GEOFISICA INTERNACIONAL

REVISTA DE LA UNION GEOFISICA MEXICANA, AUSPICIADA POR EL INSTITUTO DE GEOFISICA DE LA UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO

Vol. 16 México, D. F., 1o. de abril de 1976 Núm. 2

ON LARGE-SCALE QUASI-STATIONARY WAVES

LODOVICO LA VALLE*

RESUMEN

Se ha demostrado que las ondas casi estacionarias de gran escala dependen esencialmente de la producción de vorticidad en los niveles intermedios de la atmósfera (La Valle e Celentano, 1975).

El presente estudio de la producción de vorticidad en $P_S/2$ durante cuatro meses del año Geofísico Internacional en el hemisferio norte, indica que la orografía explica cerca del veinte por ciento de la producción de vorticidad sobre el terreno en declive, mientras que la advección de temperatura y el calentamiento diabático en la mitad inferior de la atmósfera son responsables del cincuenta por ciento de la producción de vorticidad sobre el mar o la tierra plana.

ABSTRACT

The large-scale quasi-stationary waves have been demonstrated to depend essentially on the vorticity production at the intermediate levels of the amosphere (La Valle e Celentano, 1975).

The present study of the vorticity production at $P_S/2$ during four months of the I.G.Y. over the northern hemisphere indicates that orography explains about 20% of vorticity production over sloped ground, while temperature advection and diabatic heating in the lower half of the atmosphere account for 50% of the vorticity production over or flat land.

* *Servizio Meteorologico dell'Aeronautica Militare. Piazzale degli Archivi, Roma EUR, Italia.*

1. INTRODUCTION

In 1939 Rossby (Rossby and collaborators, 1939) demonstrated the existence of free waves superimposed upon the zonal current. As the length of these waves is shorter than the distance between the permanent solenoidal fields, each solenoidal field, in cooperation with orography, will excite a system of free perturbations, which in turn can generate frontogenesis.

Ten years later Charney and Eliassen (1949) simulated fairly well the stationary waves, normal in January at 45 N at 500 mb, by introducing orographic and frictional forcing in a constant zonal current. This should indicate that the stationary waves are forced by orography and modified by friction.

Bolin (1950) improves Charney and Eliassen's study, by taking into account the north-south extension of mountains and zonal current. He concludes that the mean temperature of the 500–1000 mb layer is controlled dynamically from above, in addition to the direct thermodinamical control from below.

Sutcliffe (1951) refutes the views of Charney, Eliassen and Bolin. According to hemi it is the temperature of the 500-1000 mb layer which controls the 500 mb and not vice versa.

Saltzman (1959) proposes that the synoptical view of Sutcliffe should be modified as follows: frontogenetic areas favours baroclinic development of cyclones, which maintain the quasi-stationary waves against dissipation, by a non–linear barotropic transfer of kinetic energy.

Since 1960 it has become difficult to follow the literature about quasi-stationary waves. An excellent monograph by Saltzman (1968) classifies the main studies on the subject. Generally they are improvements and extensions of the five papers quoted above (see in particular Smagorinsky, 1953). In the work by Doos (1962) the heating from below is parameterized, rather than fixed.

The most recent studies can be classified in two groups: a few prescribe the forcing functions of models, usually linearized, of the mean motion (see in particular, Derome and Wiin Nielsen, 1971). The majority introduce orography and diabatic heating into numerical models of the general circulation (see in particular, Mintz, 1965). The recent paper by Manabe and Terpstra (1974) reviews the studies of the second group carried out at UCLA, GFDL and NCAR.

Recently Rowntree (1975) has studied the effects of the thermal forcing at low latitudes on the standing waves at middle latitudes.

In spite of so many competent efforts, the genesis and the evolution of the standing waves are not yet clear. In particular the various studies continue to indicate alternatively the prevalence of the thermal effect on the dynamic one, and vice versa.

Why this uncertainty?

One important reason is that it is difficult to parameterize orographic effects and diabatic heating.

The writer thinks, however, that the main difficulty is that the problem of the standing waves is too complex to be considered as a single problem. In other words, the standing waves are not directly influenced by the orographic and thermal forcing.

It is convenient to consider divergence (or vorticity) production between mountains or heat sources and standing waves. In fact the divergence (or vorticity) production at an intermediate level of the atmosphere can be considered as the immediate cause of the standing waves superimposed upon the zonal current (La Valle e Celentano, 1975), and as the immediate effect of a mountain or heat source.

The present study follows this line of approach.

2. BASIC THEORY

The average over one month, indicated by a bar, of the motion at the level $\sigma = p/p_s$ can be represented by the vorticity equation (in p coordinates) (1). The twisting terms have been omitted. In accord with this approximation, the vertical transport of vorticity is generally disregarded too. In equation (1) the vertical transport of vorticity due to orography (only this part of the total vertical transport) has been retained to simplify the elaborations, as the next paragraph will show.

The left side of (1) can be written $\overline{V\left(\frac{\partial}{\partial s}\right)_\sigma (\mathrm{rot}\ V + \lambda)}$.

