BUBBLE PULSES AND POWER SPECTRA OF SOME MARINE SEISMOGRAMS

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RESUMEN

Sismogramas marinos de un reconocimiento marino de refracción hecho en el Lago Superior con explosivos "geogel" se estudian en función de su contenido espectral en relación a varios tamaños de detonación (1.25-200 libras). El efecto de *oscilaciones pulsátiles de burbuja* juega un papel dominante en el análisis. Un enfoque heurístico se usa y se concluye que el uso de pequeñas detonaciones es aconsejable, cuando sea posible, en reconocimientos de refracción marina debido al espectro relativamente amplio, en comparación con los de detonaciones mayores. También, si un proceso de *deconvolución* es usado para remover los efectos *pulsátiles de burbuja,* de un sismograma con un espectro angosto, nada verdaderamente significativo permanecería para interpretación.

De las observaciones de períodos *pulsátiles de burbuja,* tamaños de detonaciones y profundidades de detonación, la constante Rayleigh Willis se determina para el ambiente del Lago Superior. Los valores calculados de 5.0 dan períodos de burbuja comparables a los que se observan. Sin embargo, este valor difiere de otros valores tales como 4.19 obtenido por Worzel y Ewing (1948) y 4.36 obtenido por un grupo del Instituto Oceanográfico Woods Hole (Shor, 1963) usando explosivos de TNT en el mar. La constante parece que es dependiente, tanto del tipo de explosivo, como del ambiente donde se produce la detonación.

ABSTRACT

Marine seismograms from a marine refraction survey done in Lake Superior with "geogel" explosives are studied in terms of their spectral content in relation to various shot sizes (1.25-200 lbs). The effect of bubble pulse oscillations plays a dominant role in the analysis. A heuristic approach is used throughout and it is concluded that the use of small shots is advisable, whenever possible, in marine refraction surveys on account of the relatively broad

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advisable, whenever possible, in marine refraction surveys on account of the relatively broad spectra in comparison with those for larger shots. Also, if a deconvolution process is used to remove the bubble pulse effects from a seismogram having a narrow spectrum, nothing very meaningful would remain for interpretation.

From observations of the bubble pulse periods, sizes of shots, and depths of detonation, the Rayleigh-Willis constant is determined for the Lake Superior environment. The calculated value of 5.0 yields bubble periods comparable to those observed. However this value differs from other values such as 4.19 obtained by Worzel and Ewing (1948) and 4.36 obtained by a group at Woods Hole Oceanographic Institute (Shor, 1963) using TNT explosives at sea. The constant seems dependent on both the type of explosive and the shooting environment.

INTRODUCTION

In 1969 a marine refraction survey (Kunar, 1970) was done to study the structure of the Lake Superior Keeweenawan basin. The entire set of seismograms were affected by oscillatory motions of bubbles formed after detonation of "Geogel" explosives in the Lake.

In this paper the value of a constant contained in the Rayleigh-Willis formula for the time period of bubble pulses as well as the power spectra of a few seismograms is determined. An attempt is made to explain the presented power spectra using a heuristic approach.

THE BUBBLE PULSE PHENOMENON

Blochmann (1898) was the first to report this phenomenon. He noticed that many distinct pressure pulses were produced on exploding a charge in water. Each pulse was delayed from the other as though there were several separate explosions.

The explanation, verified experimentally in 1941 by Ewing *et al.* (1946), is as follows: When an explosive is fired, it becomes converted into gases at extremely high temperatures and pressures. The intense pressure produces outward motion of the water at a speed greater than its acoustic velocity. Consequently, a shock wave results yielding the initial pressure pulse. After explosion, the energy becomes partitioned between the kinetic energy of the surrounding flow of water and the potential energy of the hot pressurized gases and vapours within the bubble. The bubble expands adiabatically until its internal pressure equals the ambient hydrostatic pressure. However, expansion does not cease at this point because of the inertia of the outward flowing water. The bubble continues expanding until its radial velocity is zero. At this stage the energy is in potential form symbolized by a cavity in the ocean where the internal pressure is less than the ambient pressure. The surrounding water then accelerates toward the centre decreasing the bubble volume and increasing its internal pressure. Again, because of inertial effects, zero velocity is attained at an internal pressure greater than the ambient pressure. A second pressure pulse is radiated at this point. The above process is repeated many times so that several pressure pulses are produced from a single explosion. Each successive pressure pulse is of less energy than the one preceding, on account of the energy losses by conduction, convection, radiation and hysteresis.

During the 1969 survey the explosives were thrown astern and a towed hydrophone recorded the various waves coming from the shot and the bottom of the lake. In nearly all of the hydrophone records one or two bubble pulses are identifiable. At least one can be recognised easily on each record (Figure 1). They are of simple form and seem to have relatively less high frequency components than those preceding; also, the first bubble pulse shows less high-frequency content than the initial shock wave from the explosion. On comparing the intensity levels of the shock and bubble pulse arrivals for a particular shot-size, it is observed that the ratio of the levels is variable.

