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Contribution of Birkeland Currents to the Earth's distant magnetic field

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RESUMEN

Las Corrientes de Birkeland son corrientes eléctricas en forma de hojas que fluyen hacia adentro y hacia afuera de la zona auroral; su circuito parece estar cerrado lejos de la tierra ya sea por corrientes cercanas al ecuador en el lado noche o por corrientes en o más allá de la frontera con el viento solar. Debido a la intrincada configuración de estas corrientes, se desconoce su contribución al campo magnético distante. Sin embargo, ésta es de gran interés para el modelado del campo magnético global y para la comprensión de los efectos atribuídos a estas corrientes en la posición de la magnetopausa del mediodía ("erosión") y la de los "canales" polares. Se describirá un esfuerzo inicial por derivar este campo suponiendo corrientes relativamente débiles en el fondo de un campo dipolar. El método usa los potenciales de Euler (α,β) (Stern, 1970) que satisfacen $B=\nabla\alpha \times \nabla\beta$ y supone que las corrientes fluyen en hojas delgadas. Se describirán los resultados de esta configuración.

PALABRAS CLAVE: Corrientes de Birkeland, magnetosfera.

ABSTRACT

Birkeland Currents are sheet-like electric currents flowing into the auroral zone and out of it; their circuit appears to be closed far from Earth either by near-equatorial currents on the night side or by currents in or beyond the boundary with the solar wind. Because of the intricate configuration of those currents, their contribution to the distant magnetic field is unknown. Yet it is of great interest, for modeling the global magnetic field and for understanding effects claimed for such currents, on the position of the noon magnetopause ("erosion") and on that of the polar cusps. An initial effort to derive such a field will be described, assuming relatively weak currents on the background of a dipole field. The method uses Euler potentials (α,β) (Stern, 1970), which satisfy $B= \nabla\alpha \times \nabla\beta$, and it assumes the currents flow in thin sheets. Results of applying the Biot-Savart formula to this configuration will be described.

KEY WORDS: Birkeland currents, magnetosphere.

I. INTRODUCTION

Above the Earth's atmosphere lies the magnetosphere, a complex plasma environment resulting from the interaction of Earth's own magnetism with the solar wind. The magnetospheric plasma is almost collision-free and its behavior is dominated by the global pattern of its electric and magnetic fields an of the associated electric currents.

Magnetospheric modeling is the art of mapping the magnetosphere, i.e. expressing the pattern of magnetic field and currents by analytical approximations, which can be calibrated against observations. It leads to mathematical formulas, giving a "best guess" of the magnetic field vector **B** for any point (x,y,z) in the interior, based on past measurements and depending, perhaps, on the level of magnetic activity and other variables. The magnetic field may be viewed as a superposition of fields caused by electric currents flowing within the Earth (internal field) and fields generated by currents flowing elsewhere (external field). A dipole field provides the lowest-order approximation to the internal field. Four major current systems, contribute to the external magnetic field in the magnetosphere; commonly, the contribution of each of them is modeled separately and the fields are then added to the dipole. The systems are:

(1) Magnetopause currents, responsible for cooping the Earth's field inside a cavity in the solar wind.

(2) The ring current, circling the Earth and carried by the particles of the radiation belt.

(3) The tail current, responsible for the long magnetic tail on the night side.

(4) Auroral currents (also known as Birkeland Currents), flowing into the polar cap and out of it, and associated with the polar aurora.

Figure 1 shows a schematic view of the dusk side of the Earth's magnetosphere, identifying the main external current systems.

To date, adequate mathematical formulations have been developed to represent all external current systems except the auroral (or Birkeland) currents. Thus, the improvement of existing magnetospheric models requires above all improved representations for auroral currents. Here an idealized model for the magnetic field contribution from Birkeland currents will be described.