Equation (1) can be written:

$$\overline{V\frac{\partial}{\partial s}\left(\mathrm{rot}\ V + \lambda\right) + \frac{p}{p_s} V \frac{\partial p_s}{\partial s}\frac{\partial\ \mathrm{rot}\ V}{\partial p}} = - \overline{(\mathrm{rot}\ V + \lambda)\ \mathrm{div}\ V} \tag{1}$$

$$\bar{V}\left(\frac{\partial}{\partial s}\right)_\sigma\left(\mathrm{rot}\ \bar{V} + \lambda\right) = -\lambda\ \mathrm{div}\ \bar{V} - \mathrm{rot}\ \bar{V}\ \mathrm{div}\ \bar{V} - \overline{\mathrm{rot}\ V'\ \mathrm{div}\ V'} - \overline{V'\left(\frac{\partial}{\partial s}\right)_\sigma \mathrm{rot}\ V'} \tag{2}$$

The writer has shown (La Valle e Celentano, 1975) that equation (2) can be solved with sufficient precision respect to $\bar{V}$ at the level $p_s/2$, once the right side, and the zonal average of $\bar{V}$ have been assigned.

The last term of (2) represents the contribution of the disturbances to vorticity production. This contribution is negligible as far the disturbances are large (La Valle e Celentano, 1975). Small disturbances can give, however, a significant contribution (Holopainen, 1975).

In any way, as the major contribution to the right side of (2) is given by $-\lambda\ \mathrm{div}\ \bar{V}$, it is interesting to study the correlation between $-\lambda\ (\mathrm{div}\ \bar{V})\ p_s/2$ and various factors acting on it.

In the following $-\lambda\ \mathrm{div}\ \bar{V}$ will be called the vorticity production, although it is only the major part of the vorticity production due to the mean motion.

The first purpose of the present study is to evaluate the correlation between $-\lambda\ (\mathrm{div}\ \bar{V})\ p_s/2$ and the vertical velocity forced by orography at ground level.

When averaged over one month, the continuity equation and the thermodynamic law (in p coordinates) become:

$$\frac{\partial \bar{\omega}}{\partial p} = -\ \mathrm{div}\ \bar{V} \tag{3}$$

$$\overline{\omega} = -\left(\overline{A} + \frac{\overline{Q}}{c_p}\right) \Big/ \left(\Gamma_{ad} - \Gamma_{geom}\right) \tag{4}$$

In equation (4) $\Gamma_{ad} - \Gamma_{geom}$ has been considerated constant in time.

Elimination of $\overline{\omega}$ from (3) through (4) gives

$$\operatorname{div} \overline{V} = \frac{\partial}{\partial p}\left[\left(\overline{A} + \frac{\overline{Q}}{c_p}\right) \Big/ \left(\Gamma_{ad} - \Gamma_{geom}\right)\right] \tag{5}$$

Equations (2) and (5) say that the standing waves are finally found to be dependent on the vertical change of temperature advection and diabatic heating.

The second purpose of the present study is to evaluate the correlation between $-\lambda\,(\operatorname{div} \overline{V})_{p_s/2}$ and $\overline{A}$ and $\overline{Q}/c_p$ in the lower halph of the atmosphere.

This study is expected to improve the knowledge of the large-scale quasi-stationary waves.

3. OROGRAPHIC EFFECT

The basic data are 500 and 1000 mb heights, every 6 hours during July and October 1957 and January and April 1958, in the 27 X 27 points of a squared grid covering the polar stereographic projection of the northern hemisphere. The grid mesh is 800 km at 60 N. For further information about data, see La Valle and Caponigro (1973).

On the basis of these data, the following quantities have been computed, every 6 hours at every grid point more northerly than 20° N, through use of geostrophic and hydrostatic approximation (with $\Gamma_{geom} = 0.54 . 10^{-4}$ °C/cm): $T_{1000\ mb}$, T_{p_s}, $T_{0.75p_s}$, $V_{1000\ mb}$, $V_{500\ mb}$, rot $V_{1\,000\ mb}$, rot V_{500mb}. Subsequently V_{p_s}, $V_{0.5\ p_s}$, rot V_{p_s} and rot $V_{0.5p_s}$ have been calculated from the corresponding quantities at the level 500 and 1000 mb, by a linear interpolation (or extrapolation) in the p-space.

The knowledge of wind and vorticity at the levels p_s and $p_s/2$ enables to calculate the right side of equation (6). The quantity $-\lambda$ div V at the levels p_s and $p_s/2$ has been computed every 6 hours through use of equation (6).

$$\text{div V} = -\frac{1}{\lambda}\left[\frac{\partial}{\partial t} \text{rot V} + \text{V}\left(\frac{\partial}{\partial s}\right)_\sigma \left(\text{rot V} + \lambda\right)\right] \qquad (6)$$

The vertical velocity at ground level has been computed through the equation.

$$\omega_s = V_s \frac{\partial p_s}{\partial s} \qquad (7)$$

For brevity, only the figures relative to January 1958 are shown in the paper.

Figures 1 and 2 show the contour of the 500 and 1000 mb surfaces. Figures 3 and 4 show $-\lambda$ div $\overline{V}$ at levels $p_s/2$ and p_s.

The magnitude of the divergence at the level $p_s/2$ results to be not much smaller than the divergence at ground level. This may be surprising, in consideration that the level $p_s/2$ is close to the level of non-divergence. In fact the dynamics of the standing waves is quite different from the dynamics of the transient perturbations. In the last, divergence in the upper troposphere is generally superimposed upon convergence in the lower troposphere (and vice versa); consequently the root mean square of the divergence values over a month presents a minimum along the vertical at a level close to $p_s/2$. This does not imply in any way that the monthly averaged divergence must be zero (or must have a minimum) at the same level.

