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Estimation of the design flood of Tecolutla River, Mexico, using the probable maximum rainfall

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RESUMEN

Este estudio muestra una técnica para estimar la avenida de diseño en cuencas grandes usando como variable independiente la precipitación máxima probable (pmp); para esto se utilizó el modelo del hidrograma unitario instántaneo. El modelo fue alimentado con la pmp diaria que se pueda presentar en 1000 años en la cuenca del Río Tecolutla, México, según la distribución de valores extremos de Gumbel (1958). El modelo pronostica una avenida con un gasto máximo de 31,627.7m^3/s para la estación hidrométrica El Remolino, Veracruz.

PALABRAS CLAVE: Precipitación máxima probable, avenida de diseño, Veracruz, México.

ABSTRACT

The design flood in a large catchment area was estimated for the probable maximum rainfall (pmp). The instant unit hydrograph model was used. The daily pmp that may occur in 1000 years on the watershed of the Tecolutla River, Mexico, was estimated by extreme values. The model forecasts a peak flow of 31,627.7m^3/s at El Remolino, Veracruz.

KEY WORDS: Probable maximum rainfall, design flood, Veracruz, Mexico.

1. INTRODUCTION

The relationship between rainfall and runoff is complex. According to Raudkivi (1979) and Fuentes *et al.*, (1981), the runoff is a function of features of the watershed (area, elevation, slope, orientation, soil type, drainage, storage capacity and vegetation) and of climate: type of rainfall or snowfall, evapotranspiration, rainfall intensity, duration of rainfall, spatial and temporal distribution of the rainfall and direction of storms.

Models developed to relate precipitation with runoff vary from single empirical formulas to extremely complex models (Viessman *et al.*, 1977; Raudkivi, 1979; Domínguez *et al.*, 1980; Fuentes *et al.*, 1981; Aparicio, 1989). These methods are applied to small watersheds (<400 Km^2) using time intervals of less than 24 hours. In this study, the design flood is estimated for a large catchment area located in the state of Veracruz, Mexico. The daily probable maximum rainfall is used in the instant unit hydrograph model.

2. STUDY AREA

The station El Remolino is on the Tecolutla River (Figure 1). The watershed has an area of 7,342 Km^2 up to the Gulf of Mexico. Elevation varies from 3,500 m at the edge of the Sierra Madre Oriental to zero. The upper course has very steep slopes. According to Tejeda *et al.*, (1989) the temperature decreases from the Gulf coast to the Sierra Madre Oriental. The climate varies from warm to mild, with summer rains. The average annual rainfall is 1400 mm in the lowlands, 2500 mm in the middle course and 700 mm in the highlands (Pereyra and Hernández, 1989).

3. ESTIMATION OF THE PROBABLE MAXIMUM RAINFALL

Among the probability distributions used in hydrology are the lognormal, the gamma, the log-Pearson III and the Gumbel extreme-value distribution. Gumbel (1958) was the first to use extreme-value theory for analysis of flood frequencies (Viessman *et al.*, 1977; Macmahon and Srikanthan, 1981; Shrader *et al.*, 1981).

According to Yevjevich (1972) and Miroslava (1992) the extreme value distribution gives a good fit to extreme values of rainfall. In this study the pmp was computed from the Gumbel distribution,

$$P(p) = \bar{P} - \frac{S_N}{\sigma_N}\left[P_N - \ln\ln\left(\frac{1}{1-p}\right)\right] \tag{1}$$

where P(p) is the probable maximum rainfall for a probability p, $\bar{P}$ and Sp are the mean and the standard deviation of the sample, and P_N, and σ_N are the mean and the standard deviation of the population.

The pmp of the Tecolutla watershed was estimated using the records of daily rainfall at 24 stations (Figure 1). Samples of maximum rainfall were obtained for the 11 most unfavorable storms which occurred on the watershed for the last twenty years. The maximum storms, with duration of one to five days, for each station were adjusted to Eq(1). The average maximum rainfall in the watershed was obtained using the Thiessen method (Figure 2), which considers the area of influence of each station.