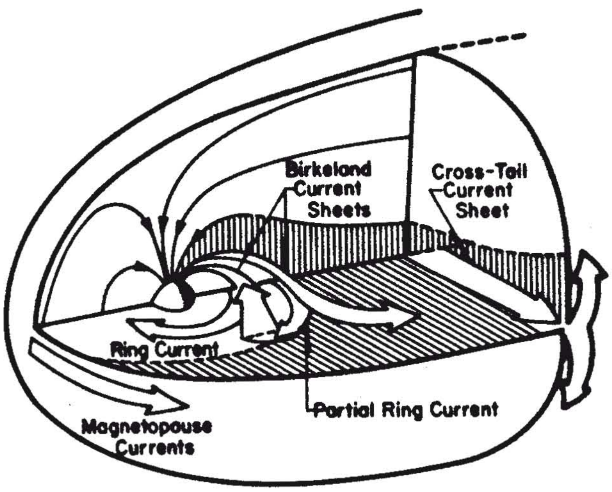

Fig. 1. Schematic view of the dusk side magnetosphere

The remainder of this paper is organized as follows. In Section II, we discuss briefly the present view of the Birkeland current system, and Section III then describes our idealized model of the auroral current system. Section IV lists some areas where Birkeland currents are expected to affect global magnetospheric properties, along with results obtained to date. The section concludes with a description of work in progress and long term objectives of this study. Finally, a summary of this work is presented in Section V.

II. AURORAL OR BIRKELAND CURRENTS

The aurora is clearly the most prominent effect originating in the distant magnetic environment of the Earth. These northern and southern lights have puzzled observers since ancient times, and the strong magnetic disturbances associated with them suggested that they were accompanied by large electric currents.

Among the first to investigate those currents in a systematic manner was Birkeland (1908) whose observations of auroral magnetic disturbances around the beginning of the century suggested that those electric currents flowed into and out of the polar ionosphere.

The present view is that Birkeland currents form sheets parallel to the magnetic field which flow into and out of the auroral zone. Their circuit appears to be closed far from Earth by near-equatorial currents on the night side, but detailed knowledge of the full current system is lacking. To make things harder, there exist two such current systems; one coming down in the morning side of the polar caps and up in the evening side, and the other one, a bit further equatorward, with opposite directions. These two systems were identified in 1974 (Zmuda and Armstrong, 1974; Iijima and Potemra, 1976a,b) and were named Region 1 and Region 2 Birkeland currents, respectively; they are obviously connected through the ionosphere.

Figure 2 below illustrates, schematically, an idealized, average configuration of the Birkeland current circuit; the sheets extend most of the way around the pole, at approximately constant magnetic latitude, but here only short sections of them are shown.

Region 1 currents descend along field lines on the morning side into the ionosphere (A), then flow toward the equator (B), and back out along the morning Region 2 current sheets (C) to the magnetosphere; there, they flow to the night side along the partial ring current (D). On the dusk side, the sequence is reversed. The currents flow back to the ionosphere along the nightside Region 2 sheets (E), across the ionosphere (F), and finally, out along the nightside Region 1 sheets (G). The closure of the Region 1 system far from Earth is not shown; some speculate that it may occur in the equatorial plane as an asymmetric ring current or an extra crosstail component while others argue that the currents map to the flanks of the magnetosphere.

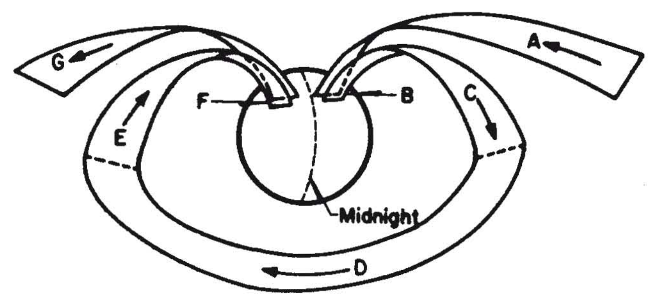

Fig. 2. Schematic view of idealized nightside Birkeland current sheets.

The intricate nature of the auroral current system makes it difficult to calculate its contribution to the distant magnetic field. However, even approximate estimates of this contribution are useful for modeling the global magnetic field, and for understanding and estimating the effects of Birkeland currents on the position of the noon magnetopause ("erosion") and on the location of the polar cusps. In the following section, we describe a simplified model for the Birkeland field and identify three specific problems to evaluate the effect of the auroral current system on the global properties of the magnetosphere.

