

CUMULUS AND SMALLER-SCALE EXCHANGE PROCESSES IN HURRICANES

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RESUMEN

Este trabajo estudia valores estimados de los procesos de intercambio que están asociados a cúmulos y a perturbaciones de menor escala en los huracanes. Dichos valores estimados se obtienen como residuo, después de evaluar los términos que envuelven valores medios de diversos parámetros en las ecuaciones de momento y de la energía termodinámica. Los resultados obtenidos confirman la idea de que la convección húmeda (cúmulos) desempeña un papel importante en aumentar el calor y el momento de la tropósfera superior. Además, los resultados indican que la hipótesis de mezcla de Prandtl, de por sí, no puede describir completamente los procesos de intercambio en los huracanes.

ABSTRACT

Estimates of the magnitude of the exchange processes associated with cumulus and smaller scale fluctuations are obtained. The estimates are obtained as residuals after computing the terms involving the mean quantities in the momentum and the thermodynamic energy equations. The results confirm the idea that cumulus convection plays an important role in increasing both heat and momentum in the upper troposphere. In addition, they show that the mixing length hypothesis alone may not be able to describe completely the exchange processes in hurricanes.

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1. INTRODUCTION

It is generally recognised that exchange processes associated with both cumulus and smaller scale disturbances are important in the formation and maintenance of tropical cyclones. In spite of this important role, however, little is known concerning the magnitude of this effect because direct measurements are difficult to make under the extremely adverse hurricane conditions. The purpose of this paper is to describe an attempt to evaluate quantitatively this effect on both momentum and temperature changes by an indirect method.

2. BASIC CONSIDERATIONS

In this section we describe the basis on which the estimates are made. The momentum exchange will be considered first. The equation which will be used as a starting point is the tangential equation of motion in a cylindrical coordinate system with pressure as the vertical coordinate. This is

$$\frac{\partial u}{\partial t} + \frac{u}{r} \frac{\partial u}{\partial \lambda} + v \frac{\partial u}{\partial r} + \omega \frac{\partial u}{\partial p} + v \left(f + \frac{u}{r} \right) = -\frac{g}{r} \frac{\partial z}{\partial \lambda} \quad (1)$$

where

- u = tangential wind component
- v = radial wind component
- ω = vertical wind component
- t = time
- r = radial distance from the center
- λ = azimuthal angle
- p = pressure
- f = Coriolis parameter
- g = gravity
- z = isobaric height

We will assume that the origin of the coordinate system is at the center of the hurricane. Multiplying by r and using the definition of absolute angular momentum (M),

$$M \equiv ur + f\frac{r^2}{2}$$

one obtains the momentum equation,

$$\frac{\partial M}{\partial t} + \frac{u}{r} \frac{\partial M}{\partial \lambda} + v \frac{\partial M}{\partial r} + \omega \frac{\partial M}{\partial p} = -g \frac{\partial z}{\partial \lambda} \quad (2)$$

Let us represent each one of the dependent variables by the sum of a mean and its deviation from the mean. Thus,

$$\begin{aligned} M &\equiv \bar{M} + M' \\ u &\equiv \bar{u} + u' \\ v &\equiv \bar{v} + v' \\ \omega &\equiv \bar{\omega} + \omega' \\ z &\equiv \bar{z} + z' \end{aligned}$$

Here, the mean represents an average over all the dependent variables - r, λ, p , and t . The interval over which the average is made in r, p , and t may be considered as arbitrary; however, the averaging over λ is for a complete circle about the center of the hurricane. After substituting these expressions in Eq. (2), one obtains

$$\frac{\partial \bar{M}}{\partial t} = -\bar{v} \frac{\partial \bar{M}}{\partial r} - \bar{\omega} \frac{\partial \bar{M}}{\partial p} + F \quad (3)$$

where

$$F \equiv \frac{\bar{u}'}{r} \frac{\partial \bar{M}'}{\partial \lambda} + \bar{v}' \frac{\partial \bar{M}'}{\partial r} + \bar{\omega}' \frac{\partial \bar{M}'}{\partial p}$$

The first two terms on the right hand side of Eq. (3) represent the effects of radial and vertical advection of the mean momentum by the mean flow. The third term, F , is the frictional effect due to velocity fluctuations whose length and time scales are shorter than the corresponding averaging intervals.

A similar derivation may be made for the equivalent potential temperature. If radiative effects are negligible, the conservation equation for the equivalent potential temperature is

$$\frac{\partial \theta_e}{\partial t} + \frac{u}{r} \frac{\partial \theta_e}{\partial \lambda} + v \frac{\partial \theta_e}{\partial r} + \omega \frac{\partial \theta_e}{\partial p} = 0$$

Introducing mean and departures from the mean in the above equation, one gets

$$\frac{\partial \bar{\theta}_e}{\partial t} = -\bar{v} \frac{\partial \bar{\theta}_e}{\partial r} - \bar{\omega} \frac{\partial \bar{\theta}_e}{\partial p} + Q \quad (4)$$

where

$$Q = \overline{\frac{u'}{r} \frac{\partial \theta_e'}{\partial \lambda}} + \overline{v' \frac{\partial \theta_e'}{\partial r}} + \overline{\omega' \frac{\partial \theta_e'}{\partial p}}$$

The terms in Eq. (4) have interpretations similar to those given for corresponding terms in Eq. (3). In particular, Q represents the effect of the velocity and the equivalent potential temperature fluctuations analogous to F for absolute angular momentum.

Eqs. (3) and (4) are the final relationships which are used in this study. Using each of these equations, we computed F and Q as the sum of the local tendency and the two advective terms. The resulting values of F and Q represent not only the effect of fluctuations in time, radial distance and vertical distance but also departures from circular symmetry. Experience indicates that the two most important fluctuations which determine exchange processes in hurricanes are the

cumulus convective scale disturbances and the small scale eddies. The former transports heat, momentum and other properties up to the upper troposphere, principally through hot towers; the latter is most effective near the earth surface.

There have been various attempts to parameterize the heat flux due to cumulus convective scale disturbances in terms of the large-scale flux. The latest parameterization techniques which have been proposed are those by Ooyama (1971) and Arakawa (1972). The corresponding parameterization of momentum flux has not been studied in as much detail.

3. DATA AND RESULTS

Analysis of Hurricane Hilda, 1964 (Hawkins and Rubsam, 1968) and Hurricane Daisy, 1958 (Riehl and Malkus, 1961) were used. In the first hurricane, both momentum and equivalent potential temperature calculations were possible. In the second, there is no momentum analysis available. The analyses of both momentum and equivalent potential temperature were based on aircraft traverses along radial and azimuthal directions forming a cloverleaf pattern in the vicinity of the storm center. The radial traverses are done simultaneously at several levels (five levels for Hilda and 2 to 5 levels for Daisy.) In general, at any one level there are about 6 equally spaced radial traverses. During the traverses, a more or less continuous measurement of the different meteorological variables is made. A complete set of traverses takes 3 to 4 hours, approximately. The observations at each level thus obtained are plotted and then subjectively analyzed. Finally, the resulting analyses are averaged along a complete circle to give a vertical cross-section. These cross-sections are used in our calculations. The method of analysis which yields the cross-section from the raw data indicates an averaging process over intervals which are difficult to specify other than that along λ which is about 360 degrees. It suffices to state that the resolution of the observations in time and height which is inherent in the vertical cross-section appears

to be lower compared to the resolution along the radial direction and the azimuthal coordinate.

