PREMONITORY ELEVATION CHANGE BEFORE AN EARTHQUAKE BASED ON DILATANCY-DIFFUSION MODEL

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RESUMEN

El modelo de difusión-dilatación, recientemente propuesto para explicar cambios premonitorios en los campos geofísicos y geoquímicos antes de varios temblores superficiales, implica la ascención de la región epicéntrica. Basado en una esfera dilatante en un semi-espacio elástico, se presentan fórmulas simples y una gráfica que pueden usarse para predecir la magnitud y profundidad del foco de un futuro temblor si la magnitud de la dilatación se conoce. Alternativamente, se puede hacer una estimación de la magnitud de la dilatación con la ayuda de la gráfica compilando información del máximo desplazamiento vertical, de la profundidad del centro y del radio de la esfera dilatante para temblores que son precedidos de dilatación.

ABSTRACT

The dilatancy-diffusion model, recently proposed to explain premonitory changes in geophysical and geochemical fields before several shallow earthquakes, implies uplift of the epicentral region. Based on a dilating sphere in an elastic half-space, simple formulas and a graph are presented which could be used to predict the magnitude and depth of the focus of a future earthquake if the magnitude of the dilatancy were known. Alternatively an estimate of the magnitude of the dilatancy can be made with the help of the graph by compiling information on maximum vertical displacement, depth of the center and the radius of the dilating sphere for earthquakes that are preceded by dilatancy.

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INTRODUCTION

It seems that premonitory changes in observed geophysical and geochemical fields before several earthquakes can be explained by a dilatancy-diffusion model (Whitcomb et al, 1973; Scholz et al, 1973). Laboratory studies on rocks have shown that as the maximum stress reaches one-third to two-thirds of the fracture stress at a given pressure, the rocks become dilatant*; that is, volume increases relative to elastic change (Brace et al, 1966). If the dilatancy occurred in the focal region before the rupture, pore pressure would drop causing a drop in the compressional wave velocity (Vp) whereas the shear wave velocity (Vs) would remain essentially unchanged. This phenomenon has been observed in the laboratory (Nur and Simmons, 1969). Several cases showing a decrease in Vp (or Vp/Vs) before shallow earthquakes have been reported in recent years (Nersesov et al, 1969; Semenov, 1969; Aggarwal et al, 1973; Whitcomb et al, 1973; Ohtahe, 1973; Wyss and Holcomb, 1973, etc.). The drop in pore pressure increases the effective stress and results in dilatancy hardening (Frank, 1965). The pore pressure eventually returns to the normal value. Since the tectonic pressure continues to build up during the dilatant period, rising pore pressure triggers the earthquake. Anomalous duration of decrease in Vp (or Vp/Vs) seems proportional to the magnitude of the earthquake (Whitcomb et al, 1973; Scholz et al, 1973). Premonitory changes have been reported for earthquakes with thrust and strike-slip fault mechanism.

One precursor directly implied by the dilatancy model is the crustal deformation around the epicentral region of the future earthquake. Changes in elevation (uplift) can be measured by gravity survey or by levelling easily and economically. An interesting and useful problem is to find the magnitude of a future earthquake and the depth of its hypocenter from the change in elevation measured by a gravimeter or from levelling and to determine what magnitude at

* Following Brace et al., (1966) we will use the term dilatancy to mean the increase of volume relative to elastic changes caused by deformation.

what depth would be detectable by a modern gravimeter It is this problem that we attempt to solve here using a simple model. As will be seen later the dilatancy phenomenon itself is not sufficiently well understood to justify a more refined model.

MODEL

We assume the elastic strain buildup and, thus the accompanying surface deformation to be a slower process as compared to the deformation caused during dilatancy. We take the earth's surface to be plane just before the dilatancy occurs in some region in the earth (stage II of Scholz et al, 1973). We further consider the dilatant region to be spherical with the same elastic properties as the surrounding medium. We assume that were the dilatancy of the sphere to occur 'freely' it would be purely radial (the same would be true if the sphere were buried deep in the half-space). Thus, in 'free-dilatancy' a point Ao initially at a distance ℓo from the center of the sphere is displaced to A at a distance ℓ (Ao and A lie on a radial vector; $\ell > \ell o$) such that

$$\ell = \ell o \ (1 + \epsilon_f) \tag{1}$$

If the sphere is buried in a half-space such dilatancy (or expansion) will cause deformation of the surface of the half-space. We need the solution of the problem as stated above.