BUBBLE PULSE PERIOD

In 1917 Lord Rayleigh showed that the period of bubble pulse oscillations is given by

$$T = B\, A_M\, (\rho/P_e)^{1/2} \qquad (1)$$

Where B = a constant
A_m = the maximum radius of the gas bubble
ρ = the density of water in the environment
P_e = the ambient hydrostatic pressure.

Willis (1941) combined equation (1) with the formula for the potential energy, Q, of a spherical void of radius A_m to obtain

$$T = (KW^{1/3}) \;/\; (D + 33)^{5/6} \quad \text{secs} \tag{2}$$

where K = a constant
W = the weight of the shot in pounds
(W is assumed to be proportional to Q)
D = the depth to the centre of the bubble in feet.

Equation (2) is known as the Rayleigh-Willis formula. This formula was used to compute the periods of bubble pulses for the 1969 "Geogel" explosives. It should be noted that the oscillations are quasi-periodic since energy is dissipated in between successive bubble minima. The value of W is effectively reduced so that the period between pulses is progressively shortened.

The mean detonation depth was calculated for 296 shots ranging in size from 1.25 pounds to 200 pounds. Using these depths and assigning a value of 5.0 to K yielded periods (Table 1) comparable to those measured. The maximum deviation was about 15%. Figure 1 shows the measured and theoretical periods, T and T_C respectively, for a few shots. The agreement is quite good.

It seems that K varies with the shooting environment and probably with the type of explosive. Worzel and Ewing (1948), using TNT explosives at sea, required a K-value of 4.19 for the best fit to their experimental values. Shor (1963) states that a group at Woods Hole Oceanographic Institute uses a K-value of 4.36 for TNT.

POWER SPECTRA

The power spectra for a few seismograms produced by various shot-sizes were determined and a study of their spectra was made. The lengths of the seismograms varied from 3 seconds to 5 seconds and a sampling interval of 1/125 seconds was chosen in order to satisfy the Nyquist frequency (62.5 Hz) criterion for the proper sampling of an analog signal.

The method used for obtaining the power spectra is similar to that outlined by Richards (1967). First, the auto-correlation was found, normalized, and smoothed by a Gaussian function to remove insignificant fine detail. The function, $g(\tau\ \ \delta)$, used was

$$g(\tau, \delta) = \left\{ 1/\delta(2\pi)^{1/2} \right\} \exp\ (-\tau^2\ /2\delta^2) \tag{3}$$

where τ refers to time and δ to the standard deviation. The latter was chosen so that 2.5δ was equivalent to one-tenth of the seismogram length. This choice gives strong statistical validity to the resulting spectra. At 2.5δ the Gaussian function has about 5% of its peak value.

Secondly, the Fourier transform of the smoothed auto-correlation function was calculated to yield the power spectrum.

Plots of the power spectra for shots of sizes 2.5, 5, 10 and 50 pounds are shown in figures 2a to 2d. In general there is an inclination for the spectra to get narrower with increasing shot size.

The shape of the power spectrum is influenced by a number of factors, namely:

(i) The multiplicity and spectra of the source pulses – shock and bubble pulses.

(ii) The filtering by sub-bottom stratigraphy.

(iii) The filtering effects of the water-air interface reflection at the shot: The shot-position filter.

(iv) The filtering effects of the water-air interface reflection at the hydrophone: the detector-position filter.

(v) The filtering by the recording instruments.

If the effects of the bubble pulses and of (iii), (iv) and (v) are removed by say, a deconvolution process, the calculated power spectrum should reflect the magnitudes of the frequency components in the initial explosive (shock) wave minus those filtered by propagation through the sub-bottom stratigraphy. In the analysis, the spectra were computed to estimate qualitatively the influence of the above factors on a seismic recording.

THE FREQUENCY-SPECTRUM OF THE SHOCK PULSE

From experimental evidence, Cole (1948) showed that the pressure-time dependence near an explosion can be approximated by an exponential function:

$$P(t) = P_m \exp(-t/\theta) \tag{4}$$

where P_m is the initial peak pressure of the shock pulse, t the time, and θ the time constant of exponential decay. The spectrum $S(\omega)$ of $P(t)$ is then given by

$$S(\omega) = \int_0^\infty P_m \exp\left\{-(1/\theta + j\,\omega)\,t\right\} dt$$

$$= P_m / (1/\theta + j\,\omega) \tag{5}$$

Hence, if the pulse length, Δt, is defined so that $P(t) = 0.01\,P_m$ for $t = \Delta t$, the time constant θ of equation (4) becomes

$$\theta = \Delta t / 4.6 \tag{6}$$

From experimental observations of the shock wave (Cole, 1948), Δt is of the order of 1 millisecond. Using this value of Δt, equations (5) and (6) yield the spectrum shown in Figure 3. This spectrum is essentially flat out to several hundred cycles.

