Geofísica Internacional (1995), Vol. 34, Num. 4, pp. 395-398

Estimation of orbital velocities of internal waves of short period and large amplitudes

A. E. Filonov[1] and K. D. Sabinin[2]
[1] *Depto. de Física, Universidad de Guadalajara, México.*
[2] *Inst. of Acoustics, Russian Academy of Sciences, Moscow.*

Received: September 29, 1994; accepted: June 19, 1995.

RESUMEN

Se expone un método para la determinación de la componente horizontal de la velocidad orbital de las ondas internas de gran amplitud desde una embarcación en movimiento. El método se fundamenta en el conocimiento de la respuesta en frecuencia del frenado de la embarcación y del análisis del registro en bitácora de la oscilación de la velocidad en el campo de las ondas internas. Se discuten datos obtenidos en la zona de unos bajos en la parte central del océano Indico.

PALABRAS CLAVE: Ondas internas, método de medición.

ABSTRACT

A measurement method is presented to determine the horizontal orbital velocity component of the internal waves of the sea with very large amplitude from a ship in motion. The method is based on the frequency response of the slowdown of the ship and the analysis of oscillations recorded on the relative velocity log while sailing across a field of internal waves. Application to data collected over some banks in the Central Indian Ocean are discussed.

KEY WORDS: Internal waves, measurement method.

INTRODUCTION

During the spring of 1987, measurements of short-period large-amplitude oscillations of the thermocline were carried out over the sill between the banks of Nazaret and Saya-de-Malya (Sabinin *et al.*, 1992) on board of R.V. "Academician Nikolay Andreyev". These measurements were made using two towed systems of measurement (TSM). Each system had three thermistors, at depths of 60 m, 90 m and 120 m on the average. Temperature distribution sensors with a length of 30 m were located between each pairs of thermistors. At the lower end of the system, at a depth of 120 m, a pressure sensor and a ballast weight were located. The time response was 1 min., and the speed of towing was 5-6 knots. One TSM was towed directly from the stern and is called "ship TSM" in this paper; the other TSM was towed 500 m behind the "ship TSM". The second array is called "wake TSM".

The ocean surface shows the presence of internal waves by long alternating strips of slicks and ripples which appear on the radar. Thus a ship with TSM can easily find and track internal waves. The internal waves are generated by tidal currents across the sill and propagate towards the open ocean in the form of wave packets. Each packet consists of 5-7 solitons with a maximum amplitude of 40 m and a maximum length between crests of 1 km.

Photographs of wave packets were taken on the screen of the radar and by TSM data. Measurements were made normally to the wave fronts, keeping continuous records of the ship's log velocity (Sabinin *et al.*, 1992).

The data obtained from the "ship" and "wake" arrays during the transects differ considerably. Variations are due to horizontal surface currents; thus the two TSMs crossed each wave structure at different speeds. When the ship travels in the same direction as the orbital currents of the internal waves, its speed is incremented and the thermistors of the TSM systems move upward by 5-7 m from their average position (Figure 1a). When the currents move against the ship, the sensors are displaced downward (Figure 1b). When sailing into the advancing wavefront the ground speed increases over the crests and decreases over the troughs. However, the net speed is not simply the sum of the speed relative to the water plus the speed of the surface current since the ship's mass is very large and introduces a phase difference due to inertia. If the inertial properties of the ship are known and if measurements of the oscillation of the relative speed of the ship are made, the amplitude and period of the orbital velocity can be determined.

METHOD

The inertia of the ship is related to its velocity by the differential equation

$$m\frac{dU_s}{dt} = F_t - kU_s^2 \quad , \tag{1}$$

where $m\frac{dU_s}{dt}$ is the inertia force, F_t is the traction, kU_s^2 is the water resistance, U_s is the velocity of the ship relative to still water, m is the ship's mass and k is the coefficient of water resistance.

