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Ray tracing in the troposphere for GPS satellite signals
Part II

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RESUMEN

El presente trabajo es continuación de la Parte I, donde propusimos un algoritmo para calcular trazado de rayos hasta el tercer término de la expansión. En este trabajo consideramos todos los términos de la ecuación para un tratamiento más preciso y más general. La ecuación obtenida admite métodos iterativos.

PALABRAS CLAVE: Troposfera, trazado de rayos, señales del satélite GPS.

ABSTRACT

This paper is based on a preliminar work (Lagori *et al.*, 1992) in which we proposed an algorithm to compute the tracing of rays arriving to a stationary or moving station by limiting a serie expansion to the third term. In the present work, we consider all the expansion terms to improve precision and gain generality. The equation obtained allows an iterative treatment.

KEY WORDS: Troposphere, ray tracing, GPS satellite signals.

DEVELOPMENT

The ray tracing geometry is shown in figure 1.

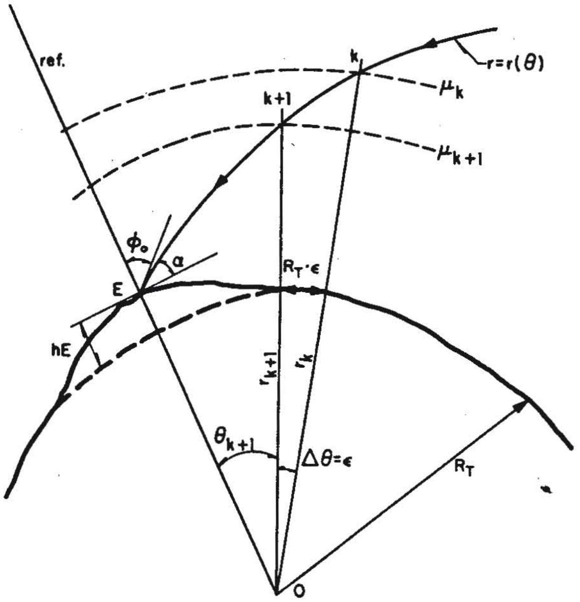

Fig. 1

In this work we consider that the troposphere presents spherical stratification and is concentric with the earth. This will allow us to assume that the refraction index is constant for each stratification level.

The ray trajectory equation will be:

$$r = r(\theta) = F(\theta) \qquad (1)$$

For $(\theta + \varepsilon)$ angle the equation (1) can be expanded as:

$$r = F(\theta + \varepsilon) =$$

$$F(\theta) + \varepsilon \frac{dF(\theta)}{d\theta} + \frac{\varepsilon^2}{2!} \frac{d^2F(\theta)}{d\theta^2} + \ldots + \frac{\varepsilon^n}{n!} \frac{d^nF(\theta)}{d\theta^n} \qquad (2)$$

If we consider points k and k + 1 on the trajectory:

$$r_{k+1} = r_k + \varepsilon \left(\frac{dr}{d\theta}\right)_k + \ldots + \frac{\varepsilon^n}{n!} \left(\frac{d^nr}{d\theta^n}\right)_k + \ldots \qquad (3)$$

Utilizing Snell's law for spherical stratification

$$n_k \; r_k \; \sin \phi_k = \text{cte} \qquad (4)$$

we obtain

$$r_k = \frac{n_o}{n_k} \, r_o \, \frac{\operatorname{cosec} \phi_k}{\sec \alpha} \qquad (5)$$

and we can show that:

$$\left(\frac{d^n r}{d\theta^n}\right)_k = \begin{cases} +r_k\left(\operatorname{cosec}^2\phi_k - 1\right)^{1/2} & \text{if } n = 4m-3 \\ -r_k\left(\operatorname{cosec}^2\phi_k - 1\right)^{1/2} & \text{if } n = 4m-1 \\ +r_k & \text{if } n = 4m-2 \\ -r_k & \text{if } n = 4m \end{cases} \tag{6}$$

where m = 1, 2, 3,...(positive integer number).

If we include equation (6) in (3), we obtain:

$$r_{k+1} = r_k\left[1 - \frac{\varepsilon^2}{2!} + \frac{\varepsilon^4}{4!} - \frac{\varepsilon^6}{6!} + ..\right] + r_k\left[\operatorname{cosec}^2\phi_k - 1\right]^{1/2}\left[1 - \frac{\varepsilon^3}{3!} + \frac{\varepsilon^5}{5!} - ..\right] \tag{7}$$

which can be written:

$$r_{k+1} = r_k\left[\sum_{j=o}^{\alpha}(-1)^{j+1}\frac{\varepsilon^{2j}}{(2j)!}\right] + r_k\left[\operatorname{cosec}^2\phi_k - 1\right]^{1/2}\left[\sum_{i=o}^{\alpha}(1)^i\frac{\varepsilon^{2i+1}}{(2i+1)!}\right] \tag{8}$$

The series of equation (8) are convergent and:

$$\sum_{j=o}^{\alpha}(-1)^{j+1}\frac{\varepsilon^{2j}}{(2j)!} \longrightarrow \cos\varepsilon \tag{9a}$$

$$\sum_{i=o}^{\alpha}(1)^i\frac{\varepsilon^{2i+1}}{(2i+1)!} \longrightarrow \sin\varepsilon \tag{9b}$$

Then, equation (8) can be rewritten as:

$$r_{k+1} = r_k\cos\varepsilon + r_k\left[\operatorname{cosec}^2\phi_k - 1\right]^{1/2}\sin\varepsilon \tag{10}$$

CONCLUSIONS

As suggested in Lagori *et al.* (1992), we utilize the same data for ray tracing model III [eq. (10)]. The results shows an identical trajectory with a better horizontal length for all data. The water vapor pressure gradient and its changes are the fundamental factors in the trajectory modification.

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