Figure 1 shows the vertical cross-section of absolute angular momentum for Hurricane Hilda as analyzed by Hawkins and Rubsam (1968). Figure 2 shows the corresponding radial and vertical velocities which, together, show the characteristic inflow in the lower troposphere, strong ascent near the center and outflow aloft. According to Eq. (3), the momentum will be conserved along the streamlines of the transversal circulation if the frictional effect, F , is zero. The momentum distribution given by Figure 1 indicates a strong lack of conservation of momentum in the layers near the surface. In the upper troposphere, however, the momentum is approximately conserved along the streamlines. This is confirmed by the results of actual computations of the radial advection ($-\bar{V} \frac{\partial \bar{M}}{\partial r}$) and vertical advection ($-\bar{\omega} \frac{\partial \bar{M}}{\partial p}$) shown in Figure 3. The local tendency ($\frac{\partial \bar{M}}{\partial t}$) is

assumed to be negligible because, during the time of observation, Hilda was near maximum intensity. Using Eq. (3) together with the values of these terms, we computed the frictional or mixing term (F) as a residual. This term is indicated as "residual mixing" in Figure 3. The F distribution shows large negative values at low levels, decreasing in magnitude with height. This sort of distribution has some resemblance in shape but not in magnitude to the tangential friction obtained by Gray (1967) (figure 19 in his paper) using an entirely different approach. In our case, the mixing term F is of the same order as the advective terms and is roughly a mirror image of the horizontal advection. Hence, the net effect of the exchange processes is to decrease the absolute angular momentum at lower levels. However, at upper levels, the effect is a tendency to increase the momentum as indicated by the computation at the 60-mile radius. These exchange processes are often represented in theoretical models with the aid of the mixing length hypothesis. Constant vertical (K_V) and horizontal (K_H) exchange coefficients of $10^5 \text{ cm}^2 \text{ sec}^{-1}$ and $10^8 \text{ cm}^2 \text{ sec}^{-1}$ are suggested by the studies of Barrientos (1964), Miller

(1965) and Rosenthal (1962). It is interesting to find out how realistic the mixing length hypothesis is in the present case. For this purpose, the quantity

$$F^1 \equiv K_H \left(\frac{\partial^2 M}{\partial r^2} - \frac{1}{r} \frac{\partial M}{\partial r} \right) + K_V \frac{\partial^2 M}{\partial z^2}$$

was computed using the momentum distribution in Figure 1. The results are shown also in Figure 3 as "comp. mix.". It may be seen that, although the sign of F^1 is the same as that of F its distribution is quite different in that it is almost constant with height. It appears, therefore, that the simple specification of the friction through constant exchange coefficient is inadequate.

Since F^1 may be considered as that portion of F which is due to small scale turbulent mixing, the difference, $F - F^1$, will be assumed to be mainly due to the effect of momentum transport by cumulus towers. These towers transport air with higher momentum from the lower layers toward the upper troposphere. Thus, the net effect is a tendency to increase the momentum in the upper troposphere. This appears to be confirmed by the fact that the quantity $F - F^1$ is shown to be positive and large at upper levels. However, the fact that it is negative at radii of 40 and 60 miles at lower levels, is rather unsatisfactory. The effect of cumulus convection should be to increase momentum at all levels. However, the negative values may be due to an underestimate of the coefficients. If the exchange coefficients are increased by about five times the values used, the effect of cumulus convection, $F - F^1$, will be positive at all levels and increase with height.

We consider next the redistribution of the equivalent potential temperature for Hurricane Hilda. The vertical cross-section of the equivalent potential temperature is shown in Figure 4. The salient features of this distribution are the minimum in equivalent potential temperature near the 650 mb-level and the relatively high values near the center. In this region the equivalent potential temperature is approximately constant with height.

Calculation of the advective terms and the term, Q , representing the effect of exchange processes were made using Eq. (4) by replacing the derivatives with finite differences. As in the momentum calculations, the local tendency was assumed to be negligible.

Figure 5 shows the results of the computations. It will be seen that the radial advection is small compared to the vertical advection. Hence, there is an approximate balance between vertical advection and the mixing term, Q . The magnitude of the mixing increases with height, attaining large positive values in the middle and the upper troposphere. The tendency for positive values may be interpreted as heating due to transport of warm surface air by cumulus convection. The negative values in the lower troposphere are rather disturbing and are difficult to be interpreted in terms of the same mechanism. These values may be nothing but indications of inaccuracies in the data.

An attempt was made to separate the mixing effect, Q , into the molecular type diffusion,

$$Q^1 \equiv K_H \left(\frac{\partial^2 \theta_e}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_e}{\partial r} \right) + K_v \frac{\partial^2 \theta_e}{\partial z^2}$$

and the cumulus convective type diffusion, $Q-Q^1$. Figure 6 shows the three curves representing Q (Total res.), Q^1 (V.M. + R.M.), and $Q-Q^1$ (T.R. - (V.M. + R.M.)). It may be seen that the magnitudes of the Q^1 are small compared to those of Q . Hence, one may conclude that the effect of cumulus convection in the redistribution of heat appears to overshadow that of molecular type diffusion.

As pointed out earlier, the data for Hurricane Daisy, 1958, provide only an analysis of the equivalent potential temperature. However, in this case two calculations of exchange processes were possible. The first calculation uses data for the interval Aug. 25-27 during which the storm developed rapidly. In this calculation, the local time tendency was computed from the equivalent potential temperatures for August 25 and August 27. The advective terms are averages of the values for August 25 and August 27. The second calculation is for data on August 27; at this time the hurricane had attained maximum

intensity so that the steady state assumption is approximately satisfied. This calculation follows exactly the same procedure. The data used in the calculations are shown in Figures 7, 8, 9 and 10, representing the equivalent potential temperature and transversal circulation patterns for August 25 and August 27. Note the increase in the magnitudes of both the equivalent potential temperature and the velocities during the two-day period of storm intensification.

The results of the computations of the various terms for the period August 25 to 27 are shown in Figure 11. It will be noted that at all three radii, the local tendency term is generally small compared to the other terms. It will be seen also that, in contrast with Hilda, the radial advection terms for Daisy are somewhat more important. Nevertheless, there is the same tendency for the mixing term to balance the vertical advection term in the middle and upper troposphere. An attempt has been made again to separate the mixing term into two effects -molecular-type diffusion and cumulus convective mixing. The results are given in Figure 12. In general, we see distributions which are similar to those of Hilda, showing tendency for cumulus convective heating to be most pronounced at upper levels.

The results of the calculations for August 27 are shown in Figures 13 and 14. The curves are very similar to those of the previous two figures and the same conclusion can be made. Therefore, no further discussion of the results is necessary. At this juncture, we will simply summarize all the calculations on the equivalent potential temperature for both Hilda and Daisy and for all radii. The summary is shown graphically in Figure 15. The distributions are generally consistent with one another. They show that exchange processes tend to heat the upper and lower troposphere. The heating at upper levels is presumably due to the effect of deep penetrative convection associated with the strong cumulonimbus activity in hurricanes. According to Yanai et al (1973), this heating is attributed to adiabatic warming in compensatory downward motions surrounding cumulus updrafts. The heating at lower levels must be due to small scale eddy diffusion of sensible and latent heat from the underlying

surface. The small values near the 700 mb level are puzzling; we have no explanation for this unexpected result.