For $F_t = 0$, equation (1) has the following solution:

$$U_s(t) = \overline{U}_s\left[1 + \frac{k}{m}\overline{U}_s(t - t_i)\right]^{-1} \quad , \tag{2}$$

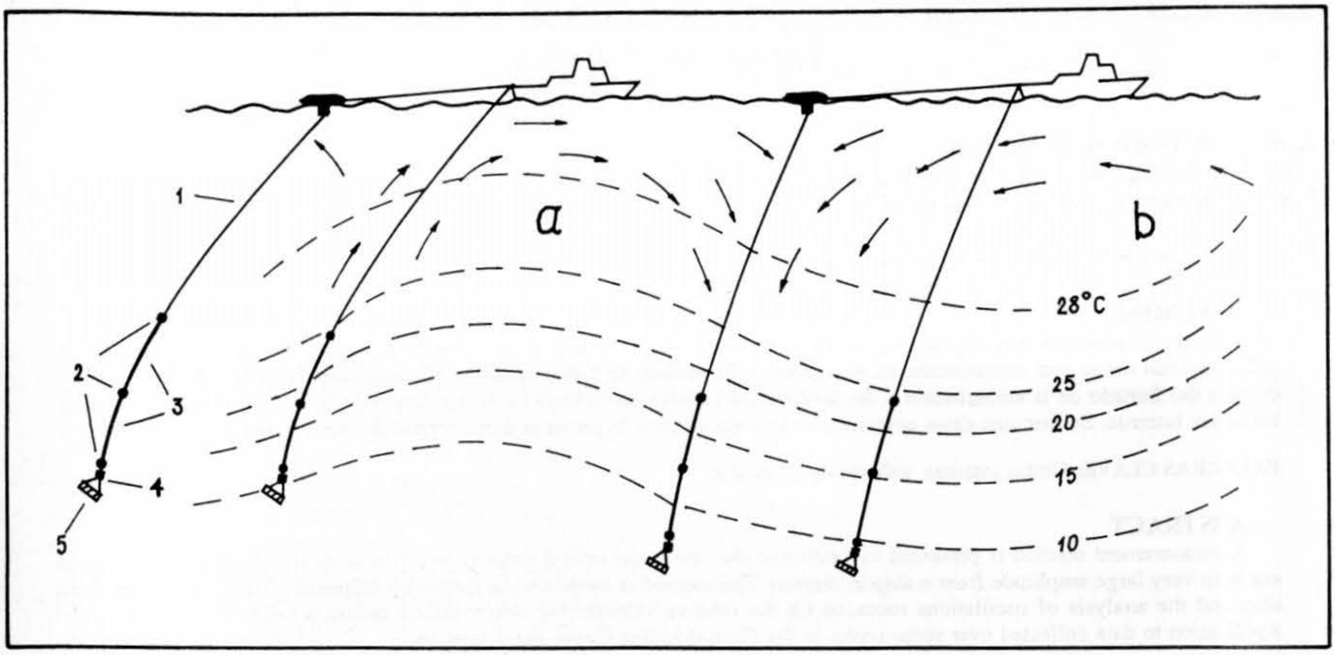

Fig. 1. Schema of the position of the towing systems TSMs on the thermocline, at the moment in which the ship is located on the field of orbital currents (a) with the same direction and (b) against the front of the internal waves of large amplitude. In the left part of the figure is shown a diagram of a towed system TSM: 1-cable, 2-thermistors, 3-sensors of temperature distribution, 4-pressure sensor, 5-submersion weight.

where $\overline{U}_s$ is the ship's velocity for slow-down time t_i, when the power is shut off.

When the speed is slow, as it was during our measurements, the dependence of resistance to motion is well described by the linear equation

$$\frac{dU_s}{dt}+\frac{1}{\tau}+U_s=0 \ , \tag{3}$$

which has the solution

$$U_s(t)=\overline{U}_s\exp(-t/\tau) \ , \tag{4}$$

where τ is the inertial lag of the ship.

The velocity recorded by the ship log is the difference between the ground speed of the ship and the surface horizontal velocity of the wave current:

$$U_l(t)=U_s(t)-U_w(t) \ , \tag{5}$$

where $U_l(t)$ is the ship log speed, $U_w(t)$ the wave orbital velocity and $U_s(t)$ is the convolution of a weighted function (characterizing the ship's inertia) with the orbital current. Let us write equation (4) in the form $U_s(t)/\overline{U}_s=\exp(-t/\tau)$. Then the ship velocity is

$$U_s(t)=\frac{1}{\tau}\int_0^\infty \exp(-t'/\tau)U_w(t-t')\ dt' \ . \tag{6}$$

Let $S_l(\omega)$, $S_s(\omega)$ and $S_w(\omega)$ be the Fourier transforms of $U_l(t)$, $U_s(t)$ and $U_w(t)$. From (6) we find

$$S_s(\omega)=(1+i\omega\tau)^{-1}S_w(\omega) \ , \tag{7}$$

where $(1+i\omega\tau)$ is the complex characteristic frequency of the ship's inertia at low speed.

