CLIMATE MODELS AND VARIATIONS IN THE SOLAR CONSTANT

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RESUMEN

Se da un resumen de los resultados obtenidos por varios autores, que usan modelos climáticos simples con promedios zonales y anuales. Se demuestra que la mayoría de estos resultados puede explicarse fácilmente usando un modelo promediado globalmente. Se demuestra también que, según la parametrización usada, el porcentaje de cambio de constante solar, relativo al valor actual, requerido en el modelo (a) para iniciar una época glacial es de -2 a -5 por ciento, (b) para producir una tierra cubierta de hielo es -6 a -14 por ciento, y (c) para iniciar el proceso de derretimiento en una tierra cubierta de hielo es de -8 a + 33 por ciento. Por lo tanto, el margen de posible error aumenta significativamente al alejarse de las condiciones actuales.

ABSTRACT

A summary is given of results obtained by a number of authors using simple zonally-and annually-averaged climatic models. It is shown that most of these results can be explained quite easily using a globally-averaged model. It is also shown that, depending on the parameterization used, the percentage change of the solar constant, relative to the present value, required in the model, (a) to initiate an ice age is -2 to -5 percent, (b) to produce and ice-covered earth is -6 to -14 percent, and (c) to start the melting process on an ice-covered earth is -8 to + 33 percent. Thus the margin of possible error increases significantly as one moves away from present conditions.

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INTRODUCTION

The history of climate modelling can be traced back to 1921 to a paper by Defant. However, it was apparently not until 1969 that the effect of fluctuations in the solar constant on global climate was discussed quantitatively in separate papers by Budyko and Sellers. Both used simple annual- and zonally-averaged models based on an equation of the form

$$Q_j (1 - \alpha_j) - I_j = \Delta F_j, \qquad (1)$$

where Q_j is the solar radiation received annually at the top of the atmosphere in the j-th latitude belt; α_j is the fraction of this returned unabsorbed to space (the albedo) and is most strongly dependent on cloud cover and the fraction of the earth's surface covered by ice and snow; I_j is the infrared radiation emitted to space, expressed as a function of the surface temperature T_j and cloud cover; and ΔF_j represents the divergence of the poleward heat transport by atmospheric and oceanic circulations. F_j is usually parameterized in terms of the poleward (surface) temperature gradient and an eddy diffusivity. By expressing the unknowns in (1) in terms of the surface temperature and applying the appropriate boundary conditions the equilibrium temperature distribution can be determined for any selected value of the solar constant.

Budyko and Sellers both obtained essentially the same results. Budyko found that when the solar constant is reduced by 1.6 percent the ice sheet expands to about 50° latitude after which it begins to advance rapidly southward all the way to the equator as a result of autodevelopment. Sellers concludes that a reduction of 2 to 5 percent in the solar constant would be sufficient to initiate another ice age and that any further decrease would result in a rapid transition to an ice-covered earth. (Öpik (1965) and Eriksson (1968) had earlier suggested that such an explosive development could take place when ice coverage extends beyond a certain latitude limit (near 50°)).

Since 1969, the models of Budyko and Sellers have been analysed

by several other investigators (Faegre, 1972; Budyko, 1972; Schneider and Gal-Chen, 1973; Dwyer and Petersen, 1973; Gordon and Davies, 1974; Held and Suarez, 1974; Chylék and Coakley, 1975; Gal-Chen and Schneider, 1975; and North, 1975a,b).

