Geofísica Internacional (1992), Vol. 31, Num. 2, pp. 131-134

Role of cold H^+ and He^+ ions and thermal anisotropy in the electromagnetic proton cyclotron instability

P. Vega*, J. Fernández* and L. Gomberoff**
* *Depto. de Matemáticas y Física, Facultad de Ciencias, Universidad de la Serena, La Serena, Chile.*
** *Depto. de Física, Facultad de Ciencias, Universidad de Chile, Santiago, Chile.*

RESUMEN

Se presenta un estudio detallado de la inestabilidad electromagnética protón ciclotrón en un plasma que contiene iones fríos de H^+ y He^+. En la presencia de iones fríos de He^+, el espectro inestable se divide en dos regiones, ramas LF y HF. Usando una expresión analítica semifría para la tasa de crecimiento convectiva, mostramos que en ambas ramas un aumento en la concentración de protones fríos corre el espectro hacia frecuencias más bajas. Sin embargo, un aumento en la población de He^+ corre el espectro a frecuencias más bajas en la rama LF y a frecuencias más altas en la rama HF. Este comportamiento conduce a que, en la rama LF, la presencia de iones fríos H^+ y He^+ retarda la estabilización del plasma debido a la disminución de la anisotropía térmica; pero en la región HF, esta estabilización se intensifica con el aumento de la población de He^+ frío y se retarda con el aumento en la concentración de H^+ frío. Así, la combinación de estos dos efectos resulta en una amplificación de estas ondas, primero el corrimiento del espectro debido a los iones fríos y después el hecho de que las ondas de frecuencias más altas son las primeras que se estabilizan.

PALABRAS CLAVE: Inestabilidad protón-ciclotrón, plasmas fríos, iones H^+ y He^+.

ABSTRACT

A detailed study of the electromagnetic proton cyclotron instability in a plasma containing cold H^+ and He^+ ions is presented. In the presence of cold He^+ ions the unstable spectrum is divided into two regions, LF and HF branches, by a stop band starting at the He^+ gyrofrequency.

By using a semi-cold analytical expression for the convective growth rate we show that in both branches an increase of cold proton concentration shifts the spectrum to lower frequency values. However, an increase in the He^+ population shifts the spectrum (1) to lower frequencies in the LF branch and (2) to higher frequencies in the HF branch. This behavior leads to the fact that in the LF branch, the presence of cold H^+ and He^+ ions slows down the stabilization of the plasma due to decreasing thermal anisotropy; but, in the HF region, this stabilization is strengthened with increasing cold He^+ population and is slowed down with increasing cold H^+ concentration. Thus, an amplification of these waves is the result of combining these two effects, first, the shift of the spectrum due to cold ions and, second, the fact that higher frequency waves are the first to be stabilized.

KEY WORDS: Proton cyclotron instability, cold plasmas, H^+ and He^+ ions.

INTRODUCTION

Electromagnetic ion cyclotron waves (ICW's) below the proton gyrofrequency in multicomponent plasmas have been studied during several years (see e.g., Gomberoff and Vega, 1987, and references therein). Since the article of Cornwall and Schultz (1971), which discussed this instability in the presence of cold plasma, various studies on the effect of cold heavy ions on the electromagnetic proton-cyclotron instability have appeared in the literature (see, e.g. Gomberoff and Molina, 1985, and references therein).

Recently, Gomberoff and Rogan (1988) have shown that the main effect of cold protons on the proton cyclotron instability is to shift the position of the unstable spectrum towards lower frequency values. They pointed out that the amplification reported by some authors is due to the fact that cold ions shift the unstable spectrum away from a stabilization or forbidden region. Even though this result is fairly general, only protons were considered as cold components of the plasma and, when cold He^+ ions are included only a qualitative study was done.

