MOMENTUM TRANSFER AND SURFACE PRESSURE 1

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RESUMEN

El mecanismo mediante el cual el intercambio de cantidad de movimiento uv en la atmósfera origina cambios de presión en la superficie es explicado y demostrado. Se demuestra que dicho mecanismo determina el perfil meridional de la presión media al nivel del mar y la intensidad de las celdas de Hadley y Ferrel. Se analizan asimismo, las implicaciones relativas a las circulaciones oceánicas y a la ecuación de la corriente de Ekman.

ABSTRACT

The mechanism by which the exchange of momentum, uv, in the atmosphere leads to surface pressure changes is explained and demonstrated. The mechanism is shown to govern the meridional profile of mean sea-level-pressure and the strenght of the Hadley and Ferrel cells. Implications with respect to ocean circulations and the Ekman drift equation are discussed.

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INTRODUCTION

Jeffreys (1926) showed that surface friction produced an exchange of momentum between earth and atmosphere and that the horizontal flow aloft of this momentum must be an important element in the mechanics of the general circulation of the atmosphere. Starr (1948) showed how momentum transfer must, of necessity, modify the shape of waves in the atmosphere, making them non-symetrical with axes titled toward NE-SW. The following year White (1949) discussed the role of mountains in the momentum balance, and Widger (1949) reported on a study of the horizontal flow of momentum. Priestly (1951) demonstrated the magnitude of momentum exchange by estimating the drag of the wind on the earth's surface and in 1952 Palmen and Alaka demonstrated the role of momentum transfer in driving the Hadley cell.

Tucker (1960) was perhaps the first to estimate a budget that balanced momentum inputs and outputs and horizontal flow over a considerable range of latitude. Starr's (1968) book discusses applications of momentum transfer equations to atmospheres in general of the earth, sun and planets, and to spiral galaxies. An excellent summary of these studies, with an extensive bibliography, is provided by Newton (1971).

A unified body of atmospheric statistical averages and variances has been provided, at long last, by Oort and Rasmusson (1971). This volume, covering a period of five years, also contains tables of the northward flow of angular momentum, averaged around full circles of latitude, for various levels in the atmosphere and for the total of all levels.

The momentum feeding into a column of air by horizontal transfer over a long period of time must equal the momentum coming out of the column by frictional drag at the earth's surface. In Figure 1 the solid line shows the meridional profile of momentum gain or loss in each 5° latitude ring of atmosphere. These values were obtained by differencing the total momentum transport (sum of yearly means of Tables C-1-a, b, c of the Oort and Rasmusson report). The dashed line in Figure 1 is the gain or loss at the earth's surface, as computed by Hellerman (1967) for ocean areas only, using wind rose data going back to the days of the sailing ships. Our dashed line is interpolated from his Figure 5.

The correspondence between these two lines offers evidence of the internal consistency of the Oort and Rasmusson data and demonstrates the momentum balance requirement of the atmosphere. It also suggests that the drag over land must be nearly equal to that over the sea. This conclusion in strengthened when the drag of the mountains, as estimated by White (1949), is subtracted from the advected momentum. This correction, indicated by the circles in Figure 1, fills the gap between the two profiles in mid-latitudes. Drag coefficients and wind speeds differ substantially between land and sea, and it can hardly be by chance or coincidence that, in the absence of mountains, they have virtually the same drag at all latitudes. A reason for the equality will be discussed in the next section.

Jeffreys (1933) showed that vertical cells in the general circulation would be negligible if they depended only on the observed temperature distributions, and Green (1970) showed how the horizontal flow of momentum was produced and how it drove the vertical cells and governed the surface winds. The surface winds, in turn, are closely related to the gradients of sea level pressure and yet this pressure must also depend on horizontal divergence of air aloft to balance that provided by the Ekman drift in the friction layer. The Oort and Rasmusson data will be used to show how the atmosphere achieves this balance and the resulting profiles will also shed light on the following questions:

1. Is the Ekman drift fully developed in the atmosphere and in the oceans?
2. Why are the sea-level-pressure patterns more strongly developed over the oceans?

