

## COMMUNICATION

### *A NUMERICAL METHOD FOR SIMULATION OF CONVECTIVE PLUME FIELD DISPERSION*

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#### RESUMEN

Al simular dispersión en situaciones complejas, la descripción simple de movimientos no resueltos en términos de una función espectral y distribución de velocidad, puede ser insuficiente debido a que se ignora la información sobre fases relativas de las diferentes componentes espectrales. Esta información de la fase se puede retener directamente generando explícitamente los vientos mediante un modelo de sistemas significativos pero no resueltos. Como ejemplo de un sistema tal, se considera aquí, el de plumas convectivas en una capa límite inestable.

Los resultados muestran que algunos de los aspectos cualitativos de la dispersión encontrada mediante la simulación física de una capa límite inestable, que no pueden ser explicados por una aplicación racional de la aproximación de difusividad de "eddies", se explican empleando el método de Monte Carlo. Es necesario un entendimiento más completo de la naturaleza física del campo de una pluma convectiva, antes de esperar obtener un entendimiento cuantitativo del problema.

#### ABSTRACT

In simulating dispersion in complex situations a simple description of the unresolved motions in terms of a spectrum function and velocity distribution may become insufficient because information on relative phases of different spectral components is ignored. This phase information may be retained most directly by explicit generation of winds given by a model of the significant, but unresolved, systems. An example of such a system, the convective plumes of the unstable boundary layer, is considered.

The results show that some of the qualitative aspects of dispersion found in a physical simulation of the unstable planetary boundary layer, which can not be explained by a rational application of the eddy diffusivity approach, can be explained by employing the Monte-Carlo approach. A more complete understanding of the physical nature of convective plume field is needed before quantitative agreement can be expected.

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In simulating dispersion it is apparent that it would frequently be desirable to explicitly handle certain "systems" or energy containing eddies as distinct entities. The flow properties accompanying such systems introduce unique dispersion characteristics. The convective plumes of the planetary boundary layer are such systems and are treated in this paper.

Basically the role of the plumes in dispersion is in transporting the pollutant particles upward in the planetary boundary layer. Outside the plumes, air subsides to compensate for this upward motion. Continuity demands convergence toward the plumes at low levels and divergence at high level.

Let a plume of radius  $R$  be surrounded by a compensating subsidence region concentric with the plume. For cylindrical symmetry the equation of continuity in cylindrical polar co-ordinates is

$$\frac{u}{r} + \frac{\partial u}{\partial r} = \frac{\partial w}{\partial z}$$

where  $u$  is the radial velocity component and  $w$  is the vertical velocity component. Solving this differential equation by the method of separation of variables with  $u=u(z).u(r)$  and  $w=w(z).w(r)$ , the equation becomes

$$\frac{u(r)}{r} + \frac{du(r)}{dr} = Mw(r)$$

and

$$\frac{dw(z)}{dz} = -Mu(z)$$

where  $M$  is a separation constant. Profiles of  $u(z)$ ,  $u(r)$ ,  $w(z)$  and  $w(r)$  need to be specified and simple analytic expressions be adopted which meet the physically determined boundary conditions.

For the radial components the boundary conditions are:

- (1)  $w(r)$  is maximum at  $r=0$ .
- (2)  $w(r)$  becomes constant at the outer edge of the subsidence region.
- (3)  $w(r)$  is positive (upward) within the plume and negative (downward) outside the plume. It is zero at the plume edge.
- (4)  $u(r)$  is zero at the plume centre.
- (5)  $u(r)$  is zero at the outer edge of the subsidence region.
- (6) The net vertical flux at any level is zero.

The differential equation and boundary conditions are satisfied by the relations

$$w(r) = J_0(\alpha r) \qquad 0 < \alpha r < 3.83171$$

$$u(r) = -M J_1(\alpha r)$$

where the  $J$ 's are Bessel functions. The  $\alpha$  is defined so that the first zero of  $J_0(\alpha r)$  falls at the edge of the plume.

Vertical profiles are constrained by the following boundary conditions:

- (1)  $u(z) = 0$  at the earth's surface.
- (2)  $w(z) = 0$  at the earth's surface.
- (3)  $w(z) = 0$  at the top of the boundary layer.
- (4)  $w(z)$  is positive.

Deardorff and Willis (1975) have simulated dispersion from an elevated point source in an unstable planetary boundary layer using a water tank. In fig. 1 their results for centerline concentration as a function of height and downstream distance are exhibited. Significant features of the water tank result are:

- a. The height of maximum concentration lowers initially as most of the pollutant is released into the subsiding air outside the plumes.
- b. After the initial lowering, the concentration maximum rises rapidly to about three-quarters of the boundary layer height. Most of the pollutant has been entrained into the convective plumes and has

been transported aloft. The region of strongest entrainment to the plumes appears to be in the lowest tenth of the planetary boundary layer. Detrainment appears to start at about half the planetary boundary layer height.

- c. The maximum lowers to the half height of the boundary layer as the pollutant is detrained from the plume, subsides and becomes uniformly mixed within the boundary layer.

This result provides information, additional to that of the boundary constraints, to aid in choosing the vertical profile. Specifically, the  $u(z)$  appears greatest toward the plume at lower levels and appears to go to zero while  $w(z)$  becomes a maximum, at about half the boundary layer height.

Fig. 2 shows a  $w(z)$  profile satisfying all the above constraints. The rapid variation in the lower part of the boundary layer makes this a difficult profile to fit with an analytical function. In addition, observation and theory do not appear well enough reconciled to allow a profile in  $z$  to be established. In this study, where the emphasis is to illustrate the approach rather than to reproduce details, the dispersion produced by a simple profile meeting only some of the criteria has been calculated.

The assumption of a constant  $R$  which is employed here has been found to be quite reasonable by Telford (1970). With the specifications above particle trajectories through the plume system can be calculated.

In application to dispersion in the unstable planetary boundary layer the motion of particles is calculated from components given by resolved winds, the plume model and the small scale turbulence model. Horizontal winds are formed from components given by the resolved and small scale turbulence components. Vertical winds use a component from the plume model in addition. Parameters from the larger scales may be required in the smaller scale parameterizations. Thus, for example, the height of the convective boundary layer used in the plume model can be obtained from the resolved scale.

The result of Monte Carlo simulation of dispersion from an infinite

line source normal to the mean wind in an unstable boundary layer with 5000 trajectories calculated is shown in fig. 3. In this simulation the mean (resolved) wind is assumed uniform in direction and speed with position, which is in accordance with the results of Telford (*loc. cit.*). Lacking definitive information on the vertical profile needed for the plume circulation specification the expression

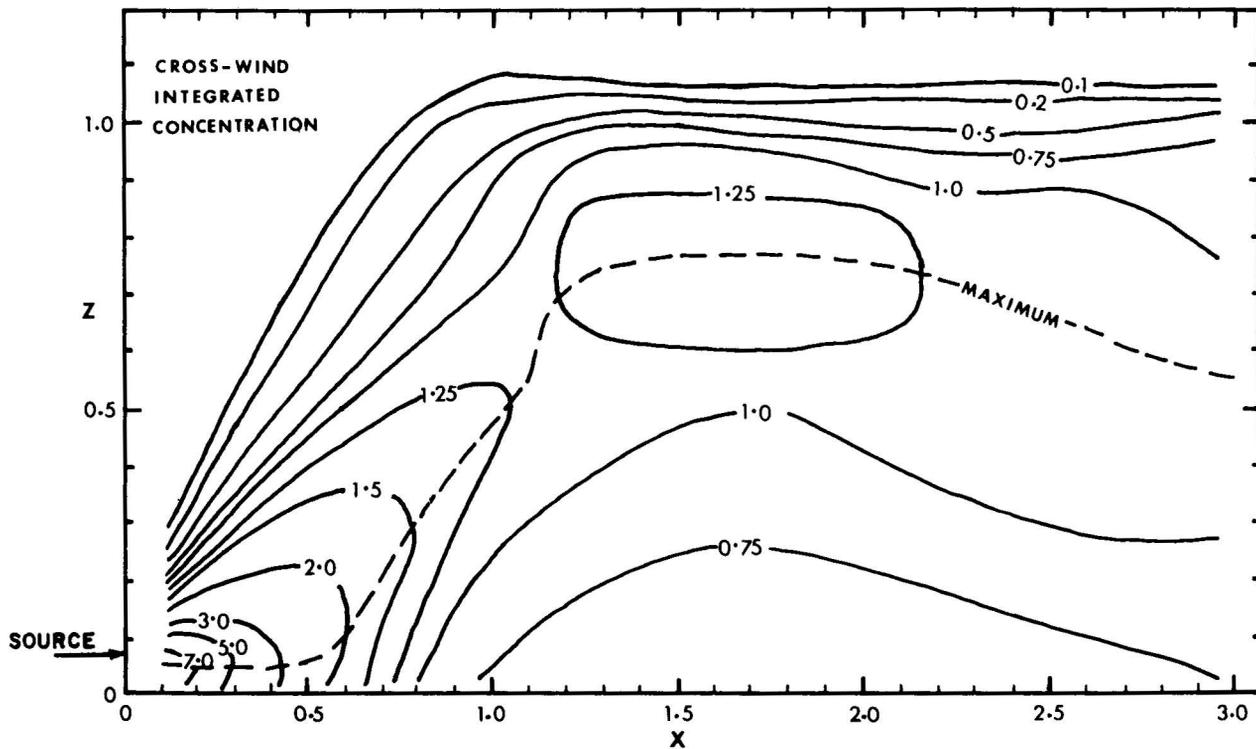
$$w(z) = 20 z (1-z) (1-z/2) \quad 0 \leq z \leq 1 \quad (z=1 \text{ at the top of the convective layer})$$

is employed, which fits boundary conditions 2, 3 and 4, and has a maximum of  $w(z)$  at about 0.4 of the convective layer height.

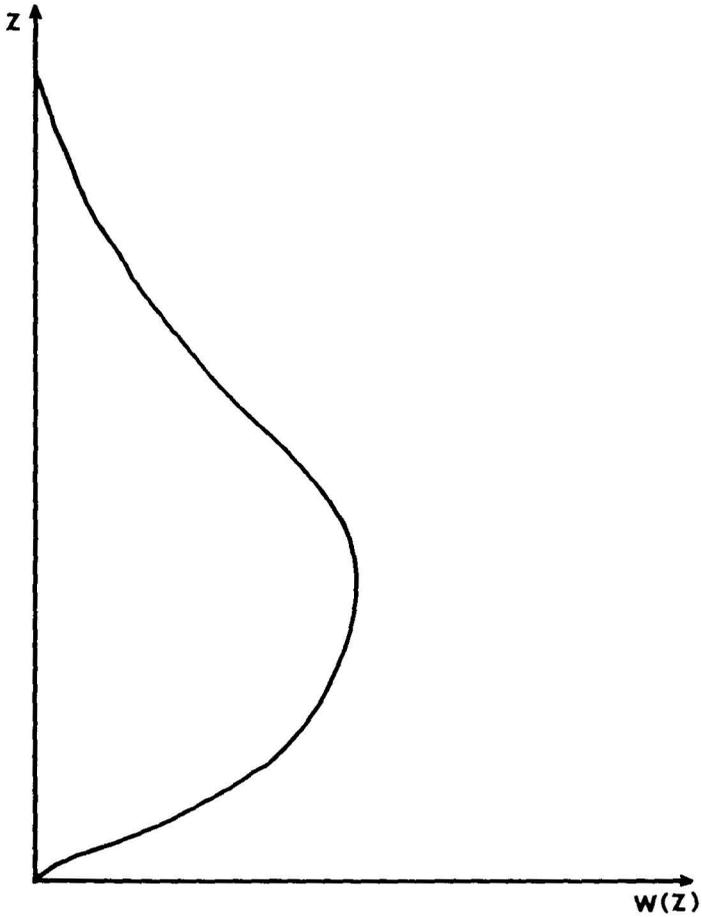
It is interesting to compare the result of this simulation with the result for cross-wind integrated concentration from a point source found in the water tank experiments simulating dispersion in an unstable planetary boundary layer of Deardorff and Willis (*loc. cit.*). Both simulations show the maximum concentration initially lowering too close to the surface. This is because most of the particles are released into the subsident layer outside the plumes. These particles gradually become entrained into the plumes which carry them aloft. Thus an elevated maximum forms in both simulations. Differences in the concentration patterns can be attributed to lack of definitive knowledge of the profile of vertical velocity in the plumes.

#### ACKNOWLEDGEMENTS

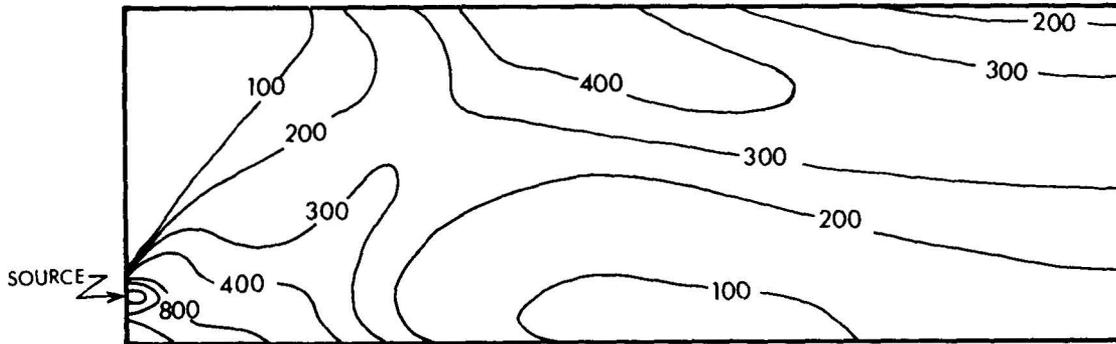
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**FIG. 1 : SMOOTHED VALUE OF DIMENSIONLESS CROSS-WIND INTEGRATED CONCENTRATION AS A FUNCTION OF HEIGHT AND DOWNSTREAM DISTANCE (AFTER WILLIS AND DEARDORFF, 1975).**



**FIG.2: PLUME VERTICAL VELOCITY PROFILE SATISFYING BOUNDARY CONSTRAINTS.**



**FIG. 3 : AXIAL CONCENTRATIONS IN THE PLANETARY BOUNDARY LAYER FOR DISPERSION FROM AN INFINITE LINE SOURCE SIMULATED BY THE MONTE - CARLO METHOD. ISOPLETH VALUES ARE PROPORTIONAL TO CONCENTRATION.**

*BIBLIOGRAPHY*

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