4. CONCLUDING REMARKS

The method which has been used to assess the contribution of exchange processes in the changes of absolute angular momentum and equivalent potential temperature requires accurate data. Although the set of data used is certainly the best which is available, it is not entirely satisfactory. The results of our study must, therefore, be interpreted with caution. Nevertheless, we believe that our findings appear to support the previous ideas concerning the crucial role of cumulus convective exchange processes in the dynamic of the hurricane. On this basis, one may conclude that it would be impossible to construct a realistic hurricane model without a satisfactory parameterization of these exchange processes as they affect the redistribution of momentum, thermal energy, and moisture.

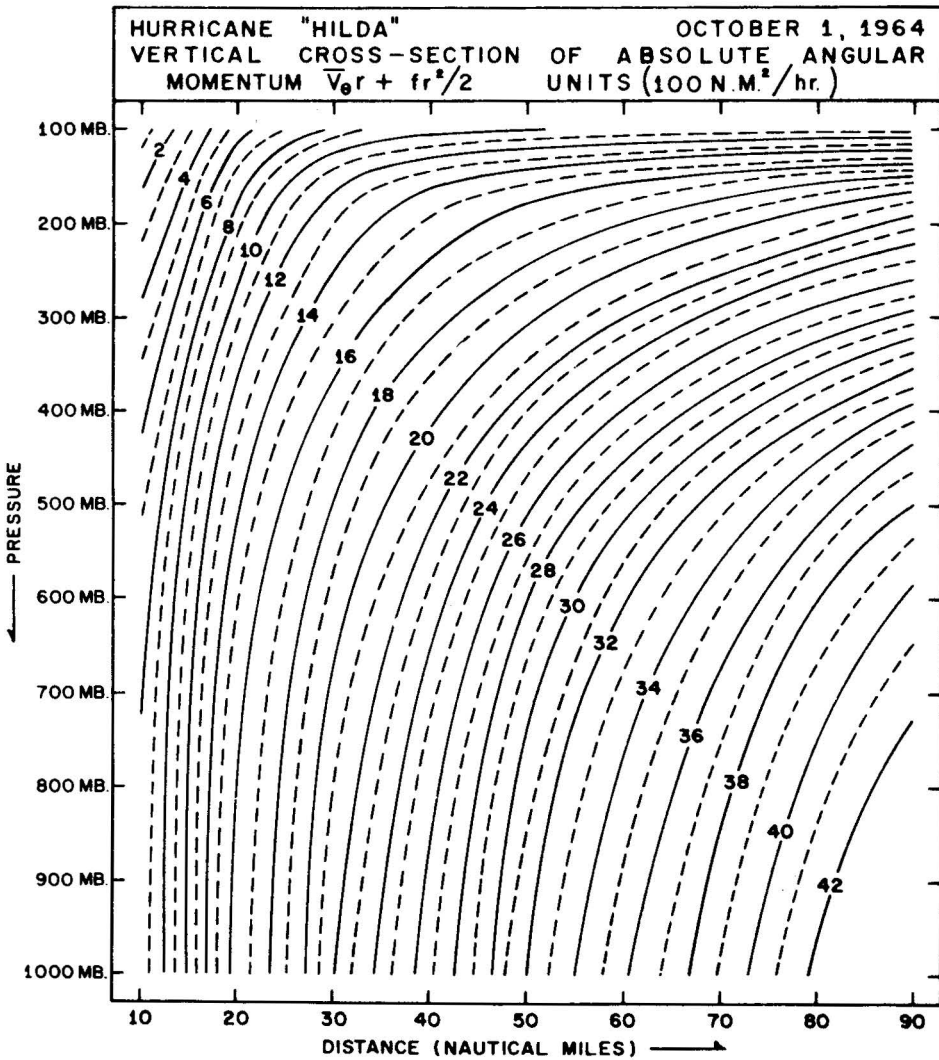


Figure 1. Vertical cross-section of absolute angular momentum for Hurricane Hilda, 1964 (after Hawkins and Rubsam, 1968).

"HILDA" OCT. 1, 1964

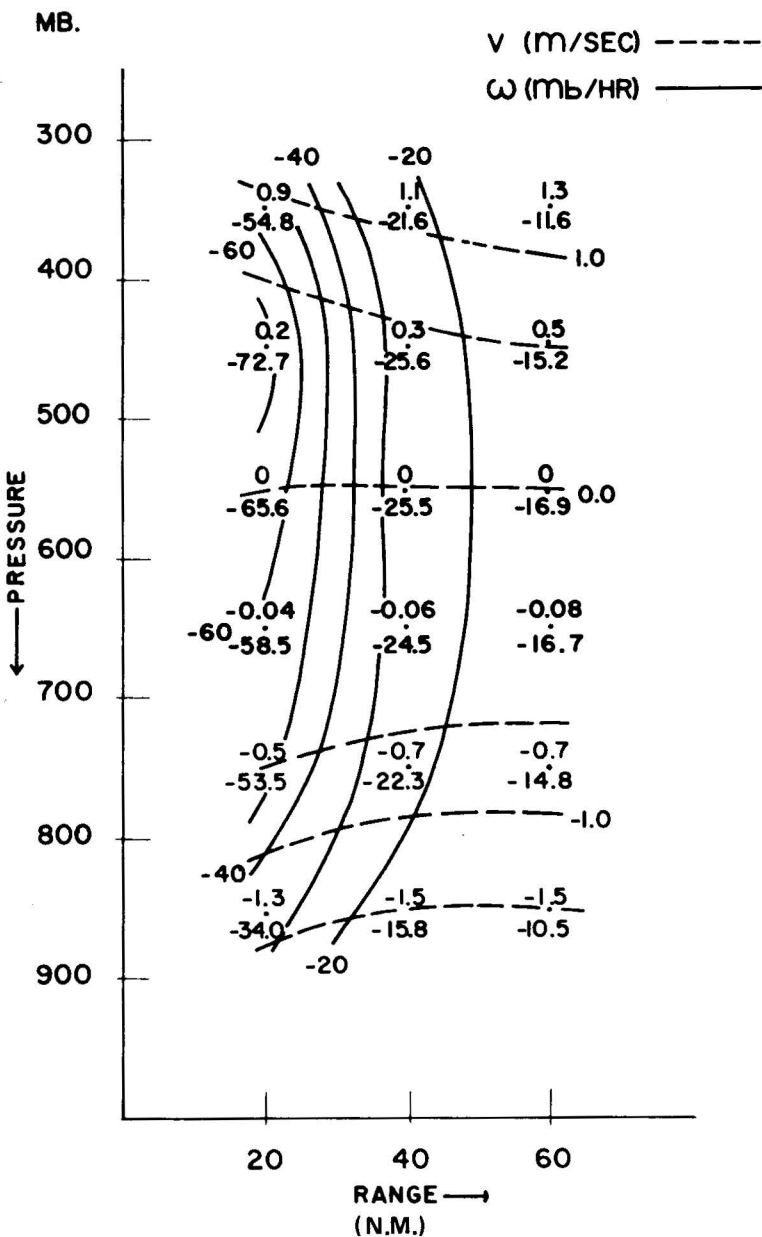


Figure 2. Radial velocities (m/sec) and vertical velocities (mb/hr) for Hurricane Hilda, 1964.