THE FREQUENCY-SPECTRUM OF THE BUBBLE PULSE

Consider the first bubble pulse following the shock wave. The pressure-time curve of bubble pulses (Epinat'eva, 1951) can be represented by

$$P(t) = P_1 \exp(-\beta^2 t^2) \tag{7}$$

where β is a coefficient dependent on the effective pulse length. The Fourier spectrum, $B(\omega)$ of this even function is

$$B(\omega) = (P_1 \sqrt{\pi}/\beta) \exp(-\omega^2 / 4\beta^2) \tag{8}$$

Again, if the pulse length, Δt_1, is defined so that $P(t) = 0.01 P_1$ for $t = \Delta t_1$, β of equation (7) is given by

$$\beta = 2.145 / \Delta t_1 \tag{9}$$

Δt_1 can be estimated in terms of the bubble pulse period T (Cole, 1948):

$$\Delta t_{1'} \simeq 0.22\, T$$

Using the bubble pulse periods typical of small (2.5-10 pounds. $T \simeq 80$ ms) and large (50-100 pounds. $T \simeq 100$ ms) shots, equation (8) gives the spectra shown in Figure 3. It can be seen that the shock wave spectrum is considerably wider than the bubble pulse spectrum. The latter is negligible beyond about 80 Hz. A comparison of the bubble spectra for the small and large shots reveals that the spectrum is narrower for the large shots.

SPECTRUM OF THE SHOCK AND BUBBLE PULSES

Consider the shock pulse together with the first bubble pulse. Accordingly, since the first bubble pulse is displaced in time by say, τ, the total spectra, $F(\omega)$, is described by

$$F(\omega) = S(\omega) + B(\omega) \exp(-i\omega\tau)$$

The first bubble pulse thus causes the spectrum $F(\omega)$ to fluctuate. Similarly, if the effects of other bubble pulses are taken into consideration, the resultant spectrum,

$$F(\omega) = S(\omega) + B(\omega) \exp(-i\omega\tau) + B(\omega) \exp(-i2\omega\tau) + \ldots \quad (10)$$

can fluctuate even more. Figure 4 indicates how the shock wave spectrum becomes modified by bubble pulses with amplitudes equal to the peak amplitude of the explosive pulse. In reality, the amplitudes of the bubble pulses are less than the amplitude of the shock pulse but this does not markedly affect the general pattern of the spectral plot. It will therefore be assumed, for simplicity, that the amplitudes of all pulses are approximately equal.

Figure 4 shows that the spectrum for the large shots have a lower minimum (in this case, zero) and seem more "spiked".

DETECTOR AND SHOT POSITION FILTERS

Two adjacent waves, for example, the direct down-going wave from the shot and the up-going wave which is reflected at the water-air interface before proceeding down-wards, can become critically refracted at the boundaries of layers beneath the bottom of the lake and arrive at the detector via the route shown in Figure 5 where they will interfere with each other. This interference effect is equivalent to filtering.

Consider the "detector-position filter", which results from interference between the adjacent rays DEF (reflected ray) and HF. Let the depth of the hydrophone be d_h, so that the extra travel time for the reflected ray is

$$T_h = 2\, d_h \;/\; V_w \cos i \quad (11)$$

where i is the angle of incidence, and V_w the velocity of compressional waves through the water layer. Since the rays are adjacent, i is small, so that equation (11) can be written as

$$T_h = 2\ d_h\ /\ V_w \tag{12}$$

Let $\delta(t)$ represent an impulsive plane wave travelling upward from the horizontal level through F, the detector position, and becoming reflected (reflection coefficient = -A) at the water-air interface before returning to the detector. This is equivalent to a filter with an impulse response,

$$F_h(t) = \delta(t) - A\ \delta(t - T_h)$$

The frequency response or transfer function of this filter is

$$F_h\ (\omega) = \int_{-\infty}^{\infty} \left\{ \delta(t) - A\ \delta(t - T_n) \right\} e^{-j\ \omega t}\ dt$$

$$= 1 - Ae^{-j\ \omega t_h}$$

Assuming $A = 1$,

$$|F_h\ (\omega)| = |\ 2\ \sin\ (\omega\ T_h\ /\ 2)\ | \tag{13}$$

Since the hydrophones were placed approximately 100 feet below the water surface, $|F_h\ (\omega)|$ shows successive maxima at 12.25 Hz and at odd multiples of 12.25 Hz. A similar process for the "shot-position filter" involving the rays ABC (reflected ray) and AG yields,

$$|\ F_s\ (\omega)\ | = |\ 2\ \sin\ (\omega\ T_s\ /\ 2)\ | \tag{14}$$

where $T_s = 2\ d_s\ /\ V_w$, and d_s is the detonation depth.

Table II shows the shot-sizes and their calculated frequency maxima lying within the recording passband. The 50 pounds shot maxima are relatively more numerous.

The above analysis is a simplified one since many other rays can contribute towards a complex interference pattern. However, it is felt that the rays dealt with play a major role in the spectral appearance of a seismogram on account of their relatively large energies.

TOTAL SPECTRA

A seismogram spectrum is the product of all the discussed spectra and those for the recording instruments and ground, consequently, its peaks are not simply related to the peaks in any one spectrum.