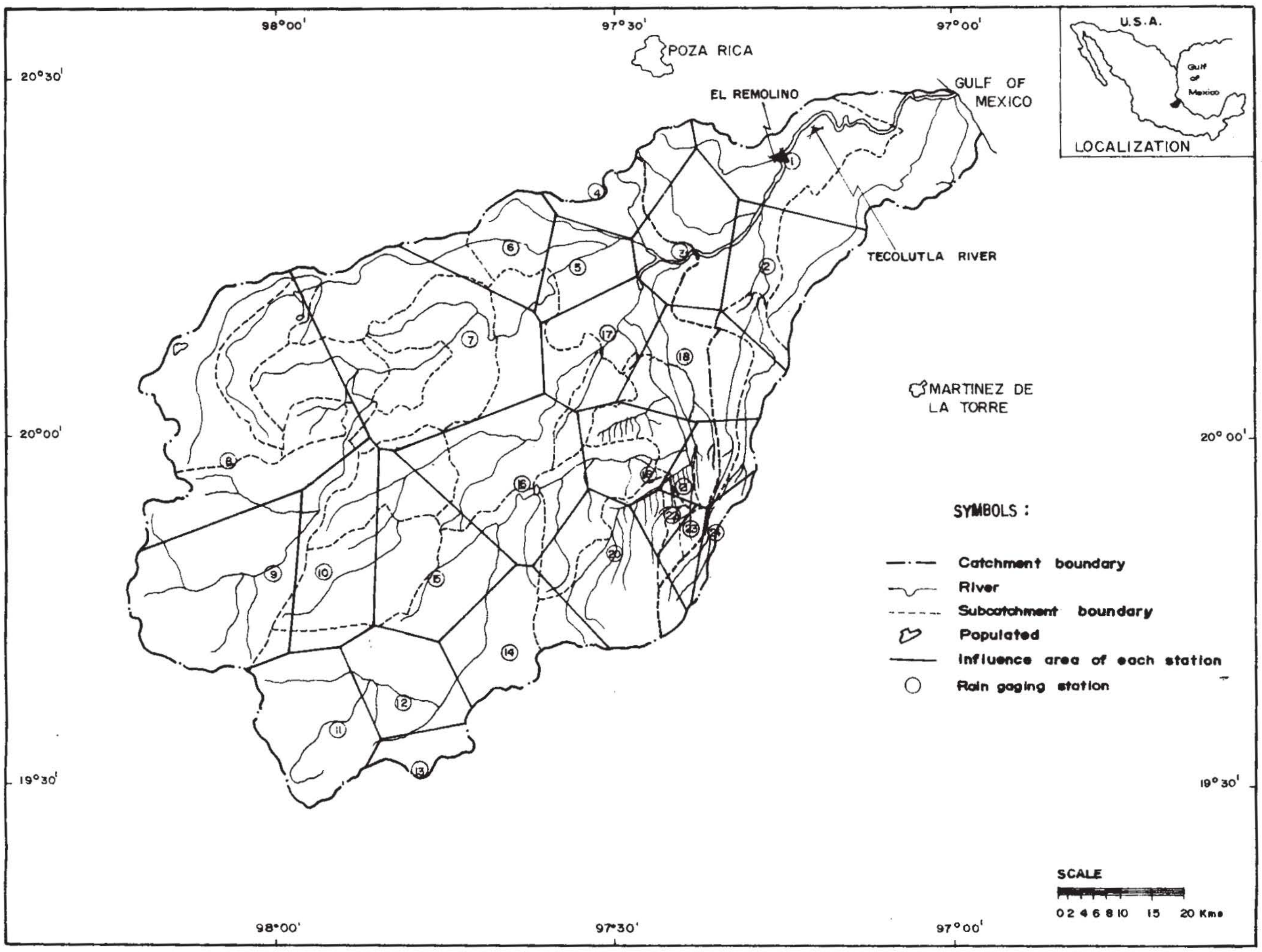

Fig.1. The Tecolutla River watershed showing the influence of each station after the Thiessen method.

4. INSTANT UNIT HYDROGRAPH MODEL

The unit hydrograph model is a technique for forecasting runoff from rainfall data. It is based on correlations between the observed volumes of runoff and rainfall. This method does not depend on physical laws but on an empirical response function.

The output from a time-invariant linear system may be represented by a convolution integral,

$$Q(t) = \int_0^t F(\tau)\, U(t-\tau)\, d\tau \qquad (2)$$

where $F(\tau)$ and $Q(t)$ represent input at time τ and output at time t, and $U(t,\tau)$ is the system response to a unit impulse.