Although there are some contradictions amoung these various studies, certain results emerge repeatedly and seem to represent real features of the models. These include the following:

a. The reduction required in the solar constant in order for the earth to become ice-covered depends on the parameterization used (Gordon and Davies, 1974; Held and Suarez, 1974; Gal-Chen and Schneider, 1975).

b. The present solar constant is compatible with at least three equilibrium climatic regimes: the present climate; an ice-covered earth; and a climate intermediate between the first two (Budyko, 1972; Faegre, 1972; Held and Suarez, 1974; North, 1975a).

c. With an ice-covered earth, the solar constant would have to be increased by 30 to 40 percent above its present value to start melting the ice and cause a jump to an ice-free earth (Budyko, 1972; Faegre, 1972; Held and Suarez, 1974; North, 1975a).

d. The results obtained are relatively insensitive to the dynamical parameterization used for ΔF_j (Gal-Chen and Schneider, 1975; North, 1975b). Held and Suarez (1974), however, find that increasing the transfer efficiency reduces the ice cover, causes a poleward shift of the latitude to which the ice must advance before it grows unstably to cover the earth with a further reduction in the solar input, and makes the ice cover more sensitive to slight changes in the solar constant.

The purpose of this note is not to add anything new to this discussion but to show that most of the above results can be obtained directly and clearly by considering a simplified version of (1). It is hoped that the following analysis will also point out some of the hazards and uncertainties involved in climate modelling.

ANALYSIS

If (1) is averaged over the globe and solved for the solar constant S, which in the following will be treated as the dependent variable (North, 1975a), we get, approximately,

$$S = 4 \bar{I}/(1 - \bar{\alpha}) \tag{2}$$

In the climate models $\bar{I}$ and $\bar{\alpha}$ are parameterized as functions of the average sea-level temperature $T_o(K)$, which becomes, in this case, the independent variable. For example, for $\bar{I}$ Budyko (1969) uses

$$\bar{I} = 0.0035T_o - 0.61 - (0.0025T_o - 0.61)n \text{ ly/min} \tag{3}$$

and Sellers (1969) uses

$$\bar{I} = \sigma T_o^4 [1 - 0.5 \tanh (19T_o^6 \times 10^{-16})], \tag{4}$$

where σ is the Stefan-Boltzmann constant and n is the fractional cloud cover. Actually, a significantly better fit to the satellite data given by Vonder Haar and Suomi (1971) is obtained using

$$\bar{I} = \sigma (T_o - \Delta T)^4, \tag{5}$$

where

$$\Delta T = 20 \, e_o^{0.2}, \tag{6}$$

and e_o is the average sea-level vapor pressure (mbs).
The planetary albedo $\bar{\alpha}$ may be estimated from

$$\bar{\alpha} = (r_a + \frac{t_a^2 \alpha_s}{1 - r_a \alpha_s}) (1 - n) + (r_c + \frac{t_c^2 \alpha_s}{1 - r_c \alpha_s}) n, \tag{7}$$

where the subscripts "a" and "c" refer, respectively, to clear and cloudy sky conditions; r and t are, respectively, the fractional reflection and transmission of the atmosphere for solar radiation; and α_s is the surface albedo. In order to obtain a tractable system, each of these variables has to be related to T_o. For r and t, the two-stream approximate solution to the multiple scattering problem given by Sagan and Pollack (1967) implies, approximately, that

$$t = (1 + a\tau + b\tau^2)^{-1} \qquad (8)$$

and

$$r = c\tau t, \qquad (9)$$

where the coefficients a, b, and c depend on the single scattering albedo $\tilde{\omega}_o$ and assymetry factor $<\cos\theta>$ of the atmosphere being considered. τ is the optical thickness of the atmosphere for solar radiation. With clear skies (and for the following illustration) τ may be crudely related to the sea-level vapor pressure e_o in millibars by

$$\tau_a = 0.2 + 0.01 \, e_o. \qquad (10)$$

In the following it will also be assumed that for cloudy skies

$$\tau_c = 50 \, \tau_a/3 \qquad (11)$$

Equation (10) is based on pyrheliometric data for Tucson, Arizona.

