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ON MOUNTAIN WAVES

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RESUMEN

El presente trabajo examina el problema de las condiciones a la frontera en el estudio de la propagación de las ondas de relieve. Se demuestra que dentro del marco de la teoría clásica, no se necesita una condición límite en la frontera superior. Al mismo tiempo se aborda el problema de la posibilidad de existencia de una onda que se propague en el sentido contrario a la corriente aérea.

SUMMARY

The problems of the upper-boundary condition and the up-stream influence in the propagation of mountain waves are studied. It is demonstrated that the controversy about the first one in the classical theory is redundant, and that under certain circumstances an up-stream influence could exist.

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INTRODUCCION

This paper examines two problems concerning the theory of mountain waves which have never been solved satisfactorily, i.e., the upper boundary condition and the hypothesis of no up-stream influence (Miles, 1968).

In the first part, we shall try to show that the general solution of Helmholtz's equation determines a boundary layer, and that travelling disturbances in the same should be considered as the source of the so-called barrier waves. In the second part it is shown that the solution for the "horizontal wave equation" can be split-up into two components travelling in opposite directions which gives a "standing wave" and if the amplitude of the up-stream wave is greater than the amplitude of the down-stream one, there is another wave travelling up-stream.

UPWARD AND DOWNWARD PROPAGATING WAVES

The vertical structure of propagating mountain waves with horizontal wave-number k and phase velocity c is governed by Helmholtz's equation:

$$\omega_{zz} + (\ell^2 - K^2)\,\omega = o \tag{1}$$

where ω (x,z) is the vertical disturbance velocity and ℓ^2 is Scorer's parameter (Covez, 1971).

The general solution of (1) is of the form:

$$\vec{\omega} = \vec{A}\, e^{imz} + \vec{B}\, e^{-imz} \tag{2}$$

The physical interpretation of (2) has been neglected by most authors. Using real functions (2) is:

$$\vec{\omega} = \vec{a} \cos \sqrt{m}\, z + \vec{b} \sin \sqrt{m}\, z \tag{3}$$

the meaning of the vectors $\vec{a}$ and $\vec{b}$ is readily ascertained for:

$$z = o$$
$$\vec{\omega}_o = \vec{a}$$
$$\vec{\omega}_{zo} = \vec{V}_o = \vec{b}\sqrt{m}$$

The vector $\vec{\omega}$ is the resultant of two vectors which can be interpreted (Booker and Bretherton, 1967) as upward and downward propagating waves. Let us now introduce the unit vectors $\vec{e}_r$ and $\vec{e}_\omega$, in this oblique co-ordinate system, the co-ordinates of a characteristic point P are:

$$\xi = r_o \cos \sqrt{m}\, z$$
$$\eta = v_o \sqrt{\frac{1}{m}} \sin \sqrt{m}\, z$$

the elimination of z gives:

$$\frac{\xi^2}{r_o^2} + \frac{n^2}{\frac{V_o^2}{m}} = 1 \qquad (4)$$

This is the equation of an ellipse in oblique co-ordinates, the co-ordinate axes being in the direction of two conjugate diameters.

We can interpret this ellipse as a boundary layer determined by the interference between upward and downward propagating modes. This result is quite similar to the general orbit of a particle moving under the influence of a quasi-elastic force (Joos, 1958). Travelling disturbances in a boundary layer is a possible source of internal waves (Townsend 1968), and accordingly of the so-called mountain waves.

We see then, that if ℓ^2 is a constant in (1) there are no propagating waves at all. For a discussion of (1) with ℓ^2 variable see (Covez, 1971).

UP-STREAM INFLUENCE

Let us consider the "horizontal wave equation":

$$\omega_{xx} + (\ell^2 - n^2)\,\omega = o \tag{5}$$

where n is the vertical wave number,
(5) may be written in a very general form as follows:

$$\omega_{xx} + f\,(x)\,\omega = o \tag{6}$$

Under the application of a certain Liouville transformation (6) assumes the form (Broer and van Vroonhoven, 1971):

$$\overset{*}{\omega}_{yy} + f^2\,(y)\,\omega^* = o$$

where

$$\frac{d}{dy} = S\,(x)\,\frac{d}{dx}$$

S(x) being a sufficiently smooth real positive function.

According to Broer and van Vroonhoven (1971) if we consider the first-order equations:

$$V_y = \alpha\,\ddot{u} + \beta\,V \qquad\qquad \omega_y = \gamma\,u + \delta\,V$$

(α and δ being complex valued functions, Im $\alpha > o$ the local wave number, Re α a modulation, and β the coupling between the waves).

$\omega = u + V$ satisfies also (5), when certain relations between these functions exist. This amounts to spliting the wave into two components travelling in opposite directions (Broer and van Vroonhoven, 1971). The superposition of the same, usually gives a "standing wave", but if the two amplitudes are unequal, there exists in addition a wave travelling in the direction of the stronger partial wave (e.g., up-stream), whose amplitude is the difference of the amplitudes of the components (Joos, 1958).

CONCLUSIONS

We think we have demostrated that the controversy about the upper-boundary condition in the classical theory of mountain waves is redundant. At the same time we have shown that under particular circumstances an up-stream influence is possible.

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