In many cases the hydrological data are available only in discrete form. Then the hydrograph is the superposition of the unit hydrographs generated by each elementary storm (Figure 3). Thus, if $U_1, U_2, \ldots, U_{NQ}$ are the unit hydrographs corresponding to a rainfall of duration Δt, the runoff generated for a storm hyetograph of ordinates $P_1, P_2, \ldots, P_{NP}$ may be estimated using eq.(2) in discrete form,

$$Q_i = \sum_{k=1}^{i} U_k\, P_{i-k+1} \quad , i = 1, 2, \ldots, NQ \qquad (3)$$

where Q_i is the direct runoff for time interval i, NQ is the total number of readings, NP is the number of ordinates of effective precipitation, and NU = NQ - NP + 1 is the number of ordinates in the unit hydrograph.

Due to the sensitivity of Eq(3) to the first ordinates of effective average rainfall (these values are less exact when estimated), and to a lesser degree due to the nonlinearity of the rainfall-runoff relation, the ordinates of unit hydrograph may be negative. According to Fuentes *et al.*, (1981) this problem can be reduced considerably if the least-square error $Z = \sum(Q_o - Q_e)^2$ of the direct runoff is minimized. Here Q_o is the observed direct runoff and Q_e is the estimated runoff. Minimization of errors generates the following equations

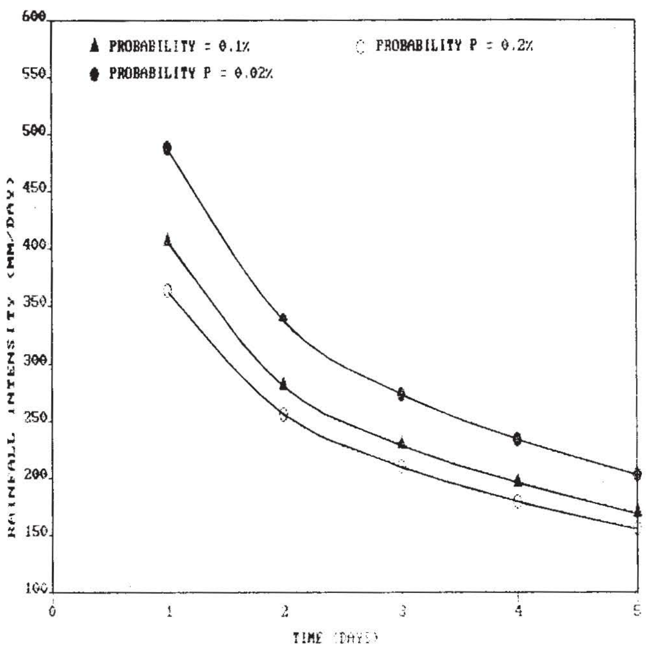

Fig.2. Probable maximum rainfall for probabilities of 0.2%, 0.1% and 0.02%.

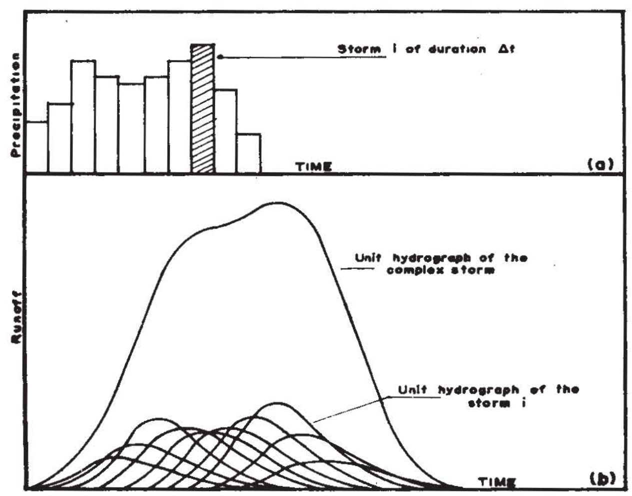

Fig.3. Unit hydrograph of a complex storm: (a) elementary storms of duration Δt; (b) superposition of runoff hydrographs for all storms.

$$\sum_{i=1}^{NQ} U_i \, \Phi PP(i-j) = \Phi PQ(j-1); \quad j = 1,2,\ldots,NU \tag{4}$$

where

$$\Phi pp(\sigma) = \begin{cases} \sum_{i=1}^{NP} p_i \, p_{i+\sigma}, & \text{for } \sigma = 0, 1, \ldots, NP-1 \\ 0, & \text{for } \sigma \geq NP \end{cases}$$

$$\Phi_{PQ}(\tau) = \begin{cases} \sum_{i=1}^{NQ} P_i Q_{i+\tau} & \text{for } \tau = 0, 1, \ldots, NU-1 \\ 0 & \text{for } \tau \geq NU \end{cases}$$

with $\phi PP(-\tau) = \Phi PP(\tau)$, and P = 0 for i > NP.