The Sea-Level-Pressure Profile

Since drag depends on the velocity of the surface wind which, in turn, depends on the pressure gradient, and since drag equals the divergence of angular momentum aloft, then it follows that the pressure grandient mus depend, in some way, on the flow of angular momentum. The sea-level-pressure gradient is a necessary intermediary between the drag term, τ, and the $\frac{\partial uv}{\partial y}$ term pictured in Figure 1.

Others have shown a mathematical development leading to Equation 3 (*c. f.* Widger, 1949, Tucker, 1960). For our purpose we will simplify the development by considering only the u component of the equation of motion.

$$\frac{\partial u}{\partial t} = -\underline{V} \cdot \nabla u - \frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_x \cdot \qquad (1)$$

Multiplying this by ρ and expanding we have:

$$\frac{\partial \rho u}{\partial t} - u\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho u\underline{V} + u\nabla \cdot \rho\underline{V} - \frac{\partial p}{\partial x} + \rho fv + \rho F_x \cdot$$

The equation of continuity shows that $u\,(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V) = 0$, and if we integrate the remaining terms in x and z around the world in a closed circle of latitude we have:

$$\int_x\int_z \frac{\partial \rho u}{\partial t}\,dxdz = \int_x\int_z \left(-\frac{\partial \rho uv}{\partial y} + \rho fv\right) dxdz + \overline{\tau}_x \cdot \qquad (2)$$

This assumes a smooth globe (no mountain effect), and the friction, F, has been considered zero in the horizontal direction in the free air, so that the term

$\int_x\int_z \rho F_x\,dxdz$ becomes $\overline{\tau}_x$ or frictional drag at the surface; the bar indicates a longitudinal average. Reversing the order of integration and differentiation we have

$$\frac{\partial \bar{u}}{\partial t} = - \frac{\partial \overline{uv}}{\partial y} + f\bar{v} + \bar{\tau}_x$$

where the bar indicates the average by increments of mass in both x and z.

In the steady-state or time average $\frac{\partial \bar{u}}{\partial t} = 0$, and $\bar{v} = 0$, (no net mass flow past the latitude into the fixed volume on either side) then

$$\frac{\partial \overline{uv}}{\partial y} = \bar{\tau}_x \ . \qquad (3)$$

as demonstrated in Figure 1.

Sverdrup (1947) developed an Ekman drift equation for the ocean. He started with the same basic equation of motion, plus the v component, and equated the friction to τ as was done in our Equation 2. Thus his method applies also to the atmosphere if the sign of τ is reversed. He defined the Ekman layer as extending to a depth where $\nabla_H p = 0$. This condition is not found in the atmosphere and the upper boundary of the atmospheric Ekman layer is not determined, but we may assume that such a boundary does exist and in the material that follows it will be labeled symply p_E.

Although in the steady state $\bar{v}$ must integrate to zero in the entire depth of the atmosphere, it is composed of two equal but opposite components: the Ekman drift and the counter-flow aloft. We will restore them to Equation 3 and examine them independently because they furnish the mechanism for the sea-level-pressure profile. Thus,

$$0 = - \frac{1}{g}\int_{p_o}^{0} \frac{\partial \overline{uv}}{\partial y}\, dp + \frac{f}{g}\int_{p_E}^{0} \bar{v}\, dp + \frac{f}{g}\int_{p_o}^{p_E} \bar{v}\, dp - \bar{\tau}_x \ . \qquad (4)$$

In equation 4 the last two terms have their effect entirely within the Ekman layer. Sverdrup's (1947) Equation 9a is

$$\frac{\partial P}{\partial x} = \lambda M_y + \tau_x$$

where $\partial P/\partial x$ is the total pressure gradient in the Ekman layer this has been averaged to zero in our case. His $\lambda \equiv f$, and his

$$M_y = \int_0^{z_E} \rho v dz = \frac{1}{g} \int_{p_o}^{p_E} v dp$$

Thus the last two terms in our Equation 4 are the same as Sverdrup's terms (with the sign changed on τ because the drag on the atmosphere becomes an acceleration on the ocean). Since these last two terms of Equation 4 sum to zero, the first two terms do also and, by Equation 3, all four terms are equal in magnitude.