Eriksson (1965) relates the surface albedo to the fraction of the earth i covered by ice using

$$\alpha_s = 0.07 \, (1 - i) + 0.65i. \qquad (12)$$

His paper also infers that a realistic relationships between i and T_o is

$$i = 24.075 - 0.15962\ T_o + 0.00026448\ T_o{}^2. \quad (296 > T_o > 240) \quad (13)$$

Using (3), (4), or (5) and (6) and (7) through (12), the solar constant in (2) can be related directly to T_o. The parameterization is crude but probably adequate for the present purpose, which is merely to illustrate the behavior of (2) without claiming any great numerical accuracy. The non-linearities neglected in going from a single point in space and time to (2) make it difficult to justify a more elaborate approach.

The values of the single scattering albedo and assymetry factor used for clear and cloudy skies are those given by Sellers (1973) and are listed in Table 1, along with the coefficients a, b, and c in (8) and (9). The latter were determined from

Table 1. The optical parameters and coefficients used in (8) and (9).

	Clear Skies		Cloudy Skies	
$\tilde{\omega}_o$	0.95	(0.63)	0.99	(0.968)
$<\cos\theta>$	0.64	(0.54)	0.84	(0.667)
a	0.3828	(0.8919)	0.1545	(0.3346)
b	0.0294	(0.3662)	0.002526	(0.01701)
c	0.2962	(0.2510)	0.1372	(0.2792)

equations also given by Sellers (1973). These optical parameters under-estimate atmospheric absorption of solar radiation by ozone, water vapor, dust, carbon dioxide, and clouds. The figures given in parentheses were obtained by using climatological estimates (Sellers, 1965) of total absorption and transmission, 20 and 75 percent, respectively, of the incoming sunlight with clear skies and 25 and 31 percent, respectively, with clouds, and working backwards to values of r and t. The absorption figures given above are higher than the value of 17 percent usually quoted in the literature (Paltridge, 1974). They have been increased in order to give a planetary albedo of 0.29 with 50 percent cloud cover and an average surface albedo of 0.094.

The sea-level relative humidity and the fractional cloud cover are assumed to remain constant at 80 percent and 0.5, respectively. Both Paltridge (1974) and Tempkin, Weare and Snell (1975) carry out calculations that indicate that cloud cover increases slowly with increasing T_o. On the other hand, Schneider and Washington (1973) obtained the opposite result working with the NCAR general circulation model.

RESULTS

The results of this study are summarized graphically in Figure 1, in which ice cover (or the equivalent latitude of the ice boundary) and the solar constant are plotted as functions of the sea-level temperature T_O. The models used to obtain curves a, b, and c differ only in the way in which the infrared emission to space is parameterized. Curve a was obtained using (3), with n = 0.5; curve b using (4); and curve c using (5) and (6). Note that curves a and b are virtually identical over a broad range of temperatures near the present (the circled point). This may help explain why Sellers and Budyko got similar results.

In each case the general shape of the curve is the same. North (1975a) divides similar curves into three phases, I, II, and III. In phase I, with an ice-covered earth, and in phase III, with little or no ice, T_o increases monotonically with increasing solar constant. In between, in phase II, T_o decreases as the solar constant increases.

Our present climate is in phase III. A decrease of the solar constant by 2 to 5 percent, corresponding to a temperature drop of 5K, would probably be sufficient to induce an ice age. For a decrease of more than 6 (curves a and b) to 11 (curve c) percent the only equilibrium climate is an ice-covered earth. These percentages are larger than those previously quoted in the literature partly because of the different parameterization used. Also, there is a tendency for the required changes in the solar constant to decrease as the sophistication of the models increases. For example, in the time-dependent continent-ocean model of Sellers (1973) a decrease in the solar

constant of only 1.7 percent would be sufficient to initiate an ice age. Whether or not this trend will continue for future models remains to be seen. It is conceivable that the parameterization of the atmospheric dynamics used in these models, which must ultimately produce a relationship between ice cover and temperature, is not quite good enough to yield accurate results. The more sophisticated the simple model is, the greater becomes the number of proceses that have to be parameterized, and the more circuitous becomes the route to the desired relationship between i and T_0.