The vertical velocity forced by orography should have the theoretical vertical profile.

$$\omega = \frac{\omega_s}{p_s^n} p^n \qquad (8)$$

(*Cf.* La Valle, 1962).

The exponent n depends on the horizontal dimensions of the mountains and on static stability; usually it amounts to a few units.

Partial derivative of ω with respect to p gives.

$$\left(\mathrm{div}\ V\right)_{p_s/2} = -\frac{n}{2^{n-1}}\frac{\omega_s}{p_s} \qquad \left(\mathrm{div}\ V\right)_{p_s} = -n\frac{\omega_s}{p_s} \tag{9}$$

Table I gives the correlation coefficients between $\lambda\overline{\omega_s/p_s}$ and $-\lambda$ div $\overline{V}$ at levels $p_s/2$ and p_s over ground sloped not less than 0.5 ‰, that is 1/4 of the hemisphere. Orography accounts for about 20% of yearly mean divergence at $p_s/2$ over sloped ground.

The value of the parameter γ, which makes zero the correlation between $\lambda\ \overline{\omega_s/p_s}$ and $-\lambda$ div $\overline{V} - \gamma\lambda\overline{\omega_s/p_s}$ furnishes the value of the coefficient n of equation (9). At level $p_s/2$, γ is found equal to 0.6. The associated n is 3.5, in accordance with theory.

Fig. 5 shows the distribution of $\lambda\ \overline{\omega s/p_s}$.

At ground level the results of this study completely disagree with theory. $\gamma = n$ is found to be equal to -1.1: there is convergence windward, and divergence leeward.

4. EFFECT OF TEMPERATURE CHANGES IN THE LOWER HALPH OF THE ATMOSPHERE

Equation

$$A = -V\left(\frac{\partial T}{\partial s}\right)_\sigma + \Gamma_{geom}\frac{p}{p_s}V\frac{\partial p_s}{\partial s} \tag{11}$$

has been applied to compute the temperature advection at level 0.75 p_s. $V_{0.75\ p_s}$ has been assumed equal to

$$V_{0.75 p_s} = \left(V_s + V_{0.5\ p_s}\right)/2 \tag{12}$$

Equation

$$B = \omega\left(\Gamma_{ad} - \Gamma_{geom}\right) \tag{13}$$

has been applied to compute the temperature change due to vertical motion at level 0.75 p_s. $\omega_{0.75 p_s}$ has been evaluated by equation (10).

$$\omega = bp + cp^2 + dp^3 \tag{10}$$

The coefficients b , c and d are determined by ω_s and the divergence at p_s and $p_s/2$ computed through equation (6).

The diabatic heating at level 0.75 p_s has been computed as residual:

$$\frac{Q}{c_p} = -A - B + \frac{\partial T}{\partial t} \tag{14}$$

$\overline{A}$, $\overline{B}$ and $\overline{Q}/c_p$ so obtained are representative of the temperature changes in the lower halph of the atmosphere. They are shown in Figures 6 to 8.

The distribution of diabatic heating obtained by the writer agrees fairly well with the results reached by the Staff members of the Academia Sinica (1958) and by Brown (1964). Some disagreements between Fig. 8 and the direct evaluation of $\overline{Q}/c_p$ by Clapp (1961) are probably due to faults of the radiative contributions as computed by this author.

Table II contains the correlation coefficients between $\overline{A}$ and $\overline{Q}/c_p$. It confirms that $\overline{Q}/c_p$ tends to compensate $\overline{A}$, especially on January.

Table III presents the correlation coefficients between $-\lambda$ (div $\overline{V}$) $p_s/2$ and $\overline{A}$, $\overline{Q}/c_p$ and $\overline{B}$, over the 3/4 of the northern hemisphere sloped less than 5°/oo .

The second line shows that diabatic heating in the lower halph of the atmosphere is mostly associated with the production of cyclonic vorticity at the intermediate levels.

$\overline{Q}/c_p$ accounts for more than 25% of the variance of $-\lambda$ (div $\overline{V}$) $p_s/2$.

Temperature advection and diabatic heating, together, account for a little less than 50% of the variance of $-\lambda$ (div $\overline{V}$) $p_s/2$.

5. CONCLUSIONS AND AKNOWLEDGEMENTS

Orography explain about 20% of vorticity production at the intermediate levels over a sloped land.

Temperature changes in the lower halph of the atmosphere account for about 50% of vorticity production over sea or flat ground.