5. INSTANT UNIT HYDROGRAPH FOR EL REMOLINO

After reviewing the hydrographs for the most unfavorable storms on the watershed of theTecolutla River for the last twenty years, the storm of 24 to 31 August 1981 was picked as representative of the region (Table 1).

The columns 3 and 4 of Table 1 suggest that NQ = 7, NP = 4 and NU = 4. Replacing these values in eq.(4) we obtain the eqs.(4a) for El Remolino station.

$$\begin{aligned} 5.3U_1 + 3.3U_2 + 1.5U_3 + 0.08U_4 &= 509.2 \\ 3.3U_1 + 5.3U_2 + 3.3U_3 + 1.5U_4 &= 467.3 \\ 1.5U_1 + 3.3U_2 + 5.3U_3 + 3.3U_4 &= 314.4 \\ 0.08U_1 + 1.5U_2 + 3.3U_3 + 5.3U_4 &= 118.7 \end{aligned} \tag{4.a}$$

The solution is $U_1 = 70.6$, $U_2 = 32.2$, $U_3 = 19.2$ and $U_4 = 0.2$, which defines the ordinates of the unit hydrograph (Figure 4). This solution was obtained using a computer program written in Turbo-Basic.

6. RESULTS AND CONCLUSIONS

The expected maximum rainfall for probabilities of 0.2%, 0.1% and 0.02% (return periods of 500, 1000 and 5000 years) according to the Gumbel model are shown in Figure 2. From the rainfall shown in Figure 2 (curve p =0.1%) the average infiltration (Φ= 32.3 mm/day), estimated by Pereyra and Hernández (1989) for this watershed, was subtracted in order to obtain the design effective rainfall (Figure 5). Using the ordinates of the unit hydrograph (Figure 4), the design flood was estimated using Eq(3). A peak flow rate of 31,627.7 m^3/s was obtained (Figure 6). This value is slightly higher than the value of 26,350 m^3/s estimated by Pereyra and Hernández (1989) using the traditional unit hydrograph model. Figure 6 suggests that the peak flow rate is reached rapidly; this is because of the steep slopes in the upper watershed. When designing a hydraulic structure, for example, the elevation and the spillway must be designed for this value.

In conclusion, it is possible to use the instant unit hydrograph for time intervals Δt of one day, for large watersheds. This enables us to use records of daily rainfall that are usually available in Mexico.

Table 1

Hyetograph and Hydrograph ordinates generated by the storm of August 1981.

Time (days)	Rainfall (mm)	Direct Runoff (m^3/s)	Effec.Precip. (mm)
23	11.1	0.0	0.0
24	33.8	0.0	1.5
25	58.1	1050.0	25.8
26	73.2	1400.0	40.9
27	86.7	2350.0	54.4
28	22.1	6900.0	0.0
29	1.8	2250.0	0.0
30	1.5	750.0	0.0
31	0.7	350.0	0.0

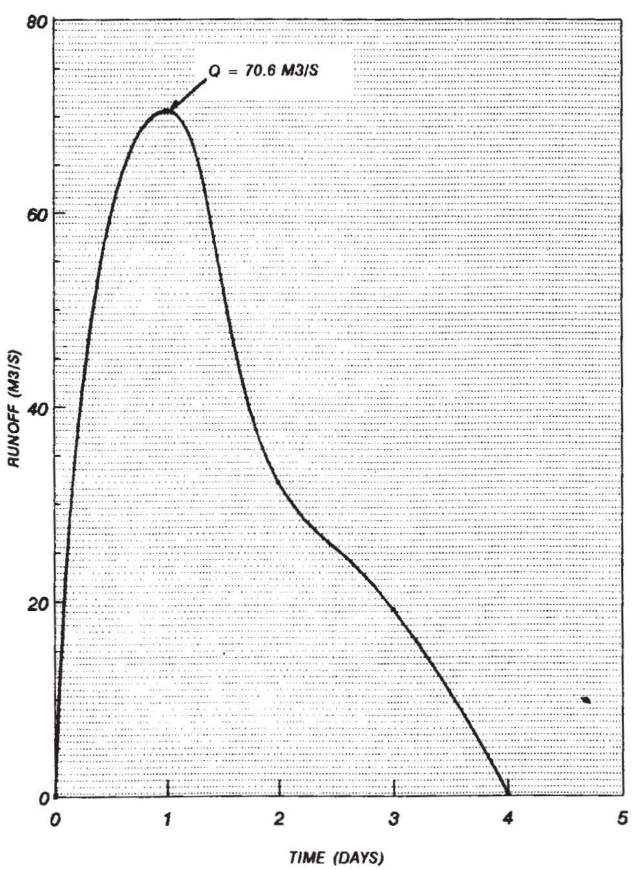

Fig.4. Unit hydrograph of the storm in August 1981.