If, for simplicity, we represent the transfer function of the recording instruments by a 0-30 Hz gate and calculate the product of all spectra excluding that for the sub-bottom layers, Figure 6(a) results. It is clear from this figure that the main spectral lobe for the 50-100 pound shots is narrower than the corresponding lobe for the 2.5 - 10 pound shots, a fact in general agreement with our observational spectra, for example 2a and 2d. For comparison, Figure 6(b) shows a similar plot when the bubble pulse pressure peaks are one-third and one-twelfth the explosive pulse peak respectively. Here the spectrum for the large shots seems wider than that for the small shots, a result contrary to observed spectra. Hence it may be concluded that the bubble pulses for the large shots must decay at a rate slower than that for the small shots. The filtering produced by the ground has been neglected in the above discussion.

It is apparent that the relatively high side lobes in figures 6(a) and 6(b) result from the assumed gate transfer function of the recording instruments. It was not possible to calculate the true response of the entire refraction recording system, but qualitative reasoning indicates that it is bell-shaped in the range 5 - 30 Hz. A bell-shaped response would certainly discriminate against the high side lobes.

CONCLUSION

Since the power spectra for the smaller shots (2.5-10 pounds) appear relatively broad compared to those for the larger shots (50-100 pounds), it seems advisable to use small shots in marine refraction surveys, for if a deconvolution process is used to remove the bubble pulse effects from a seismogram having a narrow spectrum, nothing very meaningful would remain for interpretation.

The Rayleigh-Willis constant of 5.0 obtained in Lake Superior differs from those obtained by other workers and seems dependent on the type of explosive as well as on the shooting environment.

TABLE I

Shot Size pounds	Mean detonation depth – meters	Computed period T_c – ms
1.25	59	60
2.5	57	80
5.0	64	80
10.0	55	120
25.0	116	100
50.0	145	100
100.0	141	134
200.0	156	154

TABLE II

Shot No.	Weight pounds	Depth meters	Shot position filter maxima in passband Hz
6244	2.5	48	7, 21
6222	5.0	70	5, 15, 25
4125	10.0	54	6, 18, 30
4116	50.0	133	2.5, 7.5, 12.5 17.5, 22.5, 27.5