Mindlin and Cheng (1950b), using their results on nuclei of strain in a semi-infinite solid (Mindlin and Cheng, 1950a), and based on Goodier's theory of thermoelastic stress (Goodier, 1937), give the solution of an analogous problem. They consider a sphere of radius a buried in a half-space (both sphere and half-space having the same elastic properties) to have an excess of the linear coefficient of thermal expansion α over the surrounding medium (Fig. 1). If the entire region is given a uniform temperature rise of T, displacement $\overline{U}$ in $z > 0$ and $R > a$ is given by

$$\overline{U} = \frac{a^3\beta}{3}\left[\frac{\overline{R}_1}{R_1^3} + \frac{(3-4-\nu)\overline{R}_2}{R_2^3} - \frac{6z(z+c)\overline{R}_2}{R_2^5} - \frac{2\hat{k}}{R_2^3}\left\{(3-4\nu)(z+c)-z\right\}\right] \quad (2a)$$

where $R_1 = [x^2 + y^2 + (z-c)^2]^{1/2}$ (2b)

$$R_2 = [x^2 + y^2 + (z+c)^2]^{1/2} \quad (2c)$$

$$\beta = \frac{\alpha T\,(1+\nu)}{(1-\nu)} \quad (2d)$$

ν is the Poisson's ratio, $\overline{R}_1$ and $\overline{R}_2$ are position vectors and $\hat{k}$ is the unit vector in the z direction.

If for α T in the equation 2d we substitute ϵ_f given in the equation 1 we obtain the solution of our problem. Note that ϵ_f can be written as

$$\epsilon_f = \alpha T = (\Delta V/3V)_f \quad (3)$$

where $(\Delta V/V)_f$ is the free fractional volume change.

Figure 2 shows the deformation of the surface of the sphere for c/a = 1, 2 and 12. For c/a > 3 the deformation is nearly radial.

At the surface z = 0 equation 2a becomes

$$U_r = \frac{4a^3\beta}{3}(1-\nu)\frac{r}{(r^2+c^2)^{3/2}} \quad (4a)$$

$$U_z = \frac{-4a^3\beta}{3}(1-\nu)\frac{c}{(r^2+c^2)^{3/2}} \quad (4b)$$

The tilt T of the surface z = 0 is given by

$$T = \frac{\partial U_Z}{\partial r} = 4a^3\beta(1-\nu)\frac{cr}{(r^2+c^2)^{5/2}} \tag{5}$$

It is convenient to rewrite equations 4b and 5 in the following form

$$U_Z = -\frac{4a^3\beta}{3}(1-\nu)\frac{1}{c^2}f_1\,({}^r/_c) \tag{6a}$$

$$T = 4a^3\beta\,(1-\nu)\frac{1}{c^3}f_2\,({}^r/_c) \tag{6b}$$

where $$f_1\,({}^r/_c) = \frac{1}{(1+r^2/C^2)^{3/2}} \tag{6c}$$

$$f_2\,({}^r/_c) = \frac{r/c}{(1+r^2/c^2)^{5/2}} \tag{6d}$$

Plots of $f_1({}^r/_c)$ and $f_2({}^r/_c)$ are given in Figure 3. It is obvious from equations 6 that $(U_Z)_{max}$ occurs at r/c = 0 and $(T)_{max}$ occurs at r/c = 1/2. By simple manipulation it can be shown that

$$\frac{T}{(T)_{max}} = 3.5\frac{r/c}{(1+r^2/c^2)^{5/2}} \tag{7a}$$

$$(U_Z)_{max} = -1.167\,c\,(T)_{max} \tag{7b}$$

and, $$U_Z = \frac{1}{2}(U_Z)_{max} \quad \text{at} \quad c = 1.305\,r \tag{7c}$$

If the distance at which the vertical displacement becomes half of its maximum value is known, we can apply the 'half-width' rule given in equation 7c to determine the depth to the center of the dilating sphere. If the tilt is measured, equation 7b can be used to determine (U_z) max.