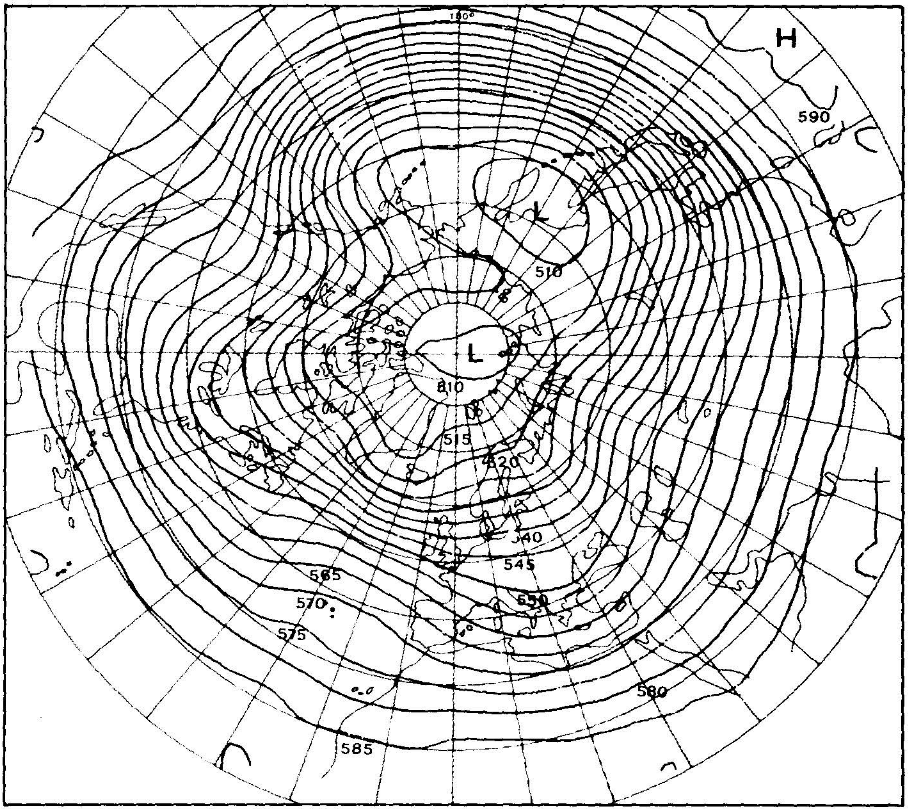
Suggestion for further researches. $-\lambda$ (div V) $p_s/2$ depends essentially on the vertical distribution of heat (equation 5). As long wave radiation, short wave radiation, condensation and sensible heat have different vertical distributions (Miyakoda, 1975), they act on quasi-stationary waves in different ways. In particular, latent heat development should play an important role in cyclonic vorticity production. Quasi-Stationary waves are expected to depend strongly on the vertical distribution of heat over some typical climatic areas. This would connect synoptic climatology with dynamical and numerical meteorology.

Dr. Romano Celentano has collaborated in paragraph 3; Mrs. Maria Rogo has collaborated in computer and plotter programmes.

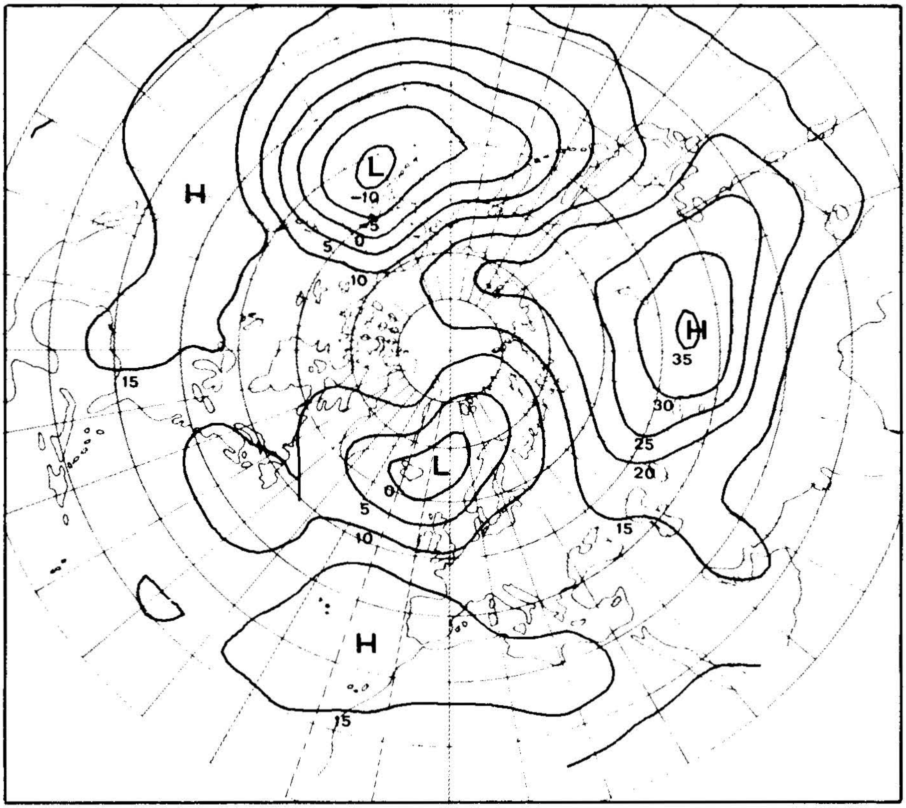
This paper is published by permission of the Chief of the Servizio Meteorologico A. M.