There is a question, however, as to whether the Ekman drift concept is acceptable in general. Sverdrup restricted its use to situations where the lateral boundary conditions kept the field acceleration terms near zero. In our case the field acceleration terms include that part of $\partial \overline{uv}/\partial y$ which lies within the Ekman layer plus the vertical flow of momentum through the top of the layer. These accelerations are not zero within the Ekman layer, but we will show by an examination of vertical motion that the Ekman drift appears to be fully developed in the atmosphere and that therefore the field acceleration terms within the layer must be small or of opposite sign. If, in the meantime, we accept the Ekman drift concept as expressing the general picture in the atmosphere we have:

$$\int_{p_o}^{0} \frac{\partial \overline{uv}}{\partial y} dp = f \int_{p_E}^{0} \overline{v} dp \qquad (5)$$

To a close approximation, the pressure at the surface is given by the weight of the overlying column of air,

$$p_o = \int_0^\infty g\rho \; dz \; .$$

Therefore, the ratio of pressure change, or "tendency", is z = w = 0 at the surface and, when we average around the globe in a ring of latitude, $\partial \rho u/\partial x$ becomes zero so that

$$\frac{\partial p_Z}{\partial t} = -\int_z^\infty g \left[\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} \right] dz + g\,(\rho w)_Z \; . \qquad \text{(Craig, 1960)}$$

$$\frac{\partial \overline{p_o}}{\partial t} = -\int_0^\infty g \frac{\partial \rho \bar{v}}{\partial y} dz = -\frac{\partial}{\partial y}\int_0^\infty g\,\rho \bar{v}\; dz = -\frac{\partial}{\partial y}\int_{p_o}^0 \bar{v}\, dp \; . \quad (6)$$

Substituting from Equation 5, we have:

$$\frac{\partial \bar{p}_o}{\partial t} = -\frac{\partial}{\partial y}\frac{1}{f}\int_{p_o}^0 \frac{\partial \overline{uv}}{\partial y} dp - \frac{\partial}{\partial y}\int_{p_o}^{p_E} \bar{v}\; dp \; . \qquad (7)$$

Equation 7 shows that mass-divergence, arising out of horizontal momentum transfer, provides one term of the pressure tendency. This is offset by a tendency from the divergence in the Ekman drift governed by τ. Figure 2 shows that the sea-level-pressure profile (solid line) does, indeed, conform to this tendency (dotted line) as estimated, in this case by differences at 5° latitude intervals. The Ekman drift provides an equal and opposite tendency which would appear to relieve the pressure build-up and provide a steady-state or average profile.

It was shown by Jeffreys (1919) that no change in surface pressure is possible in a field of pure geostrophic motion. Pressure changes must be associated with accelerations and this study shows that they are provided by momentum divergence and by drag. It is not difficult to envision the mechanics involved: The poleward flow of westerly momentum is greatest above the surface high pressure zone. Equatorward of the surface high, the westerlies are losing momentum –it goes out toward the pole, faster than it comes in from low latitudes. Thus the winds fall below geostrophic velocity and are deflected slightly downgradient, toward the pole. On the poleward side of the surface high the opposite is true; the westerlies are gaining momentum, become super-geostrophic and are deflected up– gradient toward the equator. The resulting convergence produces the surface high-pressure belt. The sea-level-pressure profile continues to develop until the Ekman drift of the resulting surface winds become just sufficient to provide an equal amount of divergence.

The ageostrophic drift aloft must be sufficient to compensate for the momentum divergence as shown by Equation 5. This drift supplies the upper limbs of the three vertical cells in the general circulation; and, in the Ferrel cell, because the momentum is converging, the drift is in the opposite direction to the normally-expected thermal drive.

It is evident from the above that the strength of the drag is a requirement of the general circulation. The fact that the drag does not vary between land and sea (as shown in Figure 1) means that the Hadley and Ferrel cells have the same strength over land as over sea. The greater roughness of the land allows the required drag to develop from lower surface winds and pressure gradients. This is why the large, semi-permanent, high-pressure cells are primarily a feature of the oceans.

All the above equations could be written in components of motion tangential and normal to the mean flow of the atmosphere. Instead of integrating around a circle of latitude, we could integrate around a closed streamline. This suggests that even small-system pressure changes may be governed by this momentum-transfer mechanism.