For curves a, b, and c and for solar constants near the present value, an ice-covered earth is the most stable solution to (2). The earth should remain in this phase until the solar constant is increased by 16 to 33 percent above its present value. For any further increase, an ice-free earth is the only stable solution. For the present solar constant, if curve a, b, or c is used, there are three possible equilibrium climates, the present, an ice-covered earth, and a climate intermediate between these two. North (1975a) shows that the latter and all of phase II represents an unstable state to the extent that in a time-dependent model after a slight perturbation of the climate in this phase away from its equilibrium point the climate does not return to its original state but shifts rather to either phase I or III. From the figure itself it is hard to see how the climate could ever get into phase II as the solar constant varies.

The model used to obtain curves c and d is the same, except that for curve d the optical parameters in parenthesis in Table 1 are used. For temperatures near the present there is little difference between curves c and d. However, for the latter the present climate is the only equilibrium solution with the present solar constant. Phase II almost disappears, because the greater atmospheric absorption of solar radiation reduces the rate at which the planetary albedo decreases as the ice cover decreases and the temperature increases. For almost all values of the solar constant this curve gives an equilibrium sea-level temperature equal to or higher than that obtained from any of the other curves.

CONCLUSIONS

Figure 1 illustrates the hazards involved in attempting to extrapolate the earth's climate beyond the known range and to predict the climate's response to changes in the solar constant. Within a few degrees of the present temperature all four curves are in reasonable agreement and suggest that, for the parameterization common to all, the ice cover-temperature feedback, a long-term decrease of the solar constant by 2 to 5 percent would be sufficient to generate an ice age. Even without considering changes in cloud cover, the transition to an ice-covered earth is not nearly as obvious, requiring anywhere from a 6 to 14 percent reduction in the solar constant. In even worse shape is the solar constant needed to start the melting process on an ice-covered earth. This may range at least from 1.80 to 2.60 ly min^{-1} (1255 to 1814 Wm^{-2}) or from 8 percent below to 33 percent above the present value. Thus the possible margin of error increases rapidly as one proceeds away from present conditions.

The true response of our climate to drastic changes in the solar constant is still not known and is not likely to be known until methods can be devised to test the accuracy of the important parameterizations involved outside the range of variations experienced by man. It has been suggested that general circulation models, developed from the physical laws that govern atmospheric motion, coupled with equally fundamental models of the ocean circulation and the hydrologic cicle, be used for this purpose. This may eventually be possible.

Atmospheric dynamics do not appear explicitly in (2), which is one reason why the results obtained with the various models are fairly insensitive to how the poleward energy transfer processes in (1) are parameterized. Also, in the cosmic sense, the dynamics are important only to the extent that they help determine the ice cover-temperature relationship in Figure 1. Even here, an increase or decrease in the efficiency of the transfer processes would probably only shift the curve to the left or right, respectively, rather than significantly change its shape.