From equation (5) we may write the equation in the frequency domain as $S_l(\omega)=S_s(\omega)-S_w(\omega)$. Using equation (7), we find the spectrum of the horizontal component of the orbital velocity

$$S_w(\omega)=S_l(\omega)/\left[(1+i\omega\tau)^{-1}-1\right] \ . \tag{8}$$

Finally, by inverting equation (8) we obtain the orbital velocity

$$U_w(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} S_w(\omega)\exp(i\omega\tau)\ d\omega \ . \tag{9}$$

Thus, if the inertial properties of the ship (τ) and the log records (U_l) are known, the horizontal orbital velocity of the internal waves in the thermocline can be found.

When the internal waves are large, the orbital current field often shows near-sinusoidal fluctuations, such as $U_w(t)=\overline{U}_w\sin(\omega t)$ where $\overline{U}_w$ is the amplitude of the current velocity. If the ship moves with average speed $\overline{U}_s$ in a field of such waves the speed of the ship may be found as

$$U_s(t) = \overline{U}_s + A(\omega)\overline{U}_w \sin[(\omega t + \varphi(\omega)] \ . \qquad (10)$$

The functions $A(\omega)$ and $\phi(\omega)$ are estimated by $A(\omega)= (1+\omega^2\tau^2)^{-\frac{1}{2}}$ and $\varphi(\omega)=\arctan(-\omega\tau)$. They represent the amplitude and the phase frequency of the inertial properties of the ship.

Substituting equation (10) in (5) and solving for $\overline{U}_w$, we obtain the expression for the amplitude of the horizontal orbital velocity

$$\overline{U}_w = \frac{U_l(t) - \overline{U}_s}{B(\omega)\sin[\omega t + \gamma(\omega)]} \ , \qquad (11)$$

where $B(\omega)$ and $\gamma(\omega)$ are given by

$$B(\omega) = \left\{1 + A^2(\omega) + 2A(\omega)\cos[\pi - \varphi(\omega)]\right\}^{1/2} \ ,$$

and

$$\gamma(\omega) = \arctan\left\{\frac{A(\omega)\sin[\varphi(\omega)]}{A(\omega)\cos[\varphi(\omega)] - 1}\right\} .$$

These functions describe the amplitude and phase changes of the velocity oscillations of the ship's log relative to the amplitude and phase of the horizontal orbital velocity of the waves.

DISCUSSION AND RESULTS

In Figure 2, we show the slowing-down response curves of R.V. "Academician Nikolay Andreyev". The calculated values of $U_s(t)$ from equation (4) are shown for the maximum initial speed $\overline{U}_s$=16,5 knots and τ = 3.5, 4 and 5 min. The best approximation of the slowing-down response curve for the section with low speed was obtained for τ = 4 min. This value was used in all calculations. In the same figure, the amplitude-phase characteristics of the ship (A (ω), $\varphi(\omega)$) and log ($B(\omega)$, $\gamma(\omega)$) are also shown for a frequency range between ∞(t = 0) and 5.2 x 10^{-2} rad/sec (t = 120 min).

The amplitudes and the phase frequency characteristics of the ship and the speed log are situated in the orbital current almost in phase opposition. Suppose that the ship steers into the advancing wave fronts, without power, and crosses a crest. The direction of the ship movement and the orbital currents match and the ship is carried by the currents so that its ground velocity increases. If the speed of the ship is greater than the phase velocity of the wave, the ship will eventually be ahead of the crest and will fall into a field of contrary orbiting currents beyond the trough; the speed of the ship relative to the ground decreases. The ship's log shows an increase and then a decrease of the ship velocity relative to the water, with some phase lag. These speed changes occur for each wavelength and depend on the mass, velocity, form of the hull, and period of the orbital currents. Figure 2 shows that the ship is little affected by currents with a period of 10s associated to the swell. Because of inertia, the ship practically does not respond to high frequency oscillations. Internal waves on long space-