Experimental data on board several satellites having shown that the electromagnetic ion cyclotron waves occur in multi-ion plasmas, one requires a detailed study of this instability when heavy ion components are included. The main aim of this article is to gain a clear understanding of the ion cyclotron instability when both cold protons and cold He^+ ions are included in the plasma.

In order to study the effect of cold ion components on the ion cyclotron instability, we analyse in detail the behavior of the convective growth rate S with varying concentrations of cold ions, δ for H^+ and η for He^+, and changing values of the thermal anisotropy.The value of $\beta_{//p}=8\pi n_p kT_{//p}/B_o^2$ and $\alpha^2_{//p} = 2kT_{//p}/m_p$ is fixed. In the presence of cold He$^+$ ions the unstable spectrum of the ion-cyclotron waves is divided into two regions by a stop band starting at the He$^+$ gyrofrequency (Gomberoff and Cuperman, 1982): a low frequency region LF and a high frequency region HF.

In 1983, Gomberoff and Neira, by using an analytical approach of the convective growth rate S, showed that the marginal mode, x_m, depends on the thermal anisotropy of the proton distribution function, $A_p = (T/_{\perp}T_{//})-1$, and is given by $x_m = A_p/(A_p + 1)$. They also showed that for some values of A_p, S is an increasing function of cold concentrations until they reach some value, $\delta = \delta_{op}$ for protons and $\eta = \eta_{op}$ for He$^+$ ions, and decrease thereafter. For these concentrations, the mode $x = \omega_r/\Omega_p$ at which S takes its maximum value is given by $x_1 = (A_p - 1)/(A_p + 1)$ and at this value, the exponent of the expression for S takes the value one, i.e.

$$Q = \frac{(1-x)^2}{\beta_{//p}\, x^2} \cdot \left\{ \frac{1+\delta}{1-x} + \frac{4\eta}{1-4x} \right\} \approx 1 \tag{1}$$

From this equation we can show that η_{op} depends, for $\beta_{//p}$ fixed, not only on the thermal anisotropy, but is a function of cold proton concentrations. The same is valid for the optimum cold proton concentrations; i.e. from eq.(1) we can show that δ_{op} depends on A_p and η.

NUMERICAL ANALYSIS. To study in detail the effect of cold ions H$^+$ and He$^+$, on the proton cyclotron instability, we consider $\beta_{//p} = 0.1$ and energetic protons with thermal energy of 25 keV (Ishida *et al.*, 1987; Gendrin *et al.*, 1984). In the following we analyse, first, the effect of cold He$^+$ ions and, second, the effect of cold protons, on the convective growth rate, S.

a. Effect of cold He$^+$ ions on proton cyclotron instability.

In Fig. 1, the convective growth rate, S, has been plotted as a function of x for $\delta = 50$ (Ishida *et al.*, 1987) and for various values of η. Each figure corresponds to a given value of A_p.

In the low frequency region (LF branch), an increase of η shifts the spectrum to lower frequency and broadens one. We can also notice that for sufficiently large values of A_p (Fig. 1a and 1b), i.e. when the marginal model x_m is far from that at which S_{max} for $\eta = 0$ occurs, the maximum convective growth rate is a very slowly varying function of the He$^+$ ion concentration. If the value of A_p is then reduced, Fig. 1c, we can see that for any η, S_{max} decreases and since S_{max} ($\eta = 0$) lies now close to x_m, S is an increasing function of η. To stress this point further, in Fig. 1d A_p has been reduced to 0.3. In this case, the decrease of S_{max} for any value of η is very large and now S_{max} ($\eta = 0$) occurs at $x < x_{He^+}$ (He$^+$ gyrofrequency), very close to the marginal mode. A comparison between the convective growth rate for $\eta = 0$ with, e.g. $\eta=10$, shows that there is a significant increase in the value of S_{max}. But if we compare this behavior with S_{max} ($\eta=0$) for a large value of A_p, e.g. $A_p = 3$ in Fig. 1a, we notice that the reduction of the termal anisotropy leads to a decrease of S_{max} for all η.