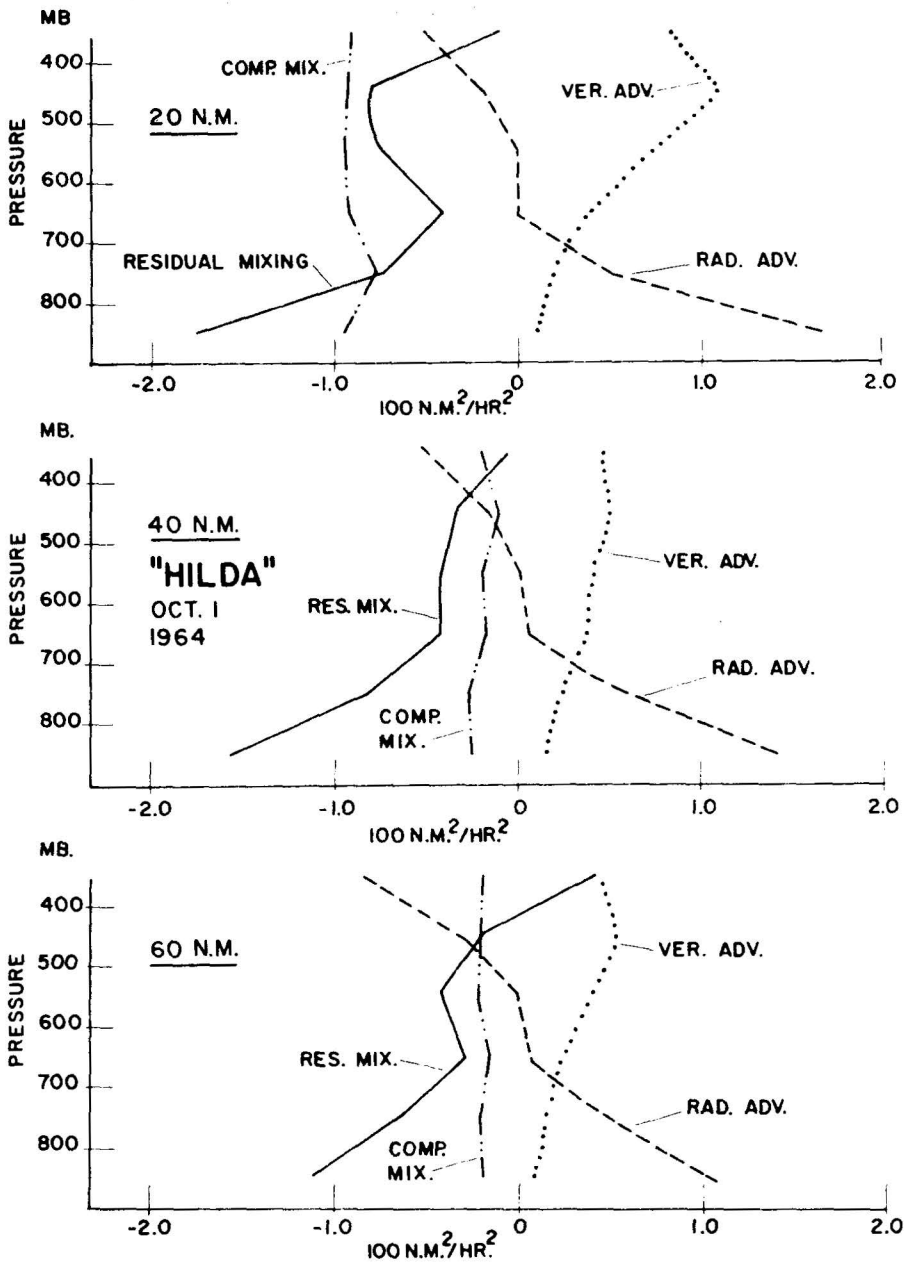


Figure 3. Advective and mixing term distributions of absolute angular momentum for Hurricane Hilda, 1964. Ver. Adv. and Rad. Adv. represent vertical advection and radial advection, respectively. The mixing which is obtained as a residual is indicated by Res. Mix. The computed mixing is denoted by Comp. Mix.

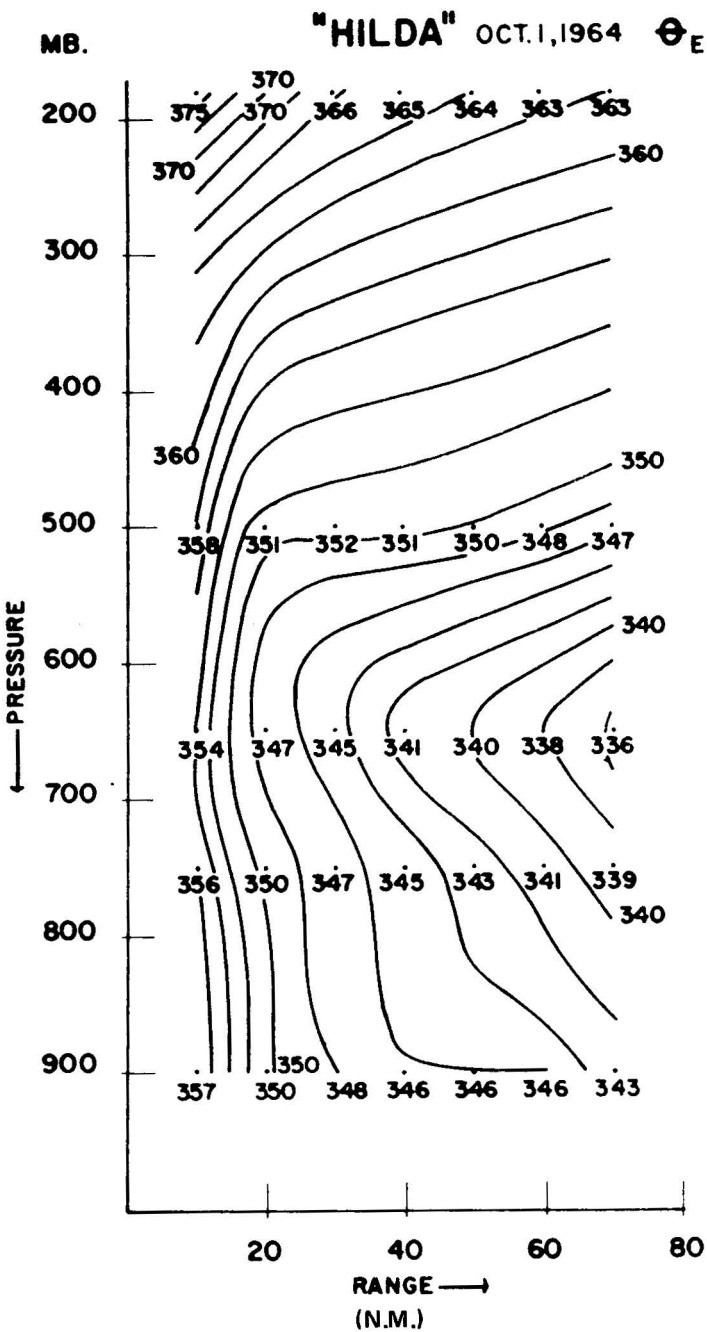


Figure 4. Vertical cross-section of equivalent potential temperature (θ_E) for Hurricane Hilda, 1964. Equivalent potential temperature values are expressed in degrees Kelvin.