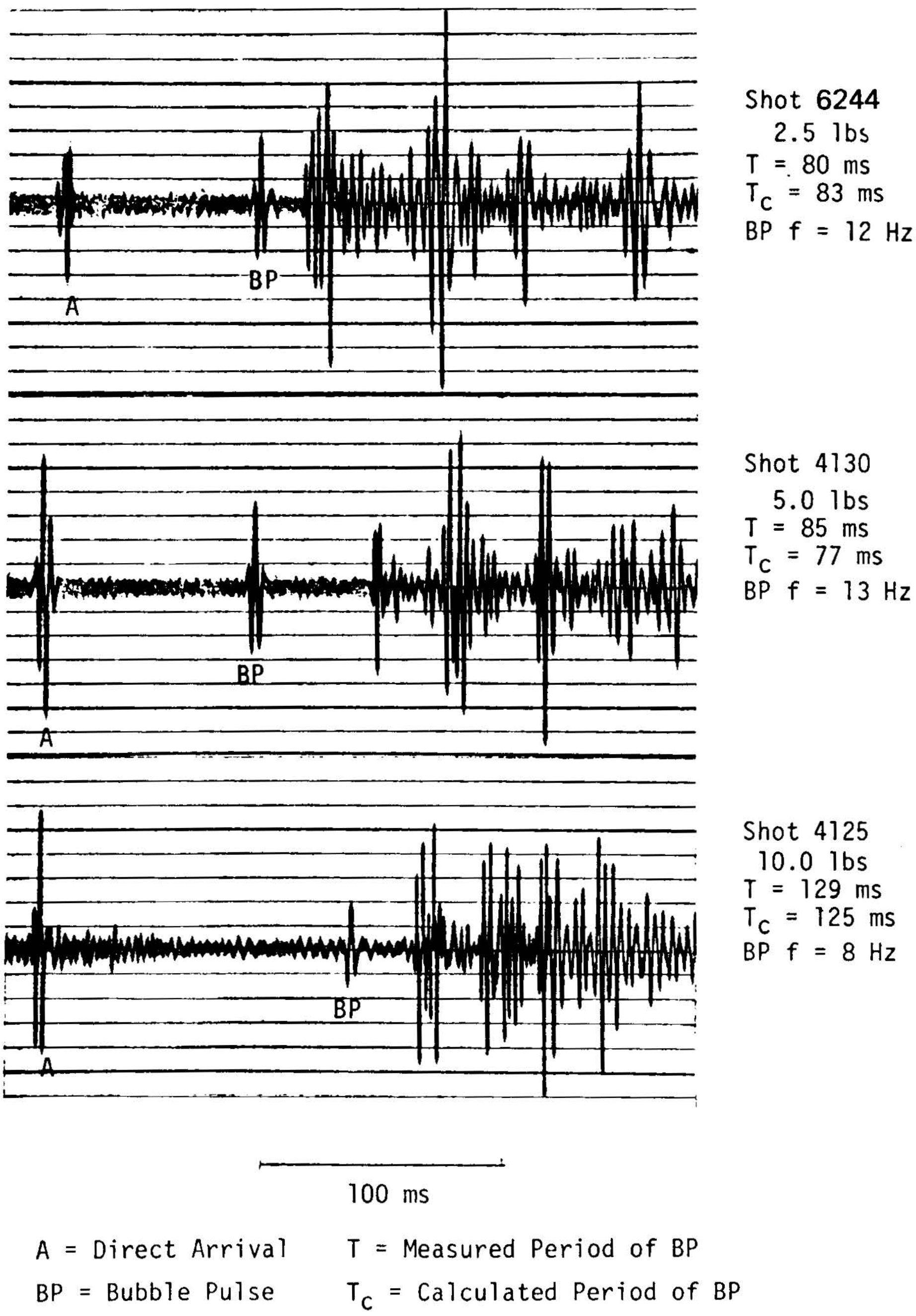

Figure 1. Towed hydrophone recordings showing bubble pulses.

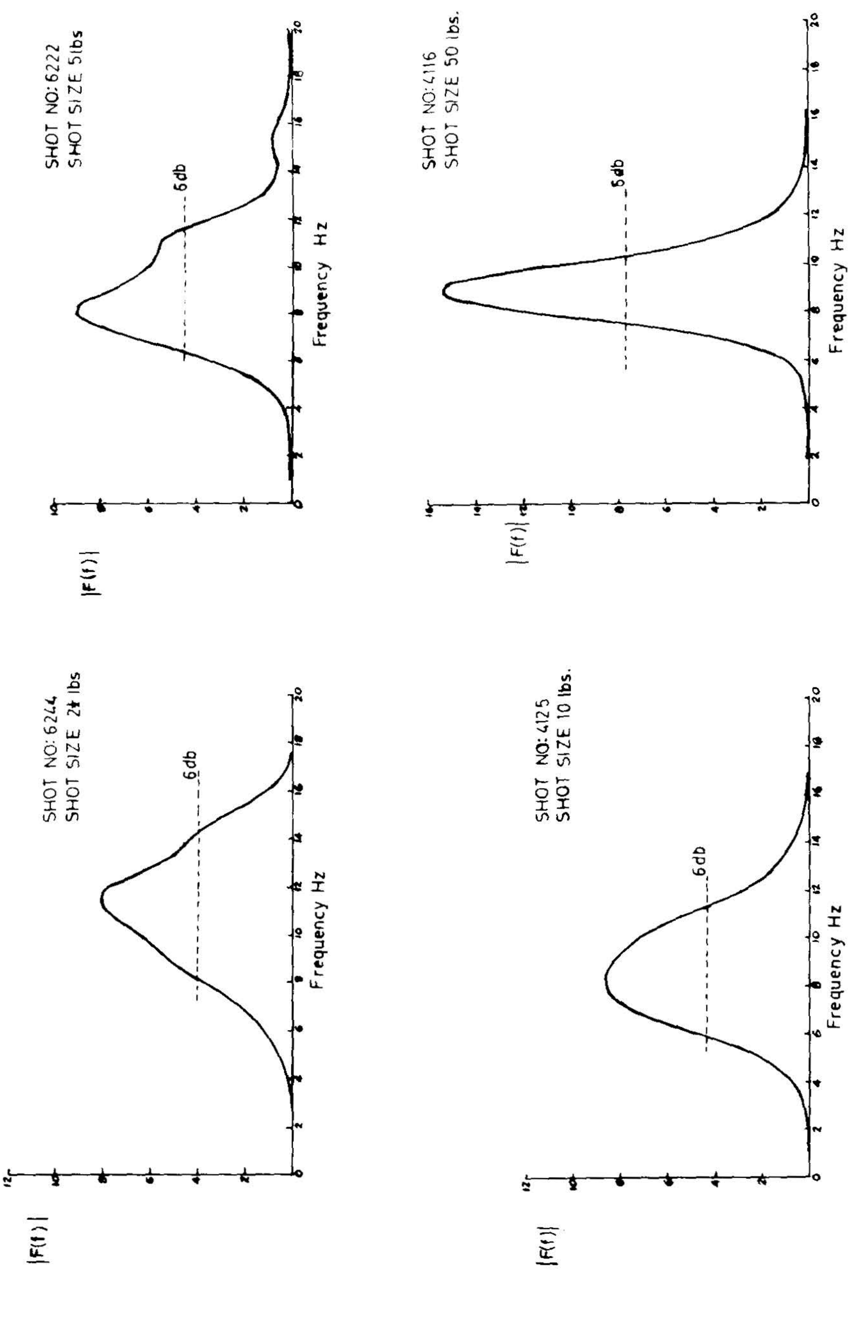

Figure 2a-2d. Spectra for shots of sizes 2.5, 5, 10 and 50 pounds.