RESULT AND DISCUSSION

The displacement $\bar{U}$ is directly proportional to $(\Delta V/V)_f$. The magnitude of dilatancy in laboratory experiments (Brace et al, 1966) is about 10^{-3} for granite and does not vary much for confining pressure of up to 8 kb. This value of dilatancy, if it were to occur close to the surface of the earth, would cause a large uplift which is not observed before earthquakes. Thus the value must be much less. We shall assume that the real value of the dilatancy before an earthquake, if it were known, wold be approximately equal to $(\Delta V/V)_f$. With this uncertainly in mind, in Fig. 4, we plot $(U_z)_{max}$ in cm as a function of a in km, for various values of c/a and for $(\Delta V/V)_f = 10^{-4}$, 10^{-5} and 10^{-6} with $\nu = 1/4$ for all cases. For example if a = 10 km and c/a = 1 (dilating sphere touching the surface of the earth) then

$$(U_z)_{max} \begin{cases} = 55.6 \text{ cm if } (\Delta V/V)_f = 10^{-4} \\ = 5.5 \text{ cm if } (\Delta V/V)_f = 10^{-5} \\ = .55 \text{ cm if } (\Delta V/V)_f = 10^{-6} \end{cases}$$

Thus if $(\Delta V/V)_f$ is known, for a given value of a and c/a, $(U_z)_{max}$ can be found from the graph in Fig. 4 [$(U_z)_{max} > 10$ cm can be easily detected by a modern gravimeter]. From the measurement of U_z on z = 0, we can compute the depth to center c (depth to the focus?) by equation 7c and a from equation 6a. Since there is some evidence suggesting that a $\cong$ L, where L is the fault length (Scholz et al, 1973) and since there are several empirical relations between magnitude M and L an estimate of M could be made from a. This all depends, however, on the precise knowledge of $(\Delta V/V)_f$ which is lacking at present.

Alternatively, the graph in Fig. 4 could be used to determine $(\Delta V/V)_f$ from observations of $(U_z)_{max}$, a and c/a before earthquakes.

It should be noted that the uplift of the earth's surface may occur due to causes other than dilatancy and therefore, the evidence of uplift only does not necessarily indicate an impending earthqueake.

The model discussed above could equally well be applied to deformation of the ground during injection or removal of magma from chambers in active volcanoes.

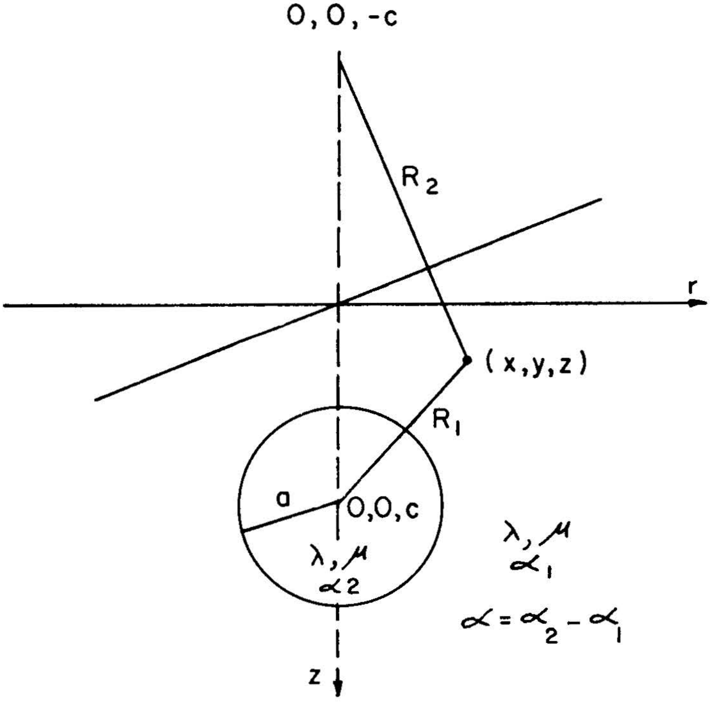

Figure 1. A sphere buried in a half-space. Elastic properties of the sphere and the half-space are the same but coefficients of linear thermal expansion are different.