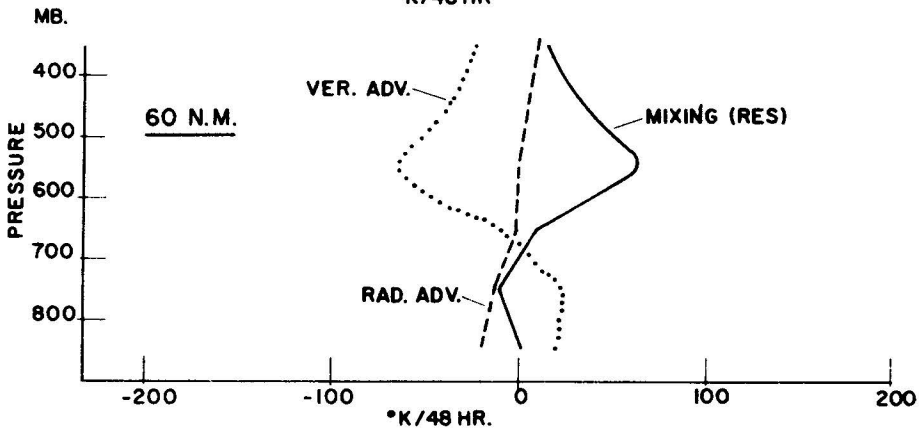
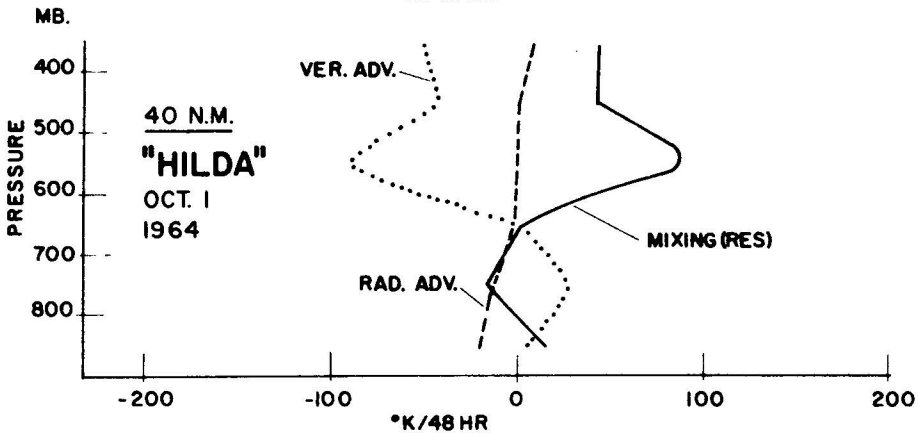
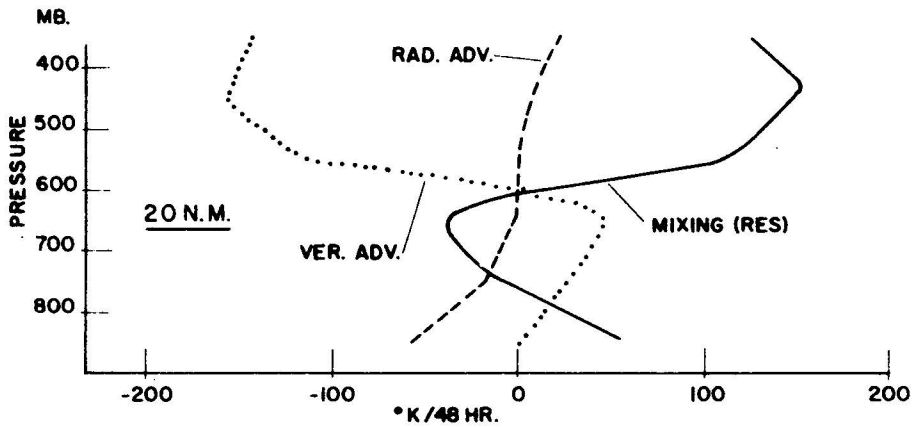


Figure 5. Advective and mixing term distributions of equivalent potential temperature for Hurricane Hilda, 1964. Ver. Adv. and Rad. Adv. indicate vertical advection, respectively. The mixing is obtained as a residual.

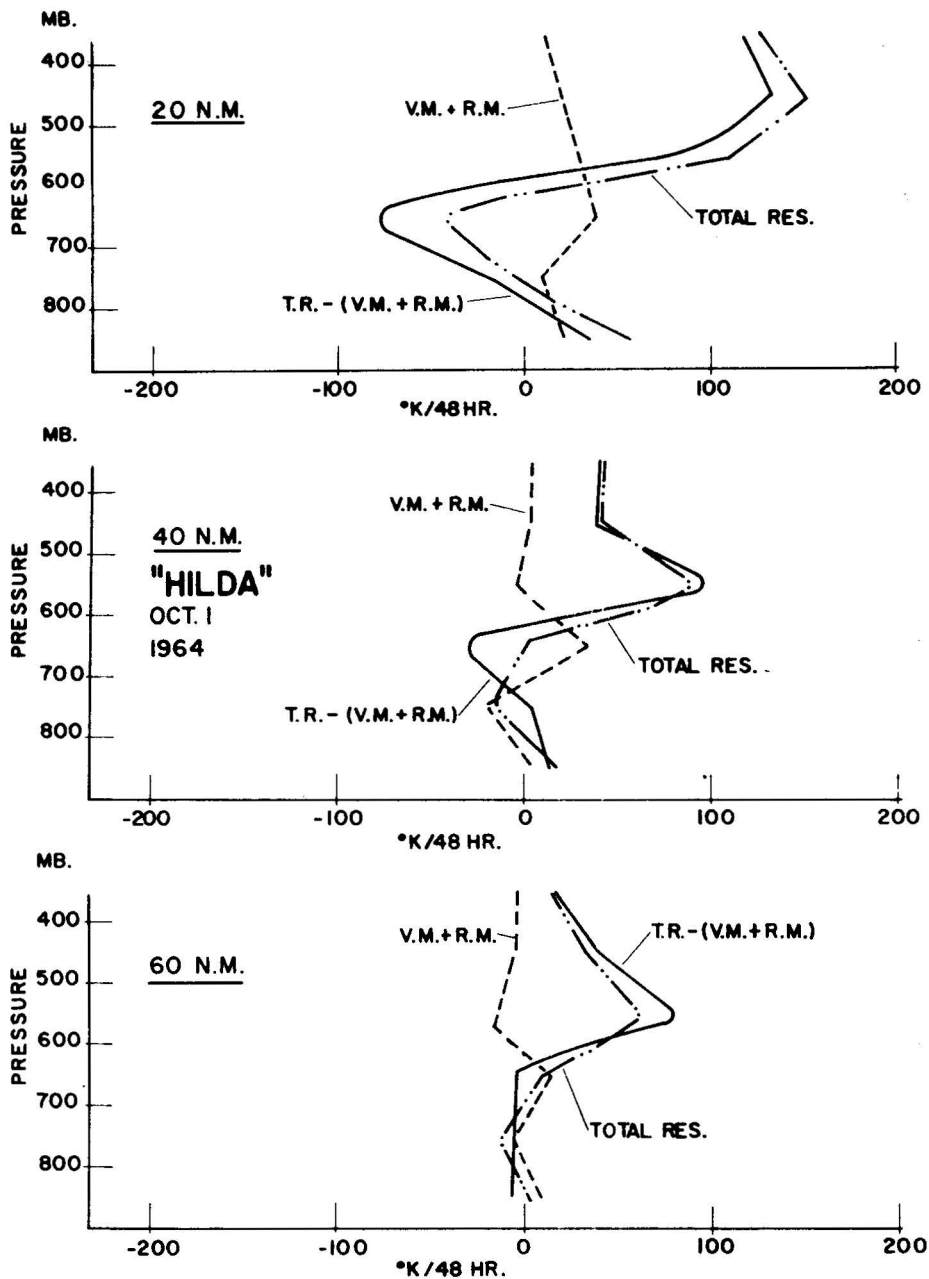


Figure 6. Total mixing, molecular-type mixing and cumulus-scale mixing distributions of equivalent potential temperature for Hurricane Hilda, 1964. The mixing in Figure 5 is denoted here as Total Res. (T. R.).