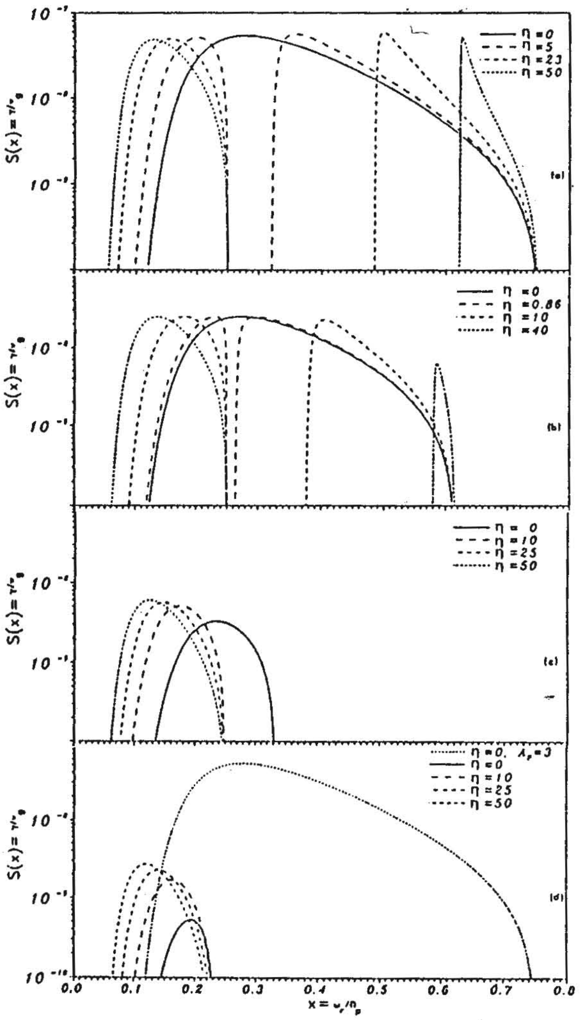

Fig. 1: Convective growth rate S(m^{-1}) vs normalized frequency lor, $\beta_p = 0.1$ and $\delta = 50$ and, (a) $A_p = 3.0$, (b) $A_p=1.6$, (c) $A_p = 0.5$, (d) $A_p = 0.3$.

Consider now the HF branch of the unstable spectrum. An increase of η shifts this frequency range of the spectrum towards higher frequencies and narrows one. We see that for large values of A_p (Figs. 1a and 1b), the maximum convective growth rate changes only slightly.

When A_p is further reduced, i.e. x_m approaches the mode at which S_{max} ($\eta=0$) occurs, this region of instability is narrowed. Fig. 1c shows this case. For the parameters considered, a slight increase of η leads to a large decrease of S_{max}. This behavior can be understood by assuming that an increase of He$^+$ ion concentrations causes the spectrum to approach a stabilization region.

It is also important to point out that the shift due to the increase of η is more effective in the HF branch than in the LF branch.

b. Effect of the cold protons on proton cyclotron instability.

In Fig. 2, we have done the same as in Fig. 1 but for $\eta = 1.0$ and for several values of δ. Notice that, in both LF and HF branches of instability, an increase of δ shifts the unstable spectrum towards lower frequencies and broadens one. This shift is more efficient in the HF branch than in the LF branch and, as shown above, a drecrease of A_p leads to a reduction of maximum convective growth rate for any value of δ.