LIST OF SYMBOLS

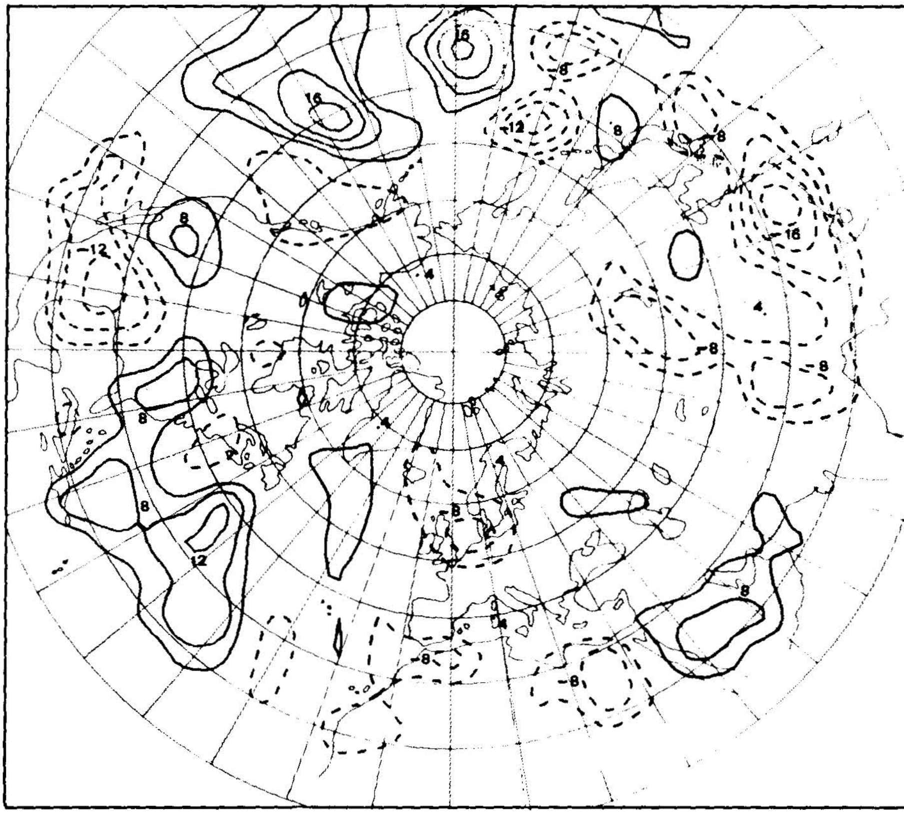
p	atmospheric pressure
T	temperature
V	horizontal wind
t	time
ω	pressure vertical velocity =
—	time average over a month
div V	isobaric divergence
rot V	isobaric vorticity
s	deplacement along streamlines
λ	Coriolis parameter
subscript $_s$	at ground level
σ	dimensionless coordinate = p/p_s
subscript $_\sigma$	at constant σ
Q	rate of heating for unit mass
c_p	specific heat of air at constant pressure
Γ_{geom}	vertical lapse rate of temperature = $\partial T/\partial p$
Γ_{ad}	dry-adiabatic lapse rate
A	temperature advection = $-V\,\partial T/\partial s$
B	temperature change due to vertical velocity = $\omega\,(\Gamma_{ad} - \Gamma_{geom})$



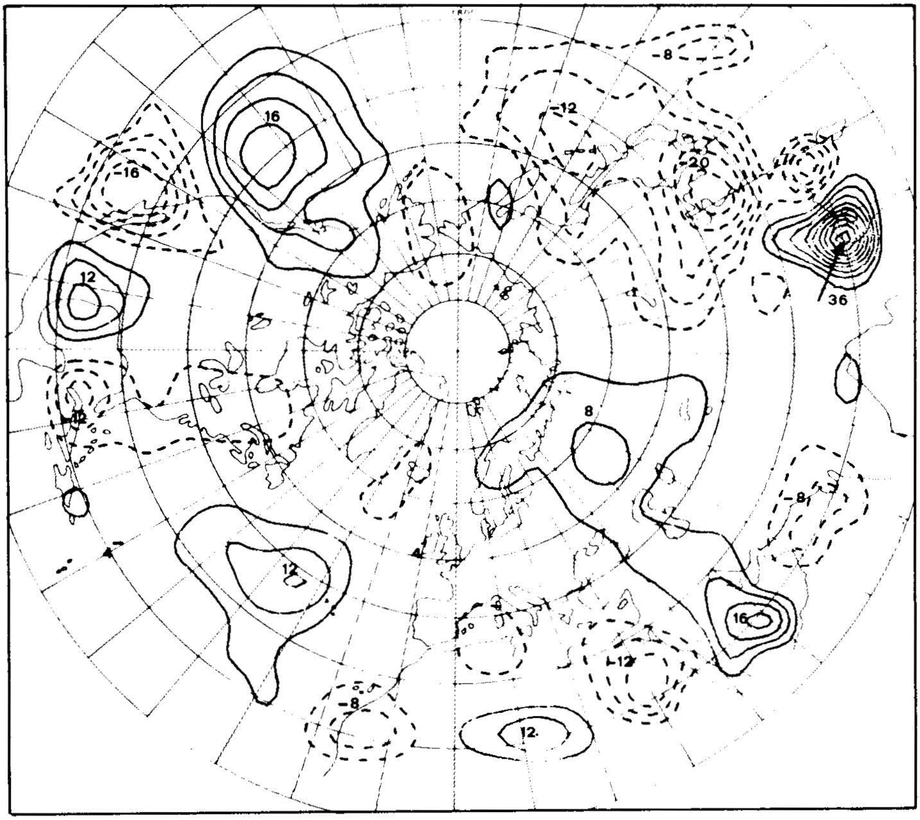
1. January 1958. Contours of the 500 mb level. Unit: 10 m.