Molecular-type vertical and radial mixings are denoted by V. M. and R. M. respectively.

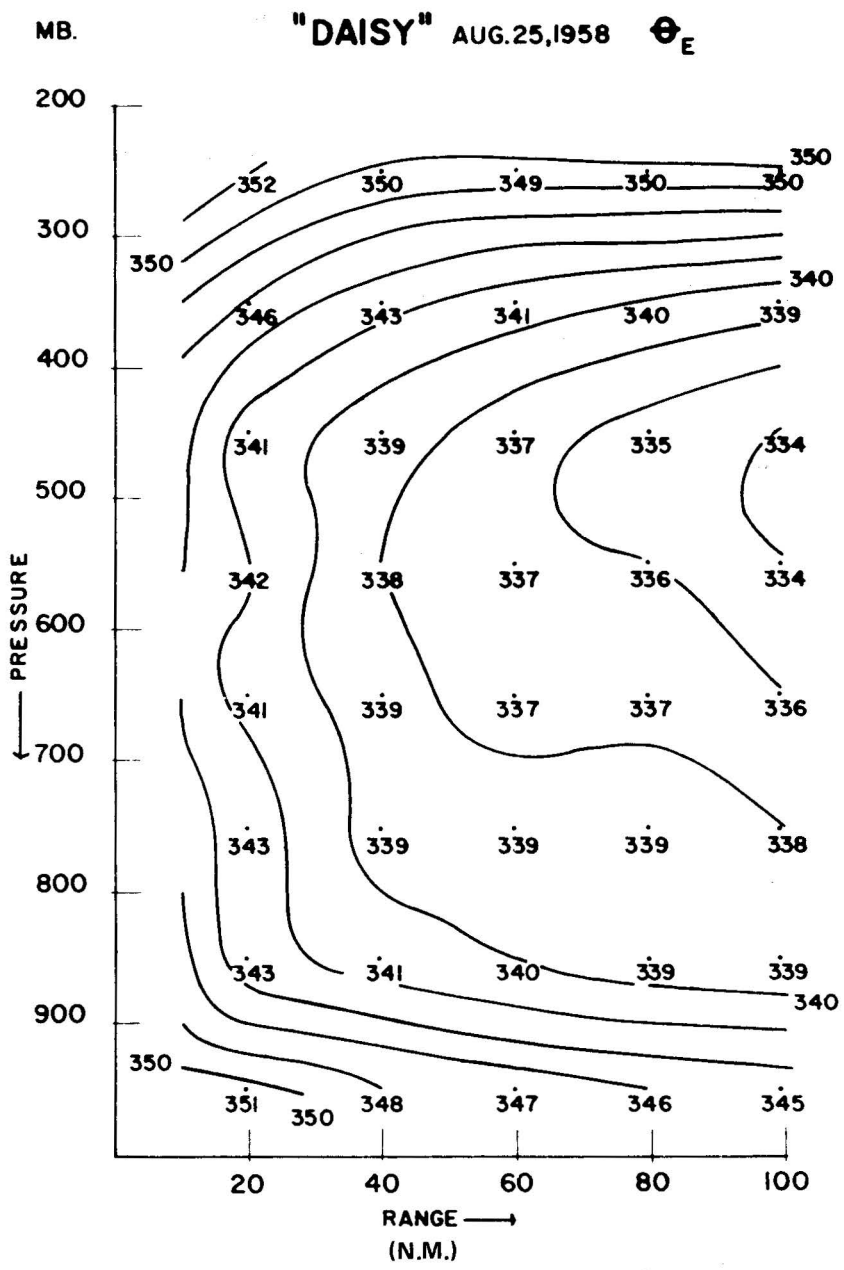


Figure 7. Vertical cross-section of equivalent potential temperature (θ_e) for Hurricane Daisy, Aug. 25, 1958. Equivalent potential temperature values are expressed in degrees Kelvin.

"DAISY" AUG. 25, 1958

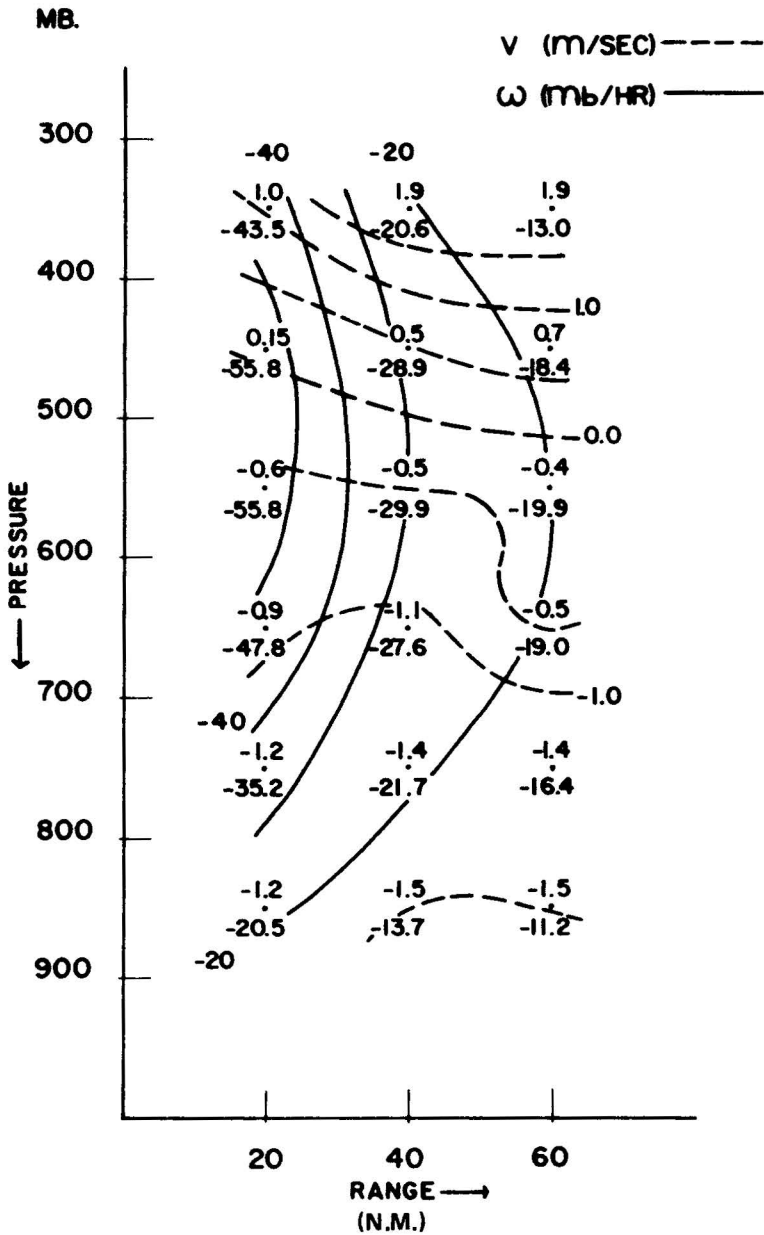


Figure 8. Radial velocities (m/sec) and vertical velocities (mb/hr) for Hurricane Daisy, Aug. 25, 1958.