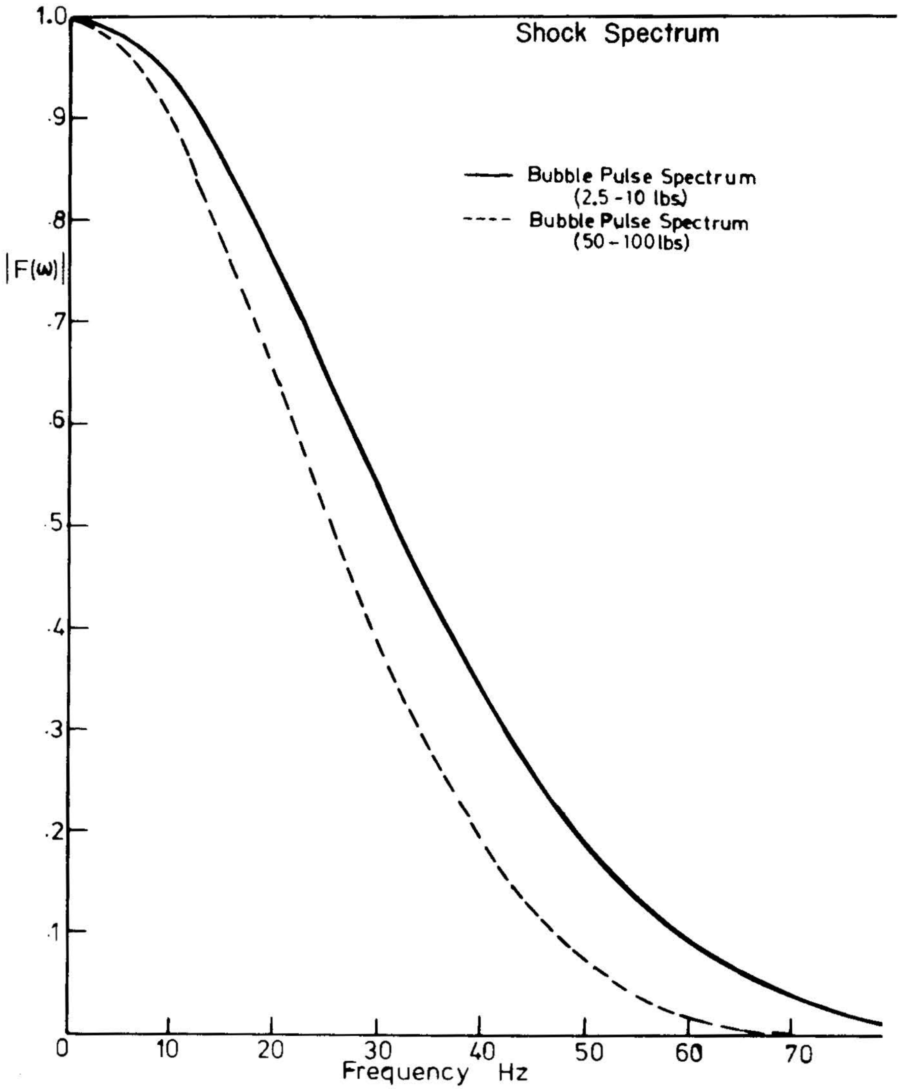

Figure 3. Spectra of explosive (shock) pulse and bubble pulses.