III. SIMPLE MODEL FOR BIRKELAND CURRENTS

As indicated before, the simplest approximation of the internal magnetic field is a dipole field. In this model, the field arising from the auroral currents will be considred to be a perturbation of a dipole configuration. The dipole field $\mathbf{B}_o$, in its turn (subscript zero will identify dipole variables) may be described either in terms of Euler potentials (α_o,β_o) [Stern, 1970] or by the gradient of a scalar potential γ_o:

$$B_o = \nabla\alpha_o \times \nabla\beta_o = -\nabla\gamma_o \qquad (1)$$

The variables $(\alpha_o, \beta_o, \gamma_o)$ form an orthogonal coordinate system; if we assume a dipole moment g = 1 and measure distances in Earth radii (R_E), they can be expressed in terms of spherical coordinates (r, θ, ϕ) as follows:

$$\alpha_o = \frac{\sin^2\theta}{r}, \quad \beta_o = \phi, \quad \gamma_o = \frac{\cos\theta}{r^2}$$

Perturbation about the dipole field

We assume that the total magnetic field, $\mathbf{B}_{tot}$ can be decomposed into $\mathbf{B}_{tot}=\mathbf{B}_o+\mathbf{B}_1$ where $\mathbf{B}_o$ and $\mathbf{B}_1$ denote the dipole and Birkeland fields and where it is also assumed that $|\mathbf{B}_o| << |\mathbf{B}_1|$. Let the perturbed dipole field be also expressed by Euler potentials:

$$B = \nabla\alpha \times \nabla\beta \qquad (2)$$

As indicated above, near Earth Birkeland currents flow in sheets parallel to the magnetic field, and one expects $\mathbf{B}_{tot}$ to have tangential discontinuities across such sheets. Such discontinuities can be modeled by making the Euler potentials discontinuous. Initially it is assumed that the perturbed Euler potentials (α,β) are related to the dipole Euler potentials by:

$$\alpha = \alpha_o + K(\alpha_o, \gamma_o) \cos\phi \qquad (3)$$

$$\beta = \beta_o + F(\alpha_o, \gamma_o) \sin\phi \qquad (4)$$

In the representation above, we have imposed at the outset a specific ϕ dependence so that maximum inflow occurs at dusk and outflow at dawn (or vice versa depending on the polarity of the system modeled -- Region I vs. Region II). The terms added to the Euler potentials above may be viewed as the m=1 mode in a Fourier series expansion; one may in principle represent asymmetries of the pattern by additional terms proportional to cos mϕ and sin mϕ, m=2,3,...In addition to the field derived here, a ring current must also be added to introduce the observed day-night asymmetry.

Motivated by the observed sheet structure of the auroral currents near Earth, we assume that the Birkeland currents are confined inside a narrow shell around the field lines with $\alpha=\alpha_1$. Mathematically, the current sheet is generated by discontinuous jumps in $[F(\alpha_o,\gamma_o), K(\alpha_o,\gamma_o)]$ at $\alpha=\alpha_1$. An additional requirement is that no current flows away from the shell into surrounding space. This requires the vanishing of the component j_α of the current density in the direction of $\nabla\alpha$, and leads to the following relation between the functions $F(\alpha_o,\gamma_o)$ and $K(\alpha_o,\gamma_o)$ [we henceforth may omit subscript zero]:

$$\frac{\partial K}{\partial \alpha} + F = \frac{\partial}{\partial \gamma}\left(B_o^2\, \rho^2 \frac{\partial F}{\partial \gamma}\right) \qquad (5)$$

where B_o represents the magnitude of the dipole field $\mathbf{B}_o$, and $\rho^2 = x^2 + y^2$.

Discontinuous jump across current sheet

We assume the currents flow inside a shell centered around the $\alpha=\alpha_1$ field lines, and divide space into three regions as illustrated in Figure 3. In regions I and III, a dipole configuration is assumed, while in region II, confined to $\alpha_1 - \Delta\alpha \le \alpha \le \alpha_1 + \Delta\alpha$, we choose for the perturbing functions $F(\alpha_o, \gamma_o)$ and $K(\alpha_o, \gamma_o)$ the form:

$$F(\alpha_o, \gamma_o) = (\alpha-\alpha_1)\, f(\gamma), \qquad (6)$$

$$K(\alpha_o, \gamma_o) = 1/2\, (\alpha - \alpha_1)^2\, k(\gamma) \qquad (7)$$

which reduces their required relationship to

$$k(\gamma) = \frac{\partial}{\partial \gamma}\left(B_o^2\, \rho^2 \frac{df}{d\gamma}\right) - f(\gamma)$$