One final point. All climate models that require energy conserva-

tion for the whole globe must show a response similar to that displayed in Figure 1. Over a finite range of values of the solar constant, usually including the present value, most will be intransitive, to the extent that more than one possible equilibrium climate will be possible for a given value of the solar constant. It is conceivable that two such equilibrium states might be close enought together to permit the climate to flip randomly from one to the other, as some data indicate. However, it seems more likely to the author that, if the sun is important in changing our climate, it is so because of either variations in its total output or in the spectral distribution of its output. Changes in the earth's orbit around the sun may also be very important.
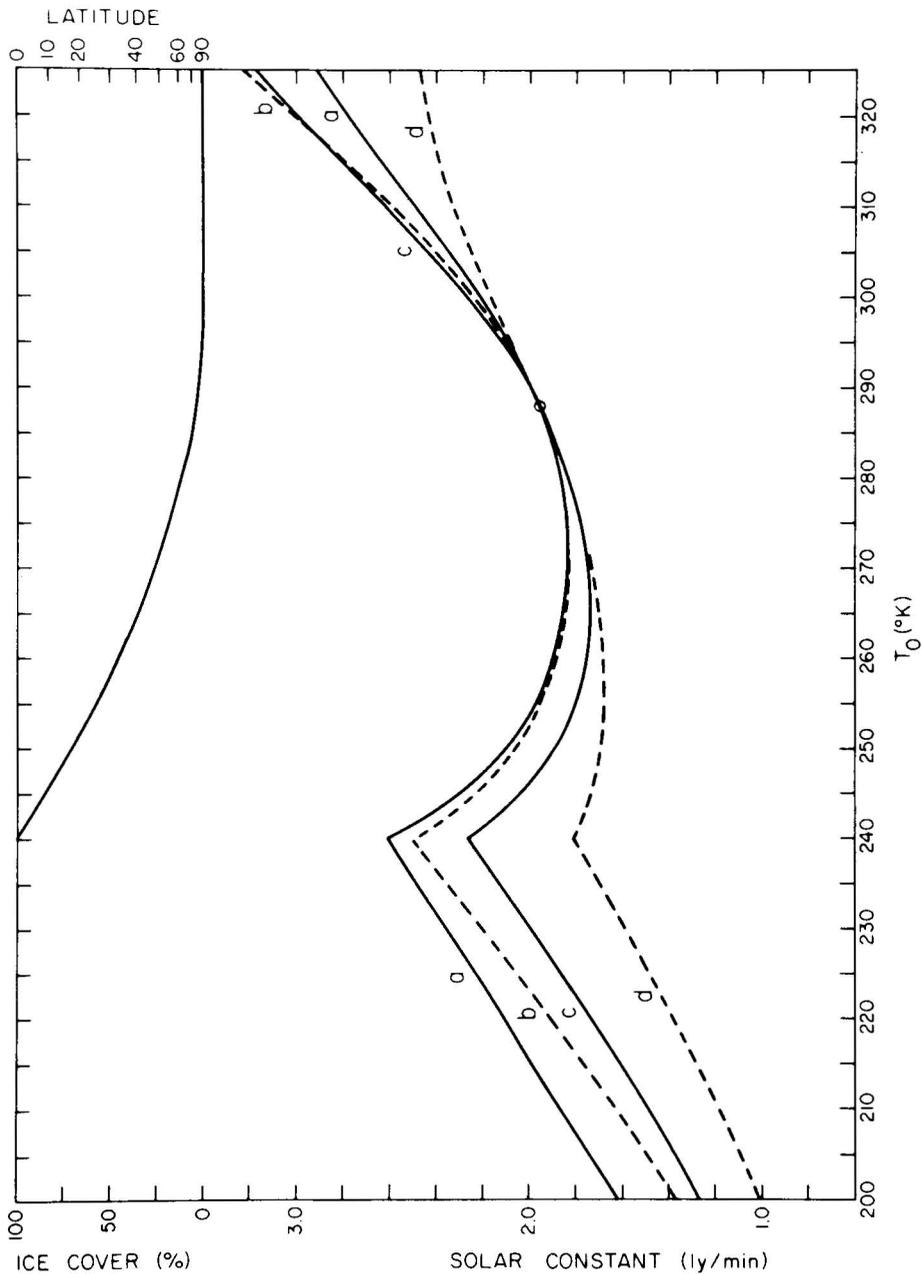
Figure 1. In the upper part of this figure is shown the relationship between ice cover and mean annual global surface temperature used in the model (see (13)). In the lower part the temperature is ploted as a function of the solar constant in langleys per minute (2 ly min^{-1} = 1395 Wm^{-2}). Curves a, b, and c use different formulations for the earth's infrared emission to space; curves c and d use different values for the solar radiation absorbed by the atmosphere. See the text for further details.

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