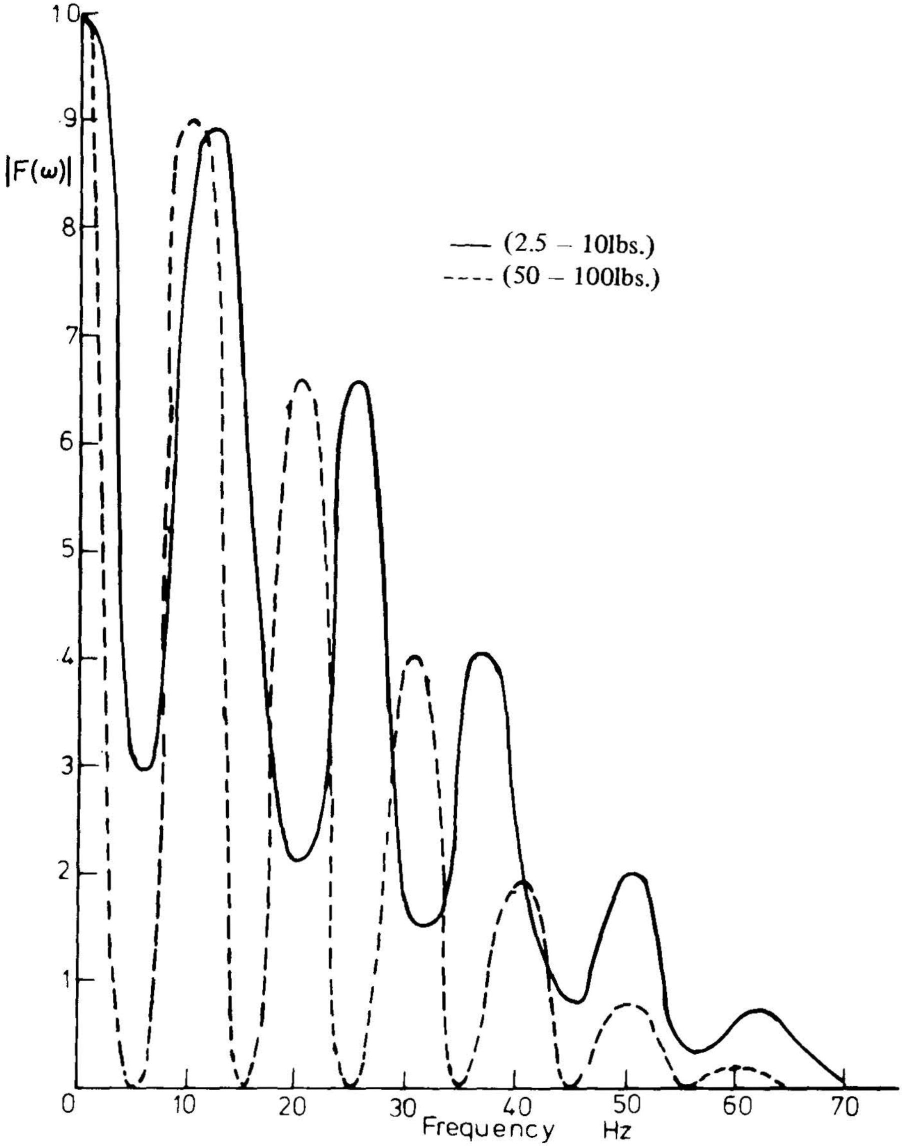

Figure 4. Spectra of explosive and bubble pulses combined - The peak pressures of all the pulses are assumed to be the same.

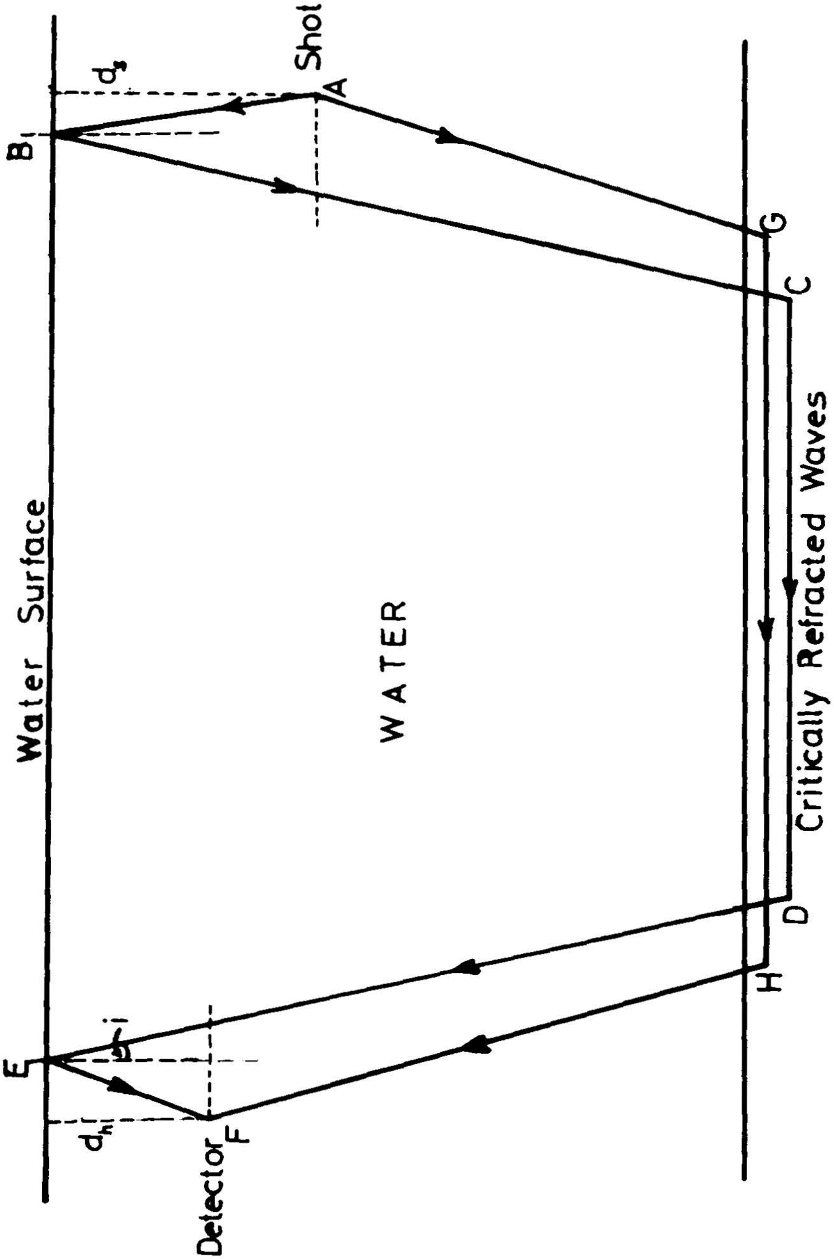

Figure 5. Diagram illustrating the detector and shot position filters.

