gap between the two branches.

The next question that emerges is whether the waves last long enough to be able to accelerate the bulk of the alpha particle distribution to velocities well in excess of the proton bulk velocity. The answer depends on how large the damping rates in the frequency region $\Omega_\alpha < \omega < \Omega_p$ are. We show that for positive proton thermal anisotropies, the waves grow in

the frequency region $0 < \omega < A_p \Omega_p / (A_p + 1)$, and are damped beyond the marginal frequency, $\omega_m = A_p / (A_p - 1)$. As A_p increases, the instability range is enhanced and damping rates decrease. We conclude that for thermal anisotropies between 2 and 3, growth rates are already fairly large in the region $\Omega_\alpha < \omega < \omega_m$, and damping rates are fairly weak ($< 10^{-2}$) beyond ω_m .

On the other hand, large thermal anisotropies seem to be the prevailing condition in fast solar wind flow, in contrast with moderate and slow solar wind flows which have negative thermal anisotropy. Therefore a small drift between the ion components can provide a feasible mechanism for preferential acceleration of alpha particles in fast solar wind conditions (Gomberoff and Elgueta, 1991).

Finally, by using a simple model, it is shown that a proton thermal anisotropy can lead to a small drift velocity between alpha particles and protons. The model assumes that a thermalized plasma is injected from the corona to a region where the magnetic field can be modeled by a tube of convergent magnetic field lines. In this region, the plasma becomes anisotropic - or it may be already anisotropic - and eventually escapes from the recombination region. The recombination region is not far from the region where the plasma is injected and, therefore, the relative drift is necessarily small.

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