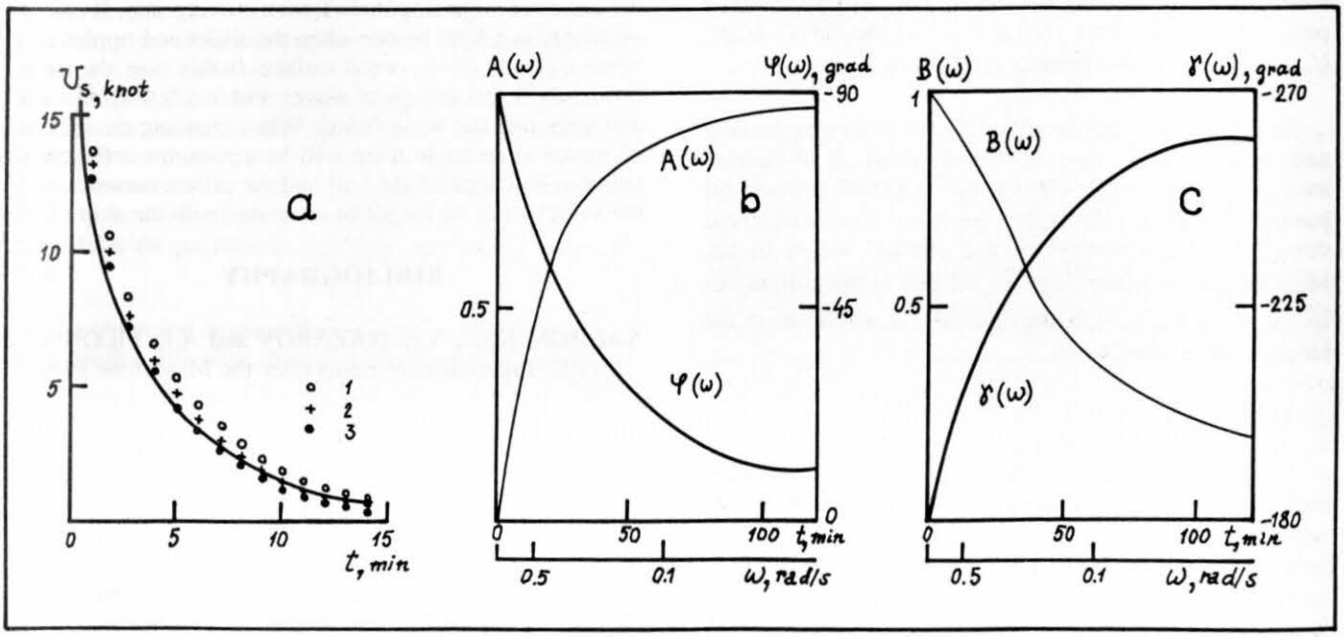

Fig. 2. (a) Experimental curve of slow-down of R.V. "Acad. N. Andreyev". The approximation of the same curve by formula (4) for τ=3.5 min (5), τ=4 min (6) and τ=5 min (7), shown by symbols. (b) Amplitude-phase frequency slowing-down response of ship's A(ω), $\varphi(\omega)$ and (c) log readings B(ω), $\gamma(\omega)$, from the period of horizontal orbital currents.