When momentum diverges within the Ekman layer it will add to or subtract from the acceleration provided by drag. The Ekman drift will be partly mixed with the momentum drift aloft. The sea-level-pressure gradient and drag will be unaffected, but the *apparent* tendency, as estimated by either term of Equation 7, will be inaccurate, and the vertical motion will be reduced or enhanced from that predicted by Equation 7.

In Figure 2 the dashed line is the vertical motion of the atmosphere averaged throughout its depth (Table A3 of Oort and Rasmusson). This vertical motion, like the dotted line profile of pressure tendency, is expressed in mb/hr. The moderately good correspondence between these two profiles suggests that only a small portion of the horizontal momentum flow can be taking place within the Ekman layer. Inspection of the Oort and Rasmusson data also indicates that, except for the deep tropics, the great bulk of the momentum flow is at high levels. Equations 5 through 7 are exact only if the term $\int_{p_o}^{p_E} \bar{v}\, dp$ is defined as the Ekman drift as specified by Sverdrup. The true drift within the Ekman layer will depart from the Ekman drift to the extent that there is horizontal momentum divergence within the layer, but Figure 2 indicates that this divergence must be small.

VERTICAL CELLS IN THE STRATOSPHERE

The vertical motion pictured in Figure 2 is the vertical average of a quantity that varies with elevation. Starting from zero at the surface it becomes much above average at the top of the Ekman layer, falls to zero again at the troposphere and becomes the opposite in the stratosphere, due, presumably, to a reversal of the momentum divergence patterns. Thus, upon each vertical cell in the mean circulation there are superimposed two additional cells, one in the troposphere, the other above it in the stratosphere. In such a pair the upper cell must rotate in a direction opposite to that of the lower cell because the two together must add to zero when averaged in the vertical. The lower cell, of course, rotates in the same sense as the over-all average cell, enhancing the vertical motion in the troposphere.

It seems likely that these additional cells, superimposed on a general circulation that is otherwise closely related to the sea-level-pressure profile, will also be a feature of individual synoptic flow patterns. In the individual case the large-scale vertical circulation in the stratosphere possibly could be determined. If these were *subtracted* from an estimated mean vertical circulation, as determined from the sea-level-pressure chart, a more nearly complete picture of tropospheric vertical motion would emerge.

OCEAN CIRCULATION

There appears to be no way to measure ocean currents with an accuracy to match what we have achieved in the atmosphere, thus it is of interest to speculate on the general circulation of the oceans based on what we have learned about the atmosphere. In the atmosphere, as suggested by Lorenz (1969) and supported by the above analysis, the flow of momentum is a cause, and the drag on the surface is an effect. In the ocean the opposite must be true since the ocean is said to be wind-driven with small contribution from the thermal drive. Thus the ocean must find a mechanism for transferring momentum to balance the drag of the winds. The major gyres, for instance, receive an almost continuous angular momentum input from the trade winds and westerlies; they apparently do not transfer this angular momentum to the solide earth since the bottom waters are relatively still, so how do they get rid of it?

Part of the answer could be provided by an inferred model of the general circulation of the oceans, a model that exactly matches that of the atmosphere with its Hadley, Ferrel and polar cells and a mass flow in each cell that exactly matches the mass flow of the corresponding cell in the atmosphere.

An equal but opposite Ekman drift in the ocean's surface layer would provide such a vertical circulation pattern, and the drift in the ocean must be equal and opposite to that in the atmosphere in order to conserve momentum at each point of the interface. (In the northern hemisphere the wind is deflected to the left by friction at the interface; the water must be deflected to the right with a mass flow exactly equal to the deflection of air so that the two components normal to the geostrophic flow of air sum to zero).

Thus the drag of the wind provides the v and w components of the general circulation of the ocean and dictates their total transport. These v and w transports regulate the thermohaline structure –the core of the gyre is composed of the less dense water that originated at the surface– and the resulting hydrostatic profile dictates the geostrophic u components. The u transport, however, does not have to match the u transport of the atmosphere, and it has the same sign only because vertical stability requires the less dense water to be at the surface. If the ocean were homogeneous in temperature and salinity the u transport would be zero at all levels and latitudes.

Sverdrup (1947) also specified that the Ekman layer extended to a depth where the horizontal pressure gradient was zero. This is the level of maximum vertical velocity, so the low-density core of the gyre will be carried to deeper levels and create deep-layer pressure gradients and circulation opposite to those of the surface layer.