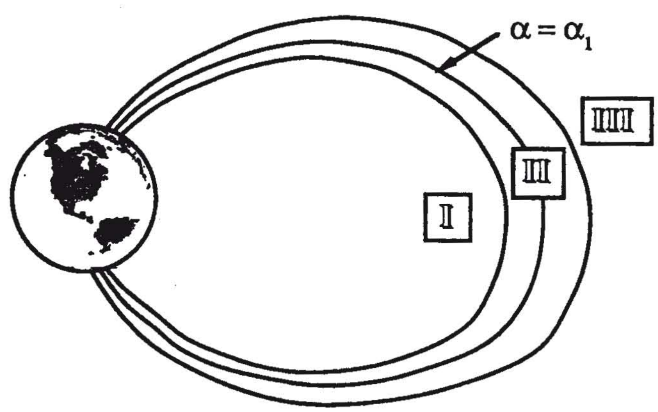

Fig. 3. Schematic representation of the current layer centered on the $\alpha=\alpha_1$ surface.

Region II is then allowed to collapse into a thin sheet; for computation purposes, we have assumed that this sheet cuts the equator at L =10. The choice of f(γ) determines the connection between the field aligned **j** and j_ϕ. We choose f(γ) so that **j** remains relatively field aligned within 3 R_E of Earth. Beyond this distance, the current density acquires an azimuthal component, but the connection between f(γ) and k(γ) assures that it will not flow out of the sheet and bleed into space. The functional form selected for f(γ) is:

$$f(\gamma) = f_o \frac{(\gamma + a)}{\left[(\gamma + b)^2 + c\right]} \qquad (8)$$

The constants a, b, and c yielding the above described radial dependence are:

$$a=0.1, \qquad b=0.0078, \qquad c=0.0015$$

The constant f_o calibrates the total current carried by the Birkeland system. The results presented here have been obtained for a strong system characterized by a total current ~1.6 x 10^6 Amp in each polar cap. In addition, by selecting f_o negative (positive), one can represent a current system with the polarity of Region I (Region II) Birkeland currents.

The circuit of Figure 3 extends all the way around the Earth, but it must still be allowed to close across the polar cap. As a simple approximation, it is assumed that this closure happens in a flat sheet, at $z=\pm 1$ R_E. The contribution to the Birkeland field from the current across the polar cap in most of the magnetosphere is relatively small and the flat sheet approximation suffices.

Magnetic field outside the current sheet

The preceding provides a reasonable approximation of the current density **j** of the Birkeland system, but what we actually seek is the perturbed field outside the current sheet. Away from the current sheet, $\mathbf{B_1}$ due to Birkeland currents is (in this model) current-free and can therefore be described by a harmonic scalar potential, Ψ, satisfying:

$$B_1 = - \nabla \Psi (r, \theta, \phi) \qquad (9)$$

$$\nabla^2 \Psi (r, \theta, \phi) = 0 \qquad (10)$$

Because of the cosϕ and sinϕ dependence built into this model, it turns out that Ψ can be described in terms of a two-dimensional function $\psi(r, \theta)$,

$$\Psi (r,\theta,\phi) = \psi (r, \theta) \cos\phi \qquad (11)$$

The boundary conditions on the scalar potential, $\Psi(r,\theta,\phi)$, are non-standard. The two-dimensional function, $\psi(r, \theta)$, is regular at the origin and at infinity, but has a jump at the current sheet, which can be expressed analytically.

Solution by Biot-Savart integration

Because efforts to express $\Psi(r,\theta,\phi)$ in terms of standard harmonic functions have failed to converge, we have resorted to Biot-Savart integration for deriving the field. According to the Biot-Savart formula, the magnetic field $\mathbf{B_1(r)}$ at position **r** is related to the current sources $\mathbf{j_1(r')}$ at r', by the integral:

$$B_1(r) = \frac{\mu_0}{4\pi} \int_V j_1(r') \times \frac{(r - r')}{|r - r'|^3} d^3V \qquad (12)$$

The assumed structure of the Birkeland currents reduces the Biot-Savart formula to an integration over the surface elements of a thin current sheet plus the contributions from the flat sheets across the polar caps at $Z = \pm 1$ R_E. Thanks to the imposed cosϕ dependence, the values of **B** in a single meridional plane allow one to derive the field in the rest of space.