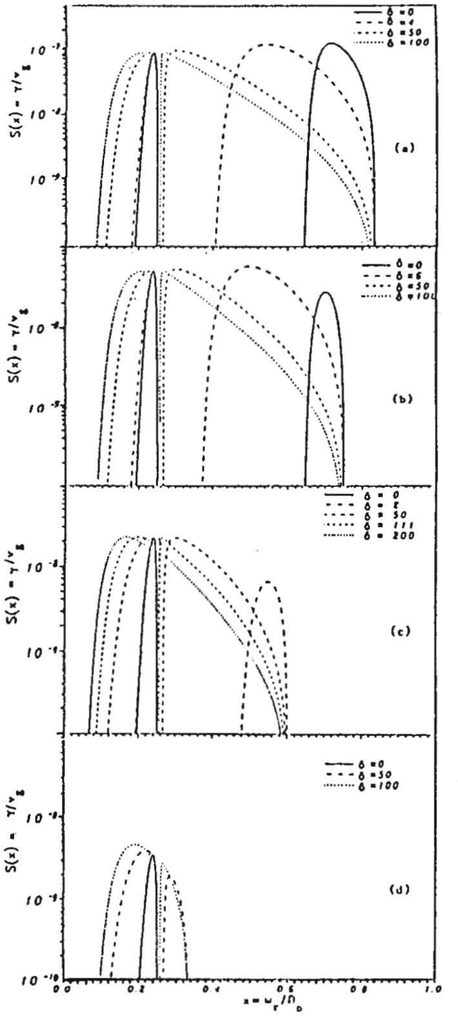

fig. 2: The same as Fig. 1 but $\eta=1.0$ and, (a) $A_p=5.0$, (b) $A_p=3.0$, (c) $A_p=1.5$, (d) $A_p=0.5$

When the value of A_p is large we see that, in LF branch, S_{max} is a very slowly varying function of the cold proton concentration. This is due to the fact that the mode at which S_{max} ($\delta=0$) occurs, is very far from the marginal mode; for instance, for $A_p=1.5$, $x(\delta=0)\approx0.21$ and $x_m = 0.6$, Fig. 2a-2c. Let us see now what happens in the HF branch. Even for a large value of A_p, S_{max} for $\delta = 0$ lies close to the marginal mode. The value of S_{max} will be almost unaltered with increasing cold proton concentration, provided that A_p is very large, i.e., as shown in Fig. 2a, where $A_p = 5.0$.

If we then reduce the value of A_p we see that, first, in the LF branch, S_{max} increases significantly when cold protons are introduced, Fig. 2d. But, even for a large value of δ, the value of S_{max} is always less than the value of S_{max} for $\delta=0$ when $A_p=5.0$. Now, in the HF branch, the situation is a bit different. Since the mode at which S_{max} ($\delta=0$) occurs is close to the marginal mode, a slight increase of δ produces a large increase of S_{max}. This occurs even for non-negligible anisotropy; we see this in Figs. 2b and 2c, where $A_p=3.0$ and 1.5. This effect is even stronger when A_p is further reduced as in Fig. 2d. In this case $S_{max}(\delta = 0)$ is smaller than $10^{-10}m^{-1}$. We point out that even though a large increase of S_{max} is produced when δ increases, the values reached are always less than that one corresponding to S_{max} ($\delta = 0$) when the value of A_p is larger, e.g. compare Fig. 2d and Fig. 2a.

In Fig. 3, we have done the same as in Fig. 1a-1c but we have increased the value of δ. This shifts the spectrum towards lower frequencies. In the LF branch, even though the effect of both ion components is added, this is more effective for $\eta=0$ than for $\eta\neq0$; thus, when δ is large, S is a slowly varying function of η, even for a small value of A_p. Now, in the HF region this produces a large increase of S_{max}. Fig. 3b shows that an increase of δ produces the appearance of the unstable spectrum which had disappeared when A_p was reduced and η was increased. (Fig. 1c).

CONCLUSIONS

A detailed study of ion cyclotron waves below the proton gyrofrequency in a multi-ion plasma with two cold component, H^+ and He^+, has been made.