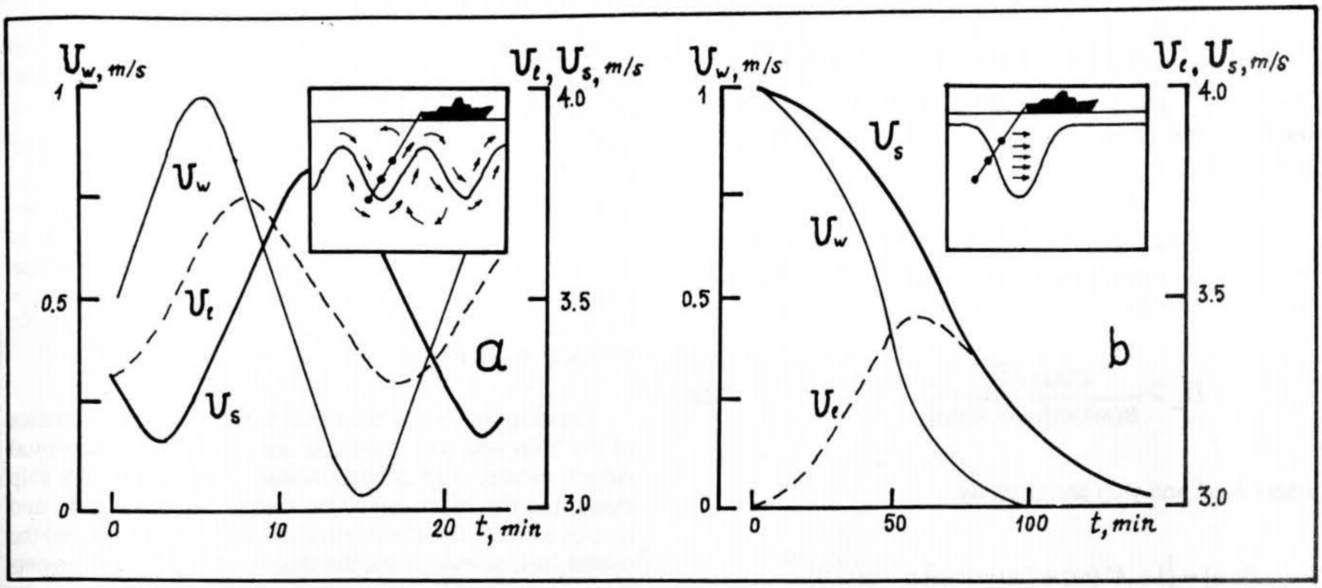

Fig. 3. Ship velocity changes (U_s), according to log (U_l) in the field of orbital currents (U_w) of the internal sinusoidal waves (a) and (b) when the ship track is normal to the internal soliton front.

time scales and orbital currents with periods of more than 5-10 minutes will strongly influence the moving ship and the parameters of the waves measured with the towed TSM.

Figure 3a shows an example of a calculation of the change in velocity of R.V. "Academician Nikolay Andreyev". It also shows the ship's log when the ship is sailing at a speed of 3.5 m/s (nearly 7 knots), in a field of sinusoidal orbital currents with an amplitude of 1 m/s and a period of 20 min. This is illustrated by the curves $U_s(t)$, $U_w(t)$ and $U_l(t)$ of the velocity oscillations.

In Figure 3b, a similar calculation is shown as the ship sails at a speed of 3m/s, across the front of an internal soliton having a profile close to $\eta(t) = \eta_o \tanh (t/2\tau w)$. All parameters for the calculations are close to those observed during the measurement of the internal waves in the Maskarene underwater ridge: $\overline{U}_w$=1 m/s is the soliton velocity, τ_w = 8.3 min. is the "period", λ_w=1000 m is the length scale of the soliton.

Thus the relative velocity (ship to soliton) is 2 m/sec, and the ship crosses the soliton field for more than 8 minutes, moving with the orbital currents. The recorded mean velocity of the ship was U_s=3 m/s. When the ship started to cross the front of the soliton, its ground velocity began to decrease sharply due to the decreasing orbital currents. However, due to the inertia, its velocity decreased less than the currents due to the soliton; at this moment, the log began to show a sharp increase in velocity (around 0.4 m/sec) which later dropped smoothly to the usual value of 3 m/s, as the ship completely moved out of the orbital currents. This shows the possibility of an original method for determining the velocity of the surface currents associated with short-period waves of large amplitudes. The advantage of this method is that the orbital velocity during the movement of the ship can be measured simultaneously with other wave parameters, using the radar and the towed instruments. This allows the determination of the spatial-temporal evolution of the parameters of interval waves in groups.

In conclusion, this method is useful to measure internal waves of large amplitude from a moving ship. It can be used only in a light breeze when the slicks and ripples can be seen clearly on the ocean surface. In this case, the radar can easily detect groups of waves within a few kilometers and steer into the wave fronts. When crossing the groups of waves at an angle there will be a complex interaction between the shape of the hull and the orbital currents, and the velocity can no longer be estimated with the ship's log.

BIBLIOGRAPHY

SABININ, K.D., A.A. NAZAROV and A.E. FILONOV, 1992. Internal-wave trains over the Mascarene Ridge. *Izvestia of Russian AN, Ph. of Atmosph. and Ocean.* *28*, 6, 625-633.

A. E. Filonov[1] and K. D. Sabinin[2]

[1] *Depto. de Física, Universidad de Guadalajara, Apdo. Postal 4-079, 44421 Guadalajara, Jal., México.*

[2] *Inst. of Acoustics, Russian Academy of Sciences, 117936, Moscow B-36, Ul. Shvernik 4.*