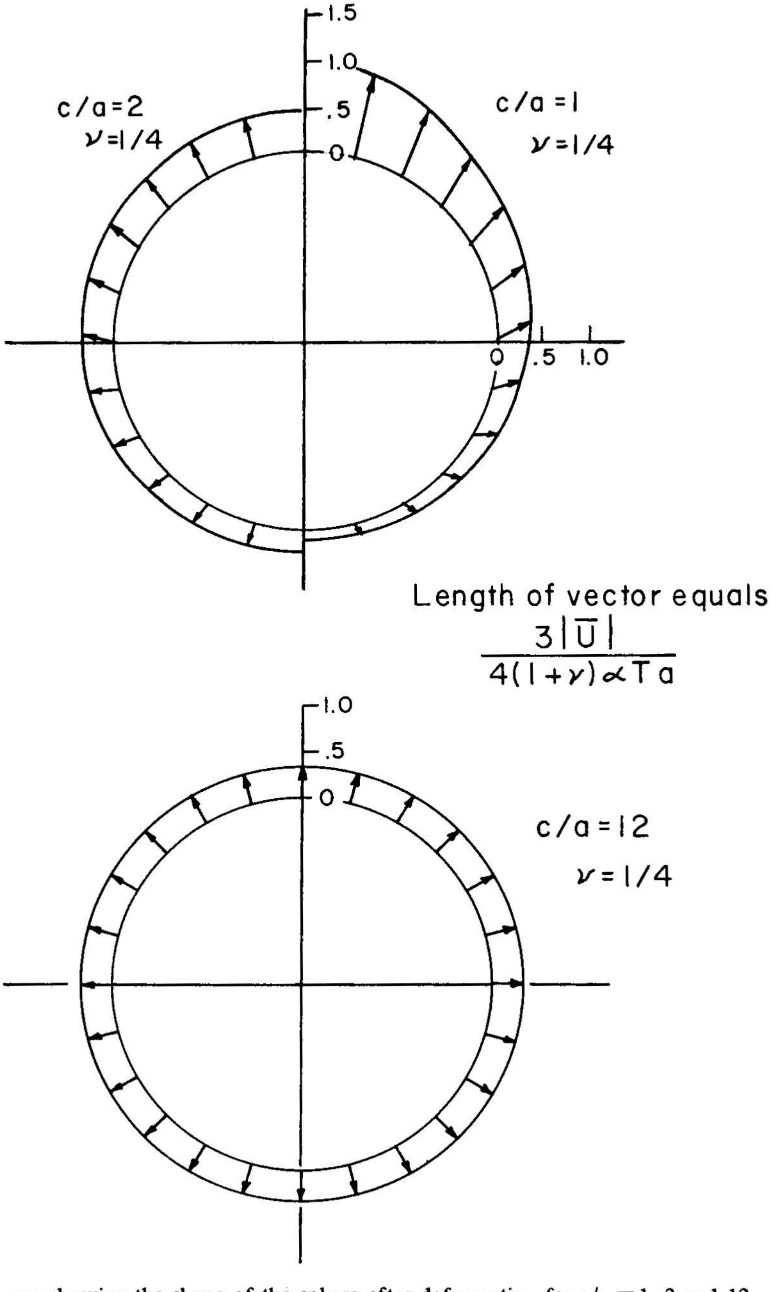

Figure 2. Diagrams showing the shape of the sphere after deformation for c/a = 1, 2 and 12. Vectors show the displacement of points on the sphere. Length of the vector is equal to $3\ |\bar{U}|/\ 4\,(1+\nu)\,\alpha\,Ta$. $\nu = 1/4$.