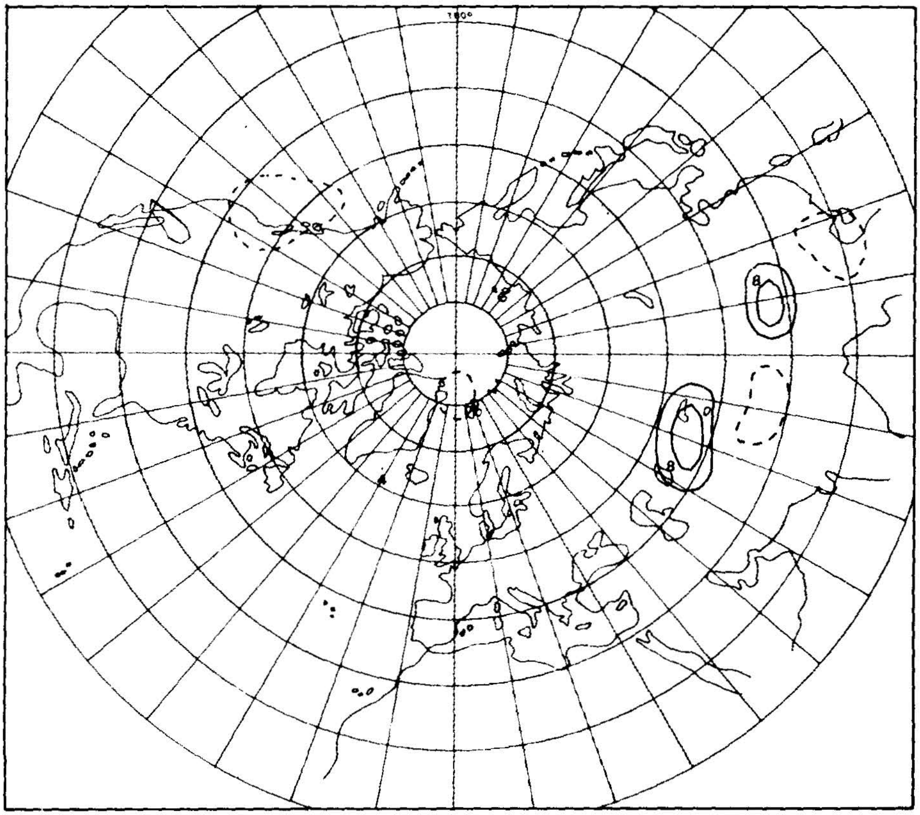
2. January 1958. Contours of the 1 000 mb level. Unit: 10 m.



3. January 1958. $-\lambda \operatorname{div} \overline{V}$ at level $p_s/2$. Unit: 10^{-11} sec^{-2}.
The levels of the contour curves plotted are $-16, -12, -8, -4, 4, 8,$ etc.

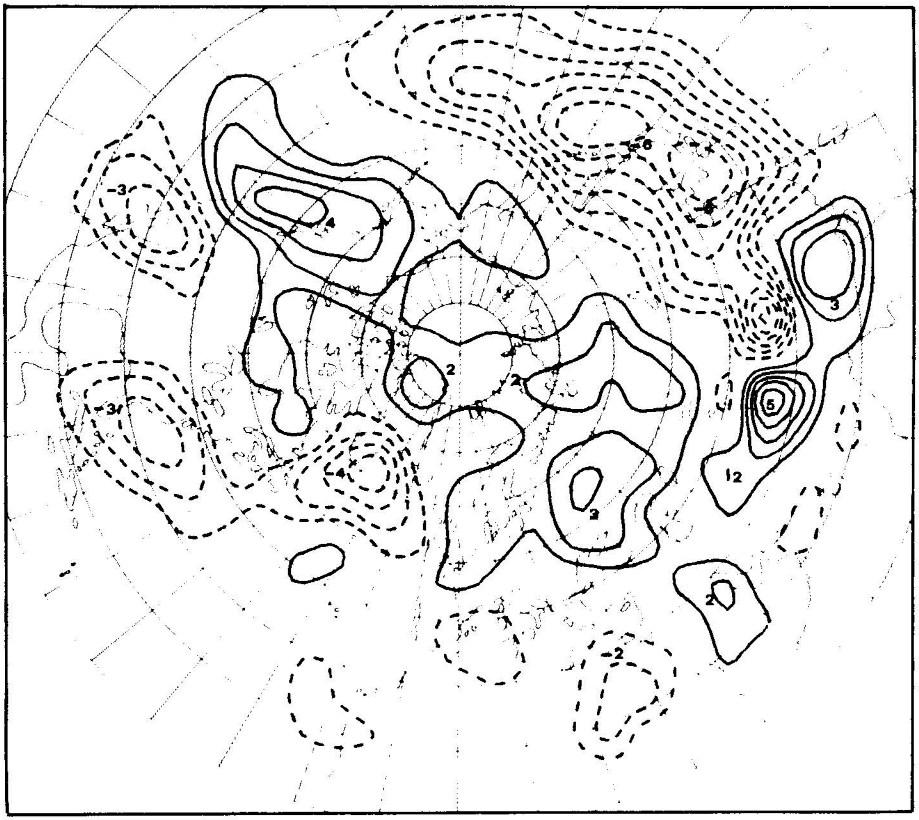


4.- January 1958. $-\lambda \operatorname{div} \overline{V}$ at level p_s. Unit: 10^{-11} sec^{-2}.
The levels of the contour curves plotted are -16, -12, -8, -4, 4, 8, etc.