We have shown that in the LF region both cold H^+ and cold He^+ ions shift the spectrum of instability away from a forbidden region and, in the HF region, first, cold H^+ ions shift the spectrum away from a stabilization region and, second, cold He^+ ions shift the spectrum towards a stabilization region. This effect combined with the fact that the reduction of the thermal anisotropy starts stabilizing higher frequency waves lead to an amplification of these waves. In other words, in the LF branch the presence of cold H^+ and He^+ ions slows down the stabilization of the plasma due to the reduction of the thermal anisotropy. However, in the HF branch, this stabilization is strenghtened with increas-

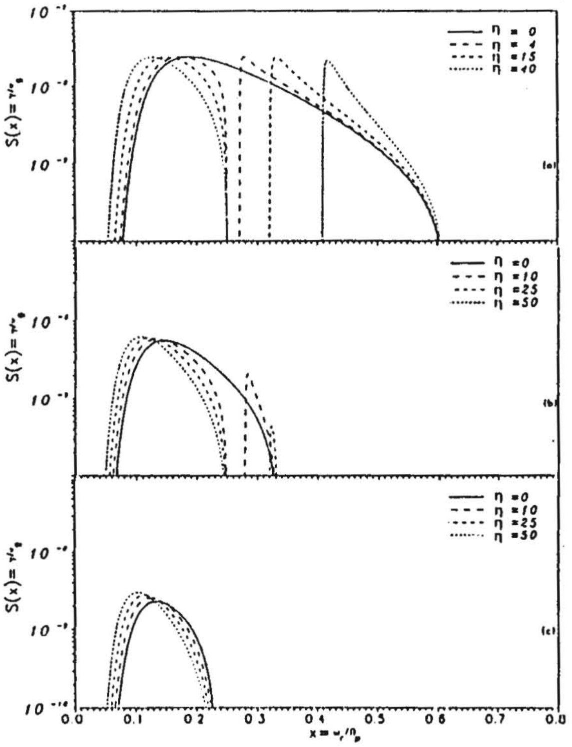

Fig. 3: The same as Fig. 1 but, (a) A_p=1.6 and δ=150, (b) A_p=0.5 and δ=250 and (c) A_p=0.3 and δ=250.

ing of cold He^+ population and is slowed down with increasing of cold proton concentration.

Since this amplification process involves a transfer of energy from particles to waves, cold particles (E<5eV) by mean of a particle-wave interaction, have not enough energy to support and amplify these waves (Young *et al.*, 1981; Roux *et al.*, 1982).

ACNOWLEDGEMENTS

This paper has been supported in part by DIULS Universidad de la Serena, grant No. 130-2-19 and FONDECYT, grant No. 90-1008.

BIBLIOGRAPHY

CORNWALL, J.M. and M. SCHULTZ, 1971 *J. Geophys. Res.*, 76, 7791

GOMBEROFF, L. and S. CUPERMAN, 1982. *J. Geophys. Res.*, 87, 95.

GOMBEROFF, L. and M. Molina, 1985. *J. Geophys. Res.*, 90, 1976.

GOMBEROFF, L. and R. NEIRA, 1983. *J. Geophys. Res.*, 88, 2170.

GOMBEROFF, L. and J. ROGAN, 1988. *Planet. Space Sci.*, 36, 207.

GOMBEROFF, L. and P. VEGA, 1987. *J.Geophys. Res.*, 92, 7728.

GENDRIN, R. *et al.*, 1984. *J. Geophys. Res.*, 89, 9119.

ISHIDA, J., S. KOKUBUN and R. McPHERRON, 1987. *J. Geophys. Res.*, 92, 143.

ROUX A. *et al.*, 1982. *J. Geophys. Res.*, 87, 8174.

YOUNG, D.T. *et al.*, 1981. *J. Geophys. Res.*, 86, 6755.

P. Vega and J. Fernández
Depto. de Matemáticas y Física, Facultad de Ciencias,
Universidad de la Serena
Casilla 554
La Serena, Chile

L. Gomberoff
Depto. de Física, Facultad de Ciencias
Universidad de Chile
Casilla 653
Santiago, Chile