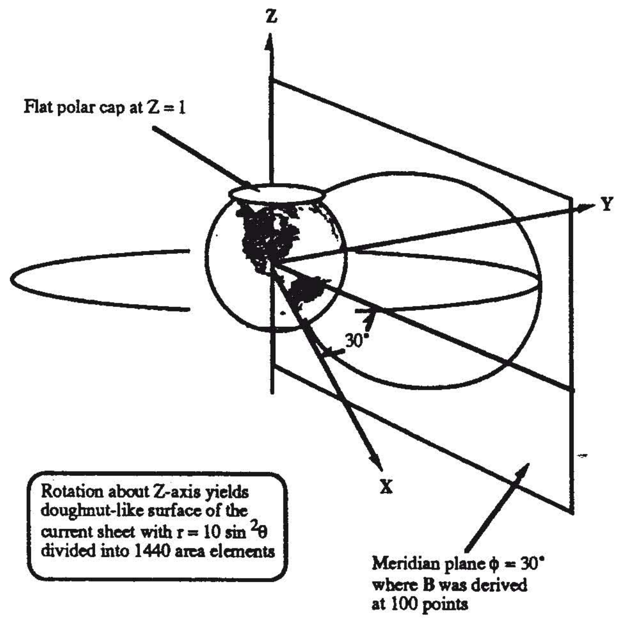

Fig. 4. Geometry for Biot-Savart integration

We have developed a program to carry out, numerically, the Biot-Savart integration. The program runs very effectively (~30 sec on MacII and ~5 sec on a VAX 8810 for a grid 10 x 10 R_E) and has successfully passed several tests--for example, results obtained in different meridional planes exhibit the appropriate symmetry and the numerical curl of $\mathbf{B_1}$ is small as required. Figure 4 illustrates the geometry used for the Biot-Savart integration described above.

IV. EXAMPLES OF BIRKELAND CURRENT EFFECTS

Thre effects of Birkeland currents on magnetospheric phenomena were investigated in three areas:

- Effects of Birkeland currents on the magnetic field observed along the orbit of a low-altitude satellite.
- Effects of Birkeland currents on the location of the polar cusp.
- Effects of Birkeland currents on the magnetic field at the noon magnetopause to estimate the "erosion" effect caused by Birkeland currents.

To address these questions, the field **B** is needed at the positions indicated in the following diagram:

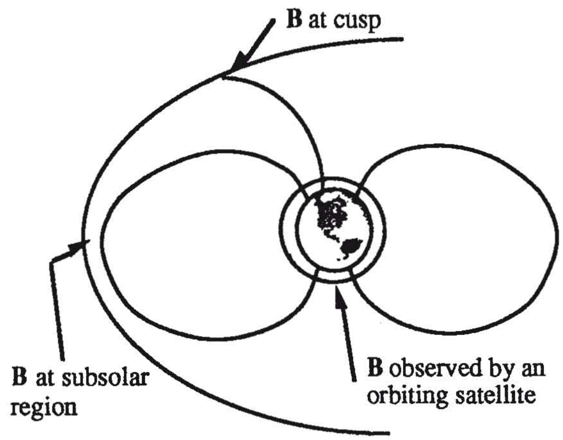

Fig. 5. Field positions for study of Birkeland current effects

Magnetic field observed by a low-altitude satellite

We have used our model to compute the anticipated jump in the magnetic field observed by a low-altitude (1000 km) satellite orbiting the Earth. The sheet contributes to **B** an azimuthal component B_ϕ which is absent in the dipole field, and as the satellite traverses the current sheet, a jump is observed in that component. Figure 6 illustrates the appropriate discontinuity in B_ϕ predicted by our Biot-Savart calculation.