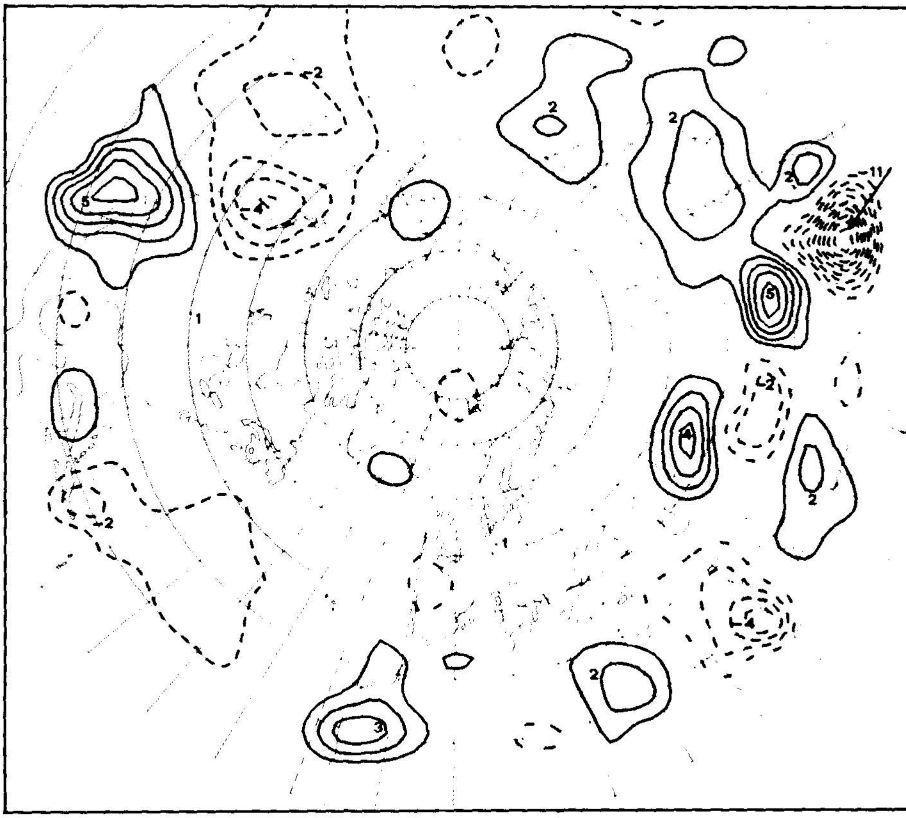


5. January 1958. $\overline{\lambda\, \omega_s/p_s}$. Unit: 10^{-11} sec^{-2}.

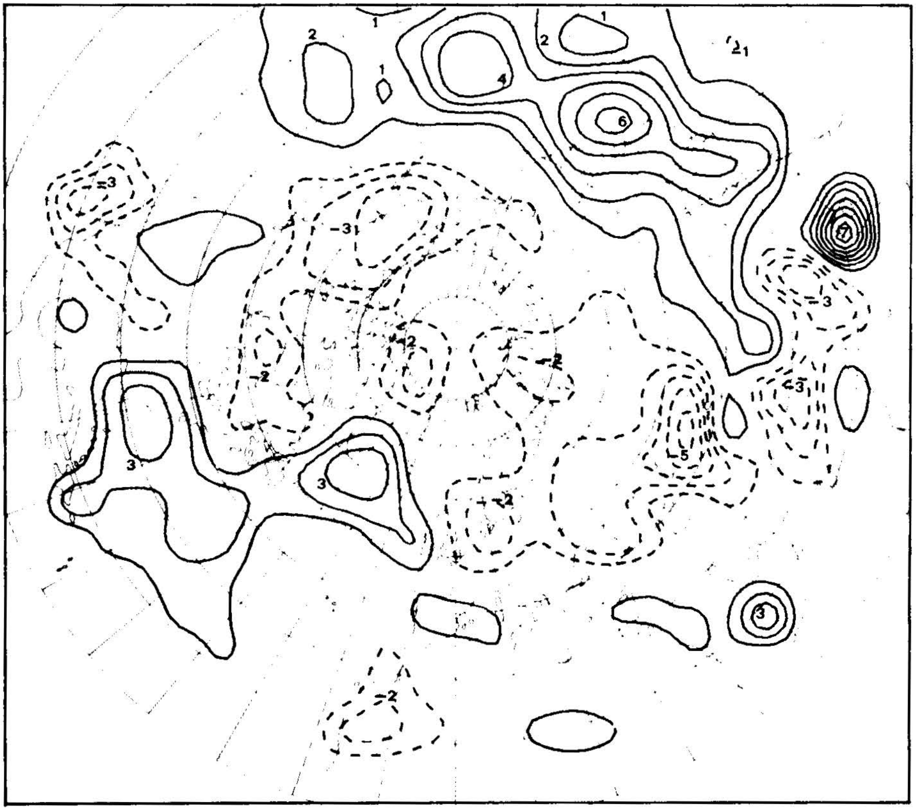
The levels of the contour curves plotted are -4, 4, 8.