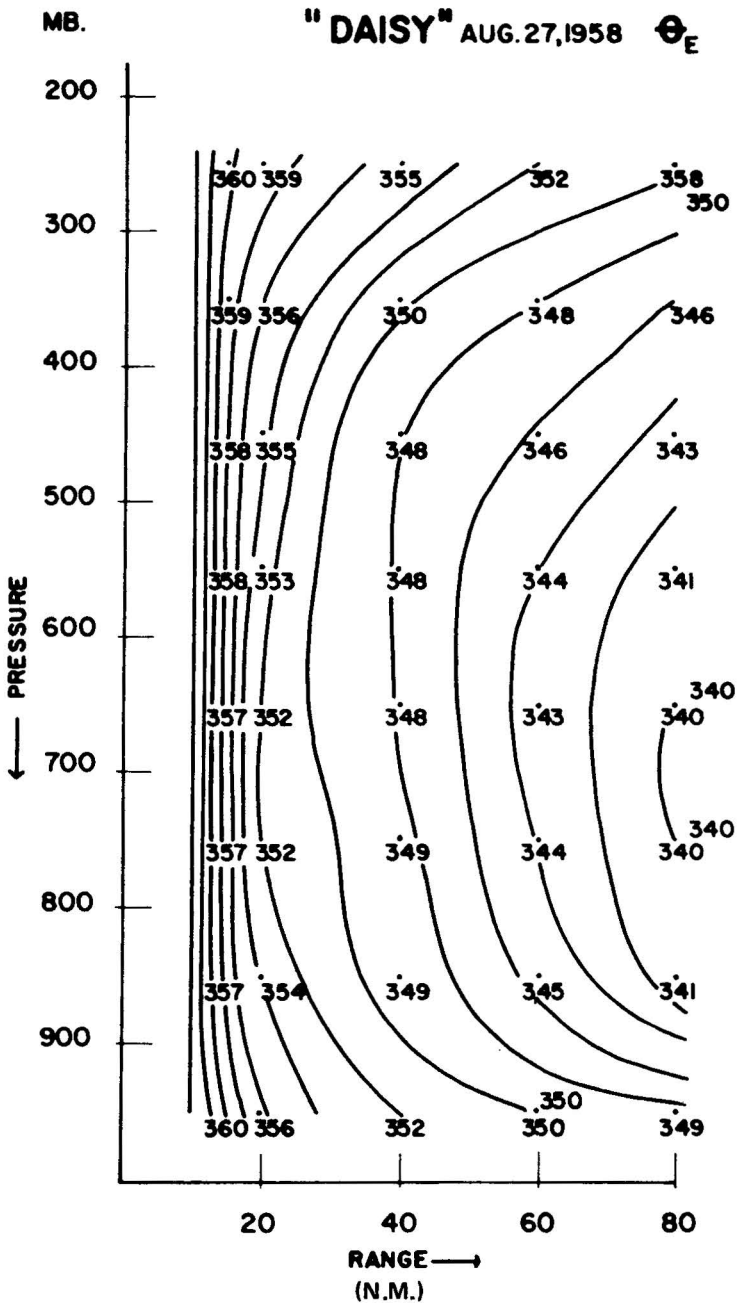


Figure 9. Vertical cross-section of equivalent potential temperature (θ_E) for Hurricane Daisy, Aug. 27, 1958. Equivalent potential temperature values are expressed in degrees Kelvin.

"DAISY" AUG. 27, 1958

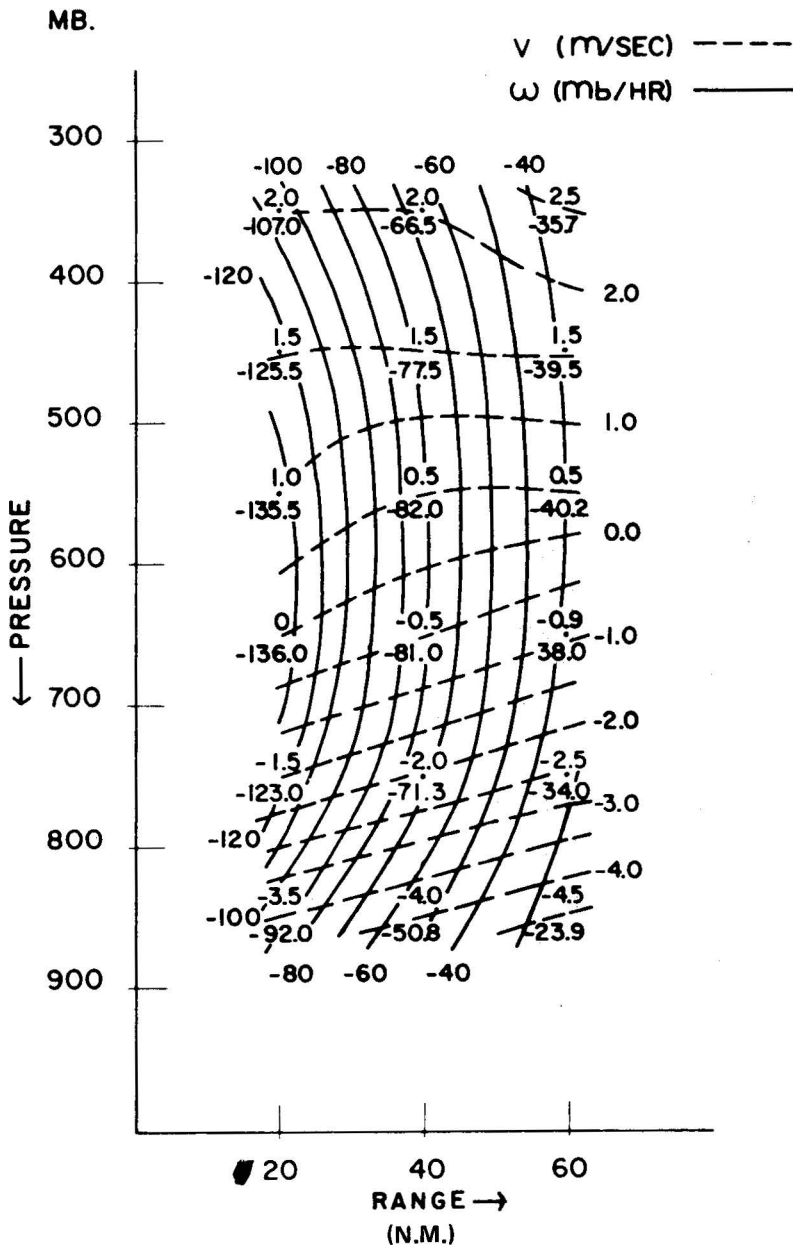


Figure 10. Radial velocities (m/sec) and vertical velocities (mb/hr) for Hurricane Daisy, Aug. 27, 1958.

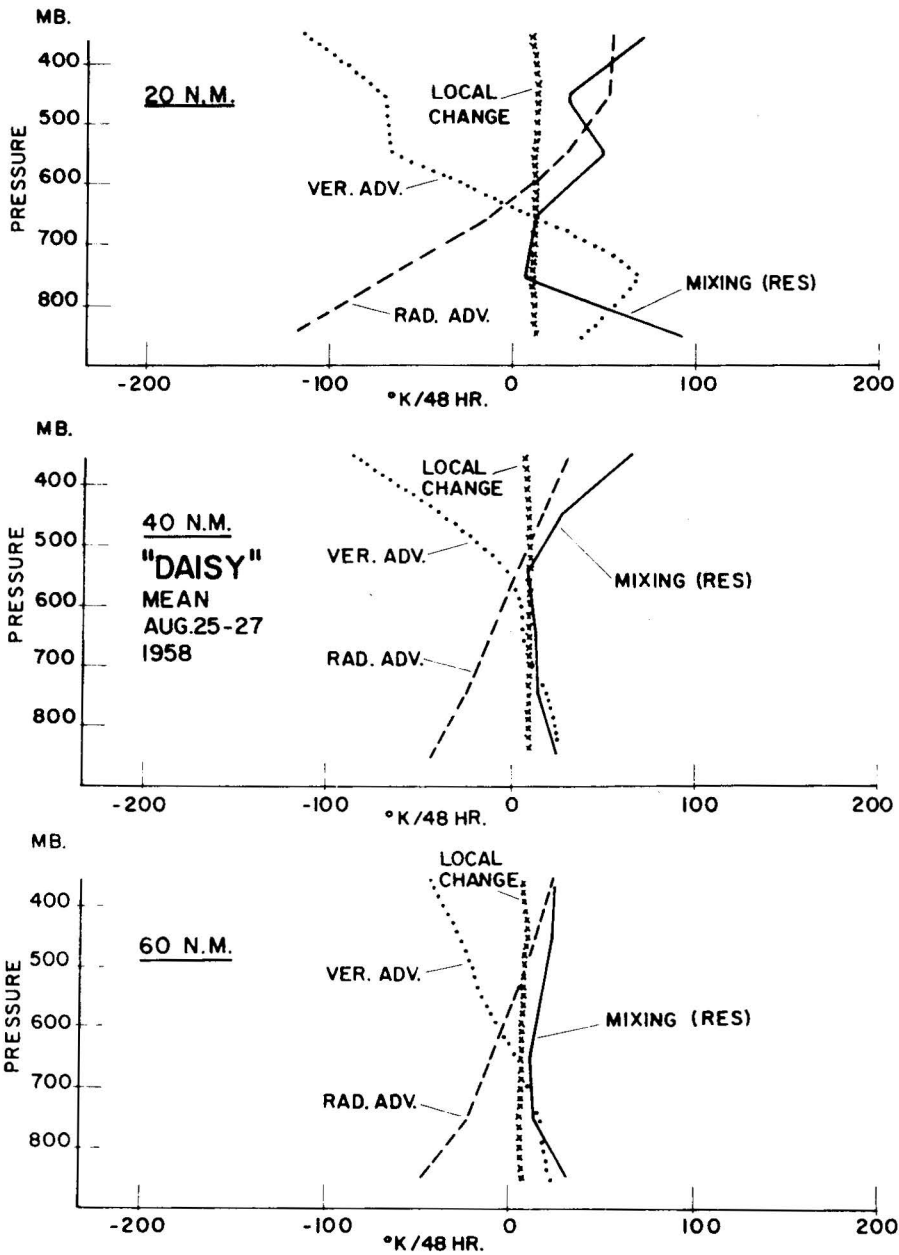


Figure 11. Local tendency, advective and mixing term distributions of equivalent potential temperature for Hurricane Daisy during the period Aug. 25-27, 1958. Ver. Adv. and Rad. Adv. indicate vertical advection and radial advection, respectively. The mixing is obtained as a residual.

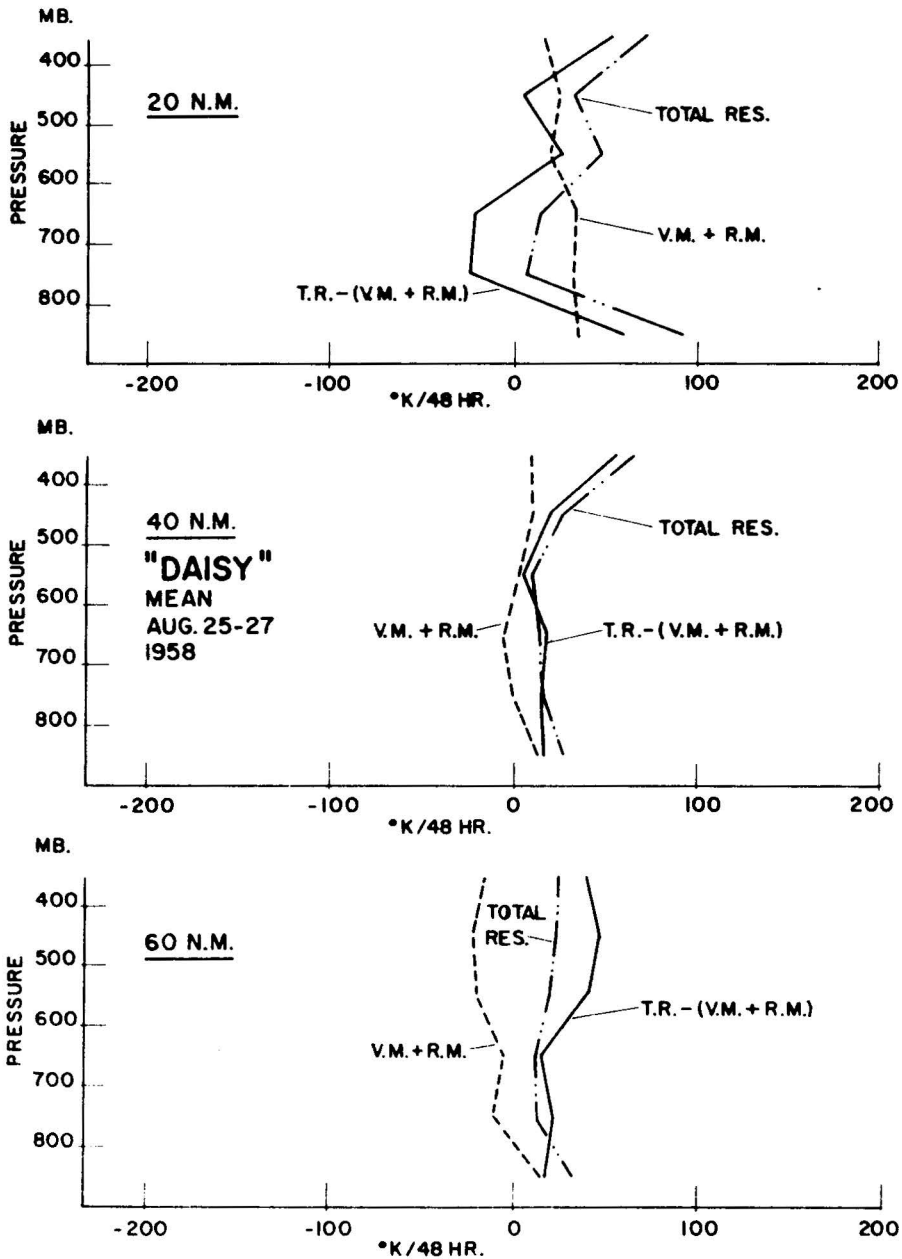


Figure 12. Total mixing, molecular-type mixing and cumulus-scale mixing distributions of equivalent potential temperature for Hurricane Daisy during the period Aug. 25-27, 1958. The mixing in Figure 11 is denoted here as Total Res. (T. R.). Molecular-type vertical and radial mixings are denoted by V. M. and R. M., respectively.