In such a model it is not difficult to visualize that the $\overline{u}\ \overline{v}$ at all levels and latitudes is negative, thus compensating in part for the mostly positive u v of the atmosphere. The remainder, u'v', must be supplied by internal waves.

SUMMARY

The sequence of cause and effect in the general circulation and, by implication, is smaller-scale circulation, is as follows:

1. Momentum is transferred by the same horizontal turbulence, large and small scale, required for the transfer of heat.
2. Divergence in this flow of momentum provides ageostrophic components that induce drift in a direction normal to the geostrophic flow.
3. Divergence in the drift provides tendencies that generate the sea-level-pressure pattern.
4. The sea-level-pressure gradients generate winds and frictional drag that lead to Ekman drift which balances the momentum induced by the drag and provides a low-level return flow to compensate for the high-level drift.

5. Divergence in the Ekman drift offsets the sea-level-pressure tendencies generated aloft and provides a mass balance.

The Ekman drift is shown to be fully developed in the general circulation of the atmosphere, and, from momentum balance considerations, we must infer that the Ekman drift is also fully developed in the oceans.

The sea-level-pressure patterns are more strongly developed over the oceans because the oceans are "smoother" than the land but must provide the same total drag.

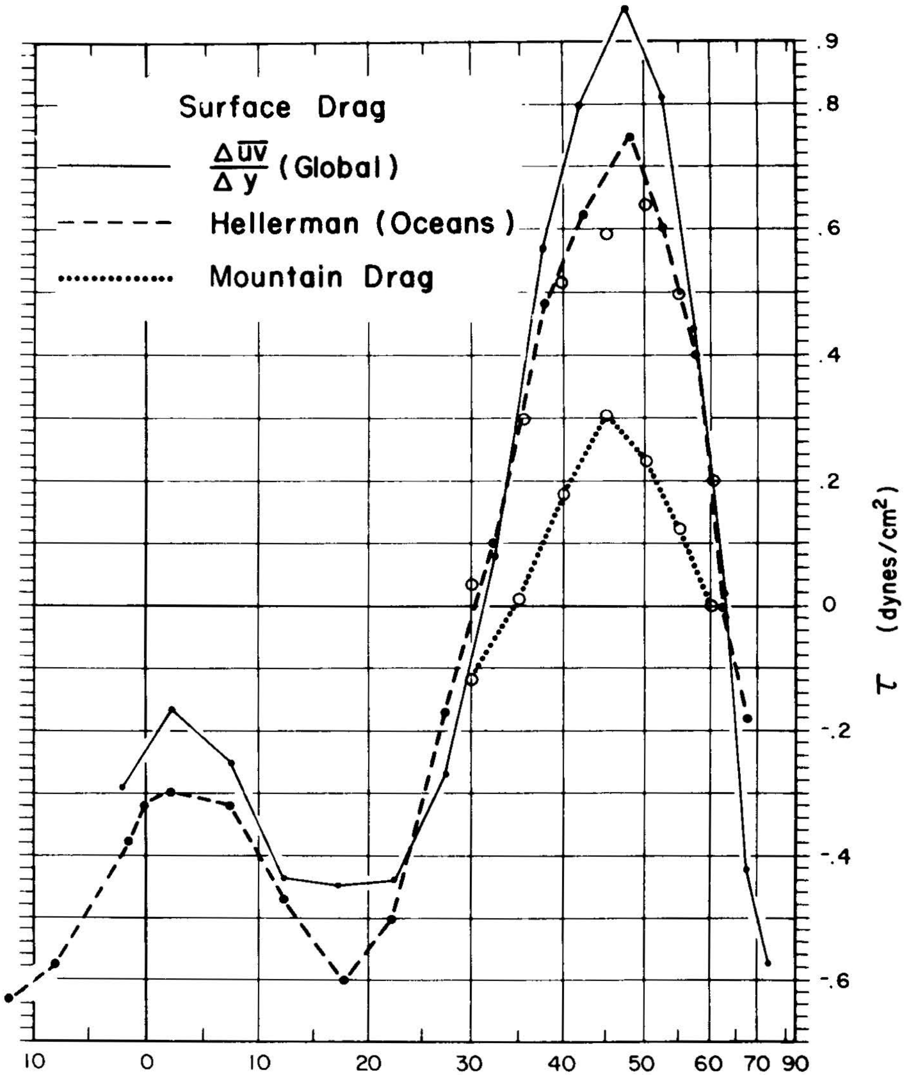

Figure 1. Advected momentum compared with surface drag. The solid line is $\Delta \overline{uv}/\Delta y$, advected momentum averaged through the depth of the atmosphere, around the Globe at 5° circles of latitude and over a five year period (from Oort and Rasmusson, 1971). Dashed line is the ocean-surface drag computed from many years of ocean wind data by Hellerman (1967). Dotted line is the torque exerted on the atmosphere by mountains (White, 1949) and the circles show this subtracted from $\Delta \overline{uv}/\Delta \overline{y}$.

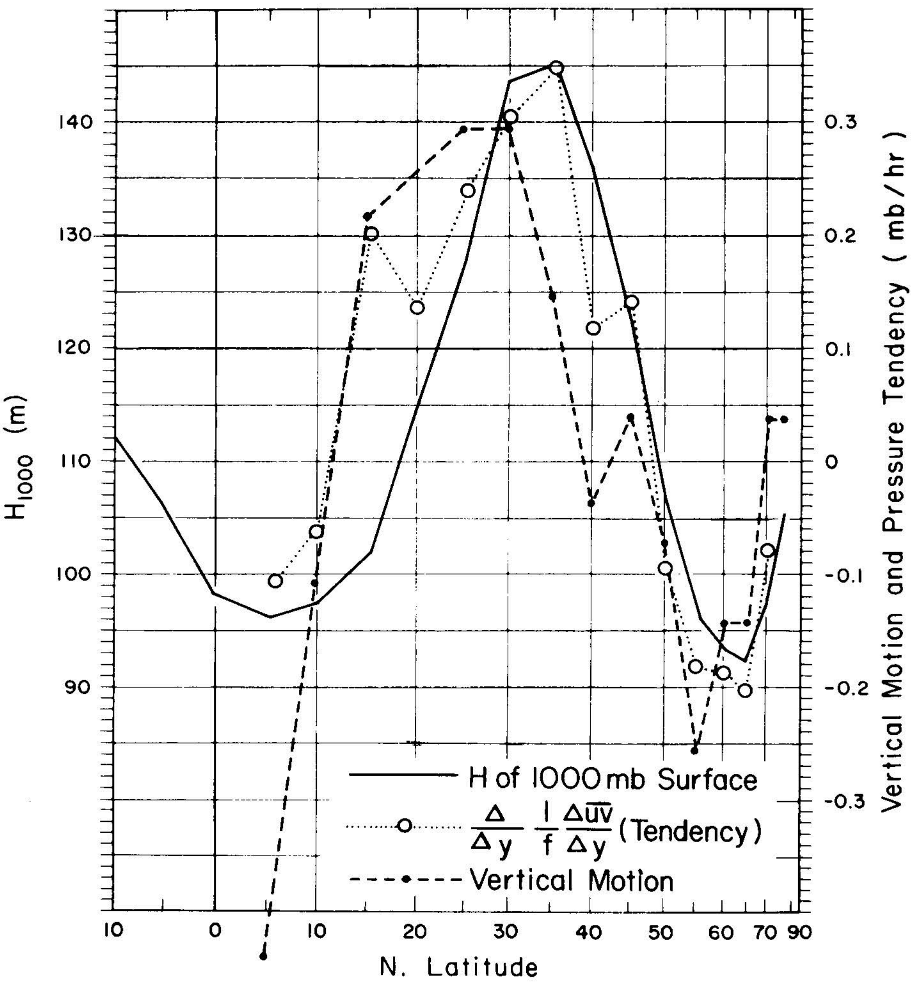

Figure 2. Surface pressure profile compared with pressure tendencies due to momentum transfer. Solid line is the height of the 1 000 mb surface (annual average) and dotted line is $\frac{\Delta}{\Delta y}\frac{1\Delta \overline{uv}}{f\Delta y}$ differenced at each 5° latitude. Dashed line is vertical motion. The dotted and the dashed line are both in mb per hour and both refer to the vertically averaged quantity. All data from Oort and Rasmusson (1971).

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