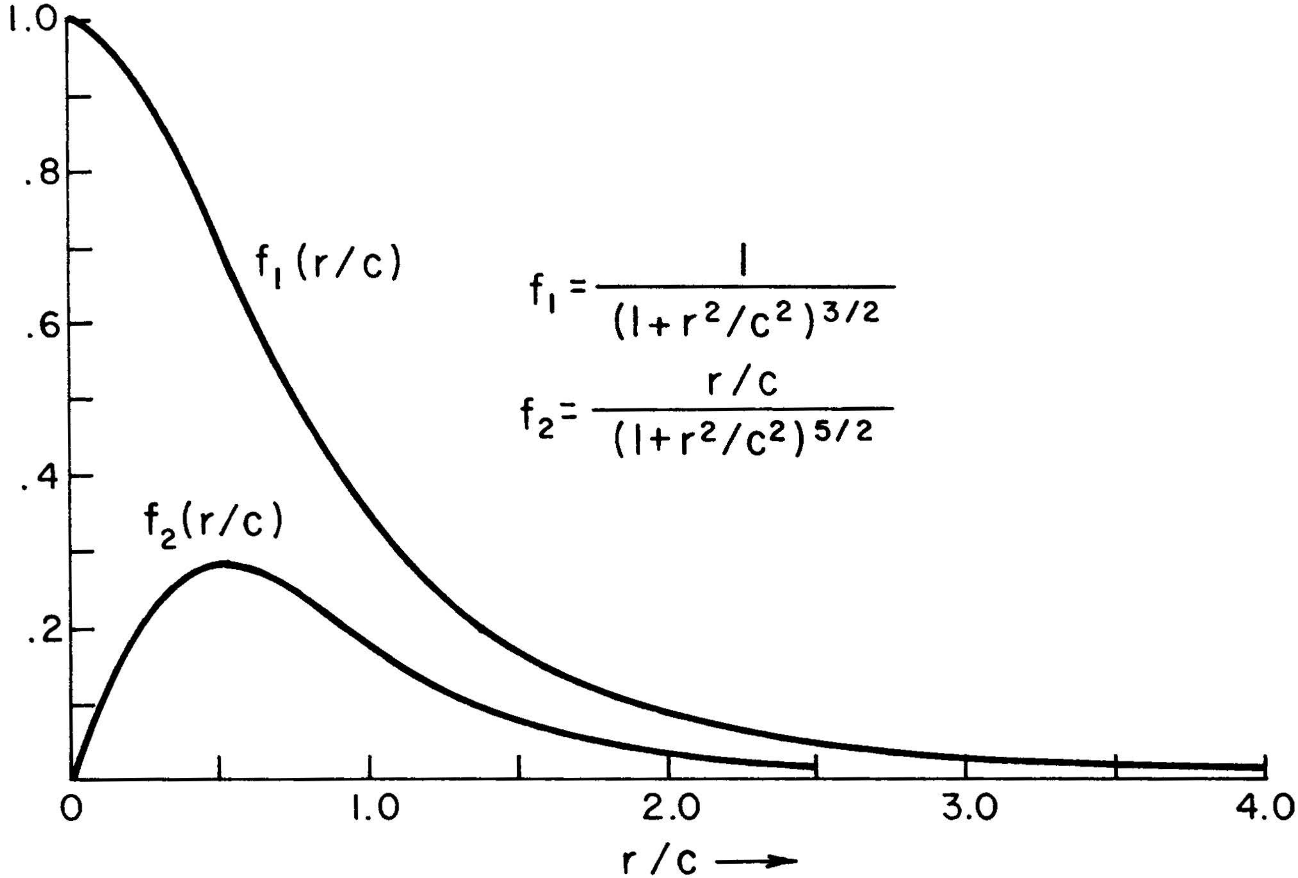

Figure 3. Change in elevation and the tilt of the surface due to a dilating sphere.