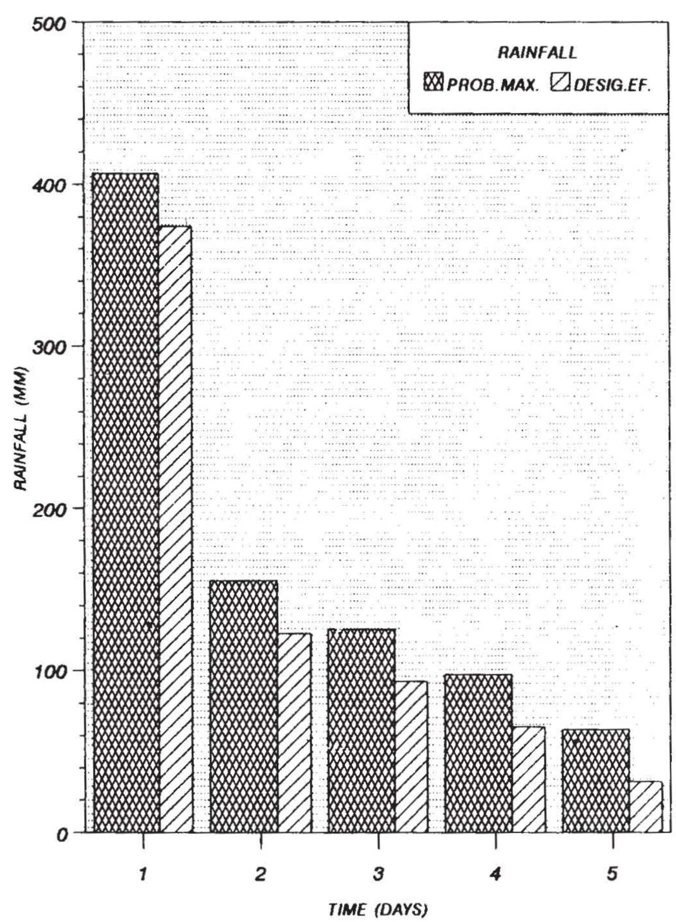

Fig.5. Probable maximum rainfall and design effective rainfall for a probability of 0.1% (return period equal to 1000 years).

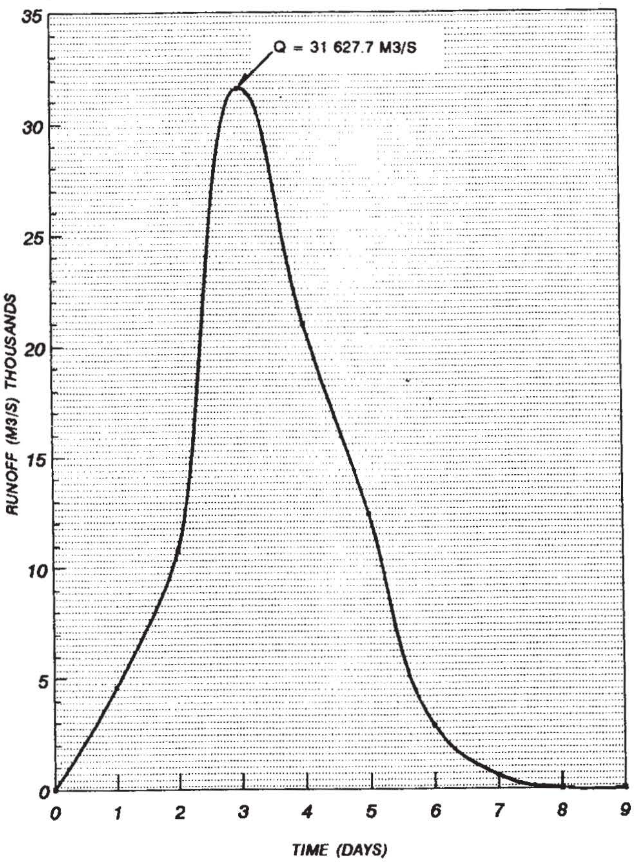

Fig.6. Design flood hydrograph for a probability of 0.1%.

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