Birkeland current effects on the location of the cusp

Magnetic field models typically include two points at which the magnetic field vanishes, known as the polar cusp; in the actual magnetosphere, the field does not vanish, but merely weakens and becomes turbulent as these regions are flooded with solar wind particles. In an analytically given model, cusps may be identified by tracing field lines in the noon-midnight meridian (Solar Magnetic Coordinates). A minimum in $|B|$ is found for lines near the cusp and a parabola fitted between the three deepest minima yields the approximate position of the cusp.

We have identified the position of the cusp with various models including: (i) a dipole field enclosed by a paraboloid, and (ii) the Tsyganenko 1987 model [Tsyganenko, 1987]. After this a ring current was added to the model Birkeland field in order to represent the proper day-night asymmetry, and the total Birkeland current field was computed. The resulting field magnitude was of order 10 nT, which suggests a rather small effect. This however is just a preliminary result and the study continues.

Effect of Birkeland currents on "erosion"

In order to address the problem of "erosion", we must incorporate an improved algorithm (currently under development) to better map **B** very close to the current sheet. If the field point where **B** is computed is located very close to the sheet, the contributions of the nearest elements in the numerical integration account for a disproportionate part of **B**, because of their small denominators, and at the same time those contributions become increasingly inaccurate. The improved model will identify field points "close" to the current sheet and use interpolation methods to calculate the contribution from the nearby current elements. Results from this computation will be detailed in a future communication.

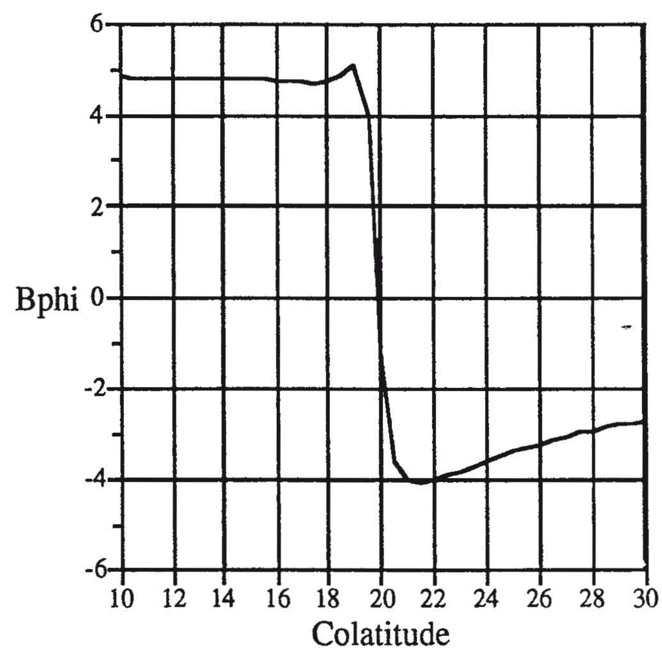

Fig. 6. Jump in the magnetic field observed by a low-altitude satellite.

Planned developments of the model

The improved model will also be used to investigate the structure of the magnetic field lines, in particular the deformations introduced by the Birkeland current contributions. After that, to make the model more realistic, it is necessary to stretch the nightside dipole field tailwards, as actually happens in the magnetosphere. The plan is to deform the pattern by stretching the dipole configuration [Stern, 1987]; the stretching could be applied here not to **B** but to the current density **j**, and then again Biot-Savart would be applied. Unfortunately, that eliminates the nice symmetries and intensive number-crunching is required. Initially, we plan to try an alternative method which is only approximate, namely stretching not the currents but the magnetic field **B**. That would deform the current sheet, but would also add spurious volume currents; the hope is that we might cancel most of those by a "Tsyganenko-type polynomial".

V. SUMMARY

We have described an idealized model to compute the magnetic field contribution due to auroral currents. The model assumes relatively weak Birkeland currents on the background of a dipole field. This dipole field is expressed in terms of Euler potentials (α,β) and the Birkeland currents are confined to flow on α=constant shells. A program has been devised to compute the Birkeland field by numerical integration of the Biot-Savart formula. Application of the model results in the expected discontinuity of the magnetic field seen by a low-altitude satellite. Preliminary results reveal that the Birkeland field contribution at the cusp (as determined by an independent field model) is of order 10 nT. Work is underway to explore the effect of Birkeland currents on "erosion" at the noon magnetopause, as well as their effect on field line structure; results from these investigations will be presented elsewhere.

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