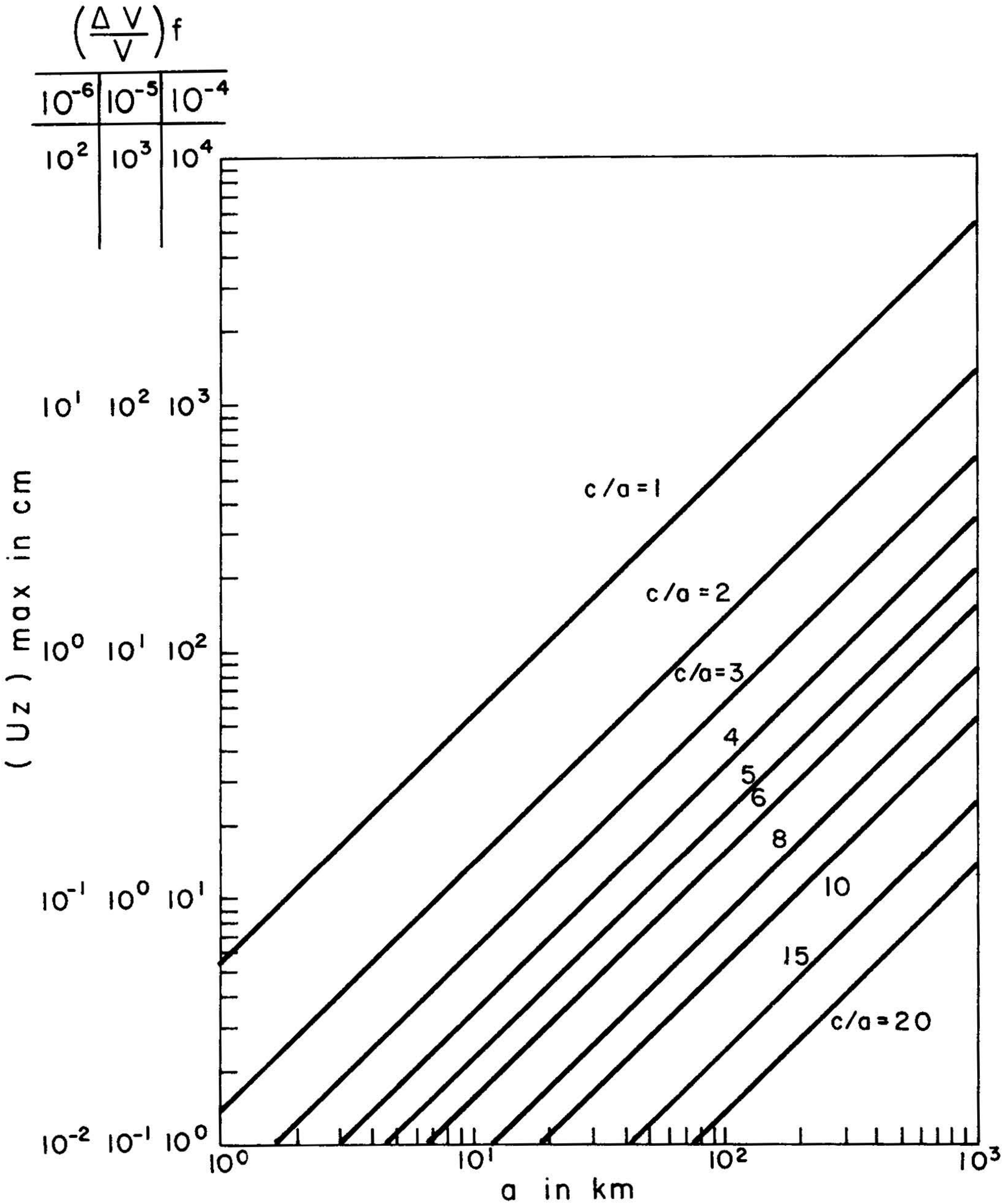

Figure 4. Graph of $(U_z)_{max}$ in cm as a function of a in km for different c/a and $(\Delta V/V)_f = 10^{-4}, 10^{-5}, 10^{-6}$. $\nu = 1/4$ for all cases. Note that the three ordinate scales refer to the three values of $(\Delta V/V)_f$.

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