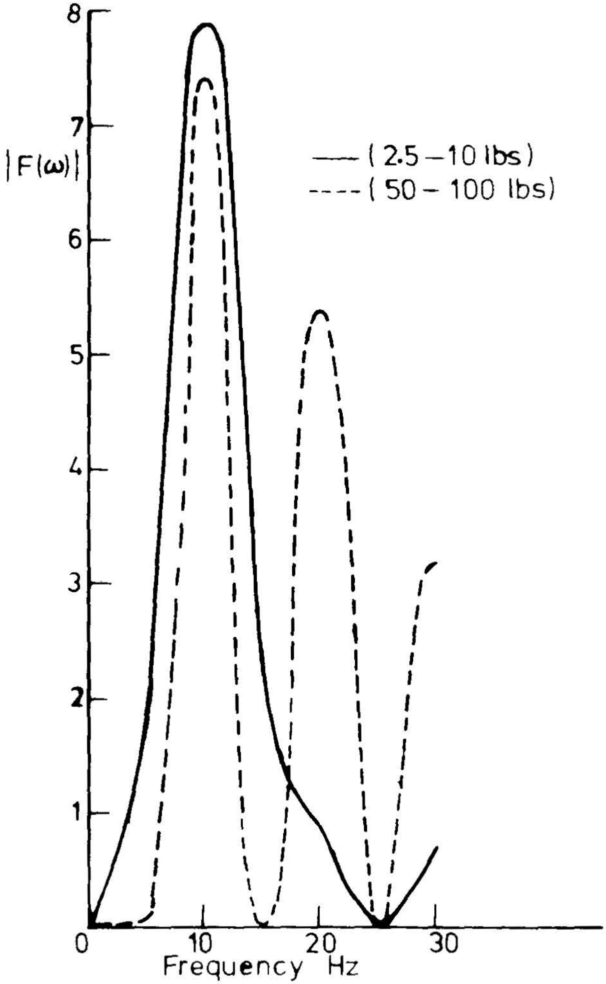

Figure 6a. Total spectra assuming the peak pressures of the explosive and bubble pulses are the same.

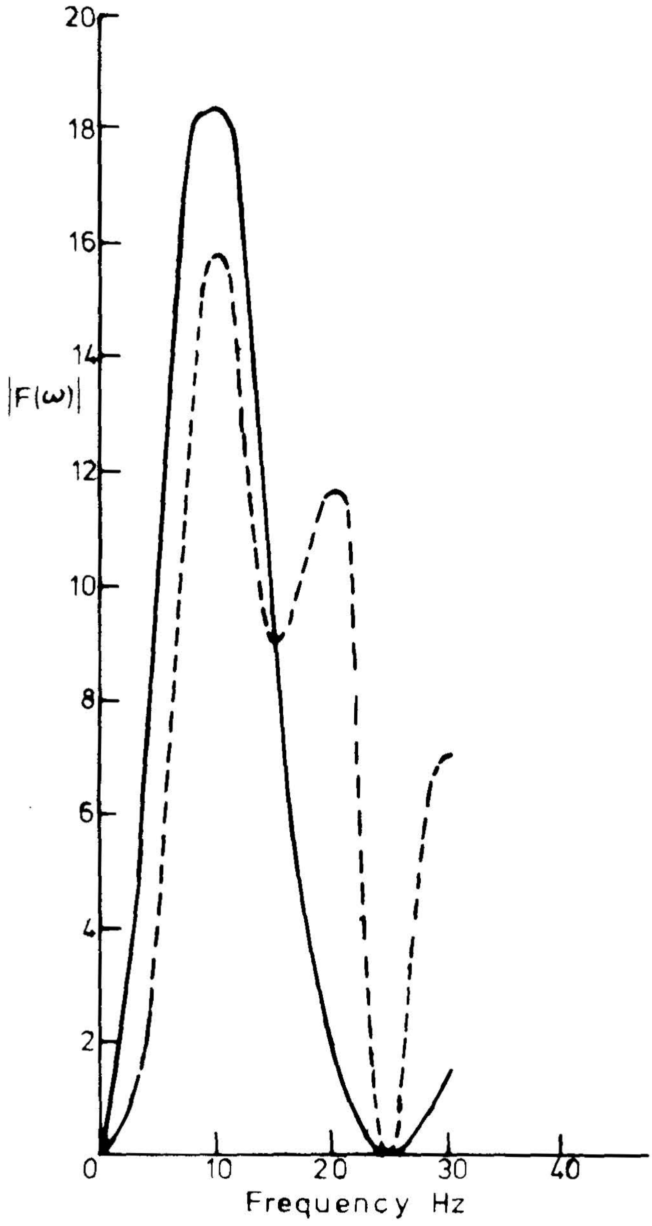

Figure 6b. Total spectra assuming that the first and second bubble pulse pressure peaks are 1/3 and 1/12 of the explosive pulse peak.

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