6. January 1958. Temperature advection at level 0.75 p_s.. Unit: °C day^{-1}.



7. January 1958. Temperature change due to vertical motion at level 0.75 p_s. Unit: °C day^{-1}



8. January 1958. Diabatic heating at level 0.75 p_s. Unit: $^{\circ}C\ day^{-1}$.

TABLE I

	JULY	OCT	JAN	APR	YEAR
$-\lambda\ (\mathrm{div}\,\overline{V})\ p_S$	−0.06	−0.58	−0.39	−0.17	−0.43
$-\lambda\ (\mathrm{div}\,\overline{V})\ p_S/2$	0.03	0.20	0.41	0.14	0.43

Correlation coefficients between $-\lambda$ div $\overline{V}$ and $\lambda\ \overline{\omega_S/p_S}$, where slope $\geqslant 0.5\ {}^{\circ}/_{\infty}$.

TABLE II

JULY	OCT	JAN	APR	YEAR
−0.37	−0.58	−0.82	−0.39	−0.51

Correlation coefficients between $\overline{A}$ and $\overline{Q}/c_p$, where slope $< 0.5\ {}^{\circ}/_{\infty}$.

TABLE III

	JULY	OCT	JAN	APR	YEAR
$\overline{A}$	0.25	0.09	0.08	0.12	0.12
$\overline{Q}/c_{p_S}$	0.51	0.53	0.37	0.55	0.51
$\overline{B}$	−0.68	−0.69	−0.70	−0.60	−0.66

Correlation coefficients between $-\lambda\ (\mathrm{div}\overline{V})p_S/2$ and $\overline{A}$, $\overline{Q}/c_p$ or $\overline{B}$, where slope $< 0.5\ {}^{\circ}/_{\infty}$.

BIBLIOGRAPHY

ACADEMIA SINICA, STAFF MEMBERS, 1958. On the general circulation over Eastern Asia (III). *Tellus,* 10: 299-312..

BOLIN, B., 1950. On the influence of the earth's orography on the general character of the westerlies. *Tellus,* 3:184–195.

BROWN, J. A., 1964. A diagnostic study of the tropospheric diabatic heating and the generation of available potential energy. *Tellus*, 16:371–388.

CHARNEY, J. G. and A. A. ELIASSEN, 1949. A numerical method for predicting the perturbations of the middle latitude westerlies. *Tellus,* 1:38–54.

CLAPP, P. F., 1961. Normal heat sources and sinks in the lower troposphere in winter. *Monthly Weather Review,* 89: 147–162.

DEROME, J. and A. WIIN-NIELSEN, 1971. The response of a middle-latitude model atmosphere to forcing by topography and stationary heat sources. *Monthly Weather Review,* 99: 564–576.

DOOS, B. R., 1962. The influence of exchange of sensible heat with the earth's surface on the planetary flow. *Tellus,* 14: 133–147.

HOLOPAINEN, E., 1975. Diagnostic studies on the interaction between the time-mean flow and the large-scale transient fluctuations in the atmosphere. Department of Meteorology, University of Helsinki, *Report No. 8,* 14 pp.

LA VALLE, L., 1962. Variazione della pressione nei grandi moti atmosferici. *Rivista di Meteorologia Aeonautica,* 22.

LA VALLE, L. and R. CAPONIGRO, 1973. Sources and sinks of wind vorticity at 500 mb over the whole globe. *Annalen der Meteorologie,* 6: 197–205.

LA VALLE, L. e R. CELENTANO, 1975. Dipendenza delle onde quasi stazionarie della sorgenti e dai pozzi di vorticità. *Rivista di Meteorologia Aereonautica, 35:* 31–38.

MANABE, S. and T. B. TERPSTRA, 1974. The effects of mountains on the general circulation of the atmosphere as identified by numerical experiments. Journal of Atmospheric Sciences, 31: 3–42.

MINTZ, Y., 1965. Very long term global integration of the primitive equations of atmospheric motion. *WMO Technical Note No. 66:* 141–167.

MIYAKODA, K., 1975. Weather forecasts and the effects of the sub-grid scale processes. Seminars on scientific foundation of medium range weather forecasts. Part II. European Centre for Medium Range Forecasts: p. 448.

ROSSBY, C. G. and collaborators, 1939. Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semipermanent centers of action. *Journal of marine research,* 2: 38–55.

ROWNTREE, P. R., 1975. Thermal and orographic forcing of the northern hemisphere winter circulation in a numerical model. WMO No. 421 (Proceedings of the WMO/IAMAP symposium on long-term climatic fluctuations. Norwich 1975): 355–364.

SALTZMAN, B., 1959. On the maintenance of the large-scale quasi-permanent disturbances in the atmosphere. *Tellus*, 9: 425–431.

SALTZMAN, B., 1968. Sufarce boundary effects on the general circulation and macroclimate: a review of the theory of the quasi-stationary perturbations in the atmosphere. *Meteorological Monographs*, 8, No. 30: 4–19.

SMAGORINSKY, J., 1953. The dynamical influence of large-scale heat sources and sinks on the quasi-stationary mean motions of the atmosphere. *Quarterly Journal of the Royal Meteorological Society*, 79: 342–366.

SUTCLIFFE, R. C., 1951. Mean upper contour patterns of the northern hemisphere: the thermal-synoptic view point. *Quarterly Journal of Royal Meteorological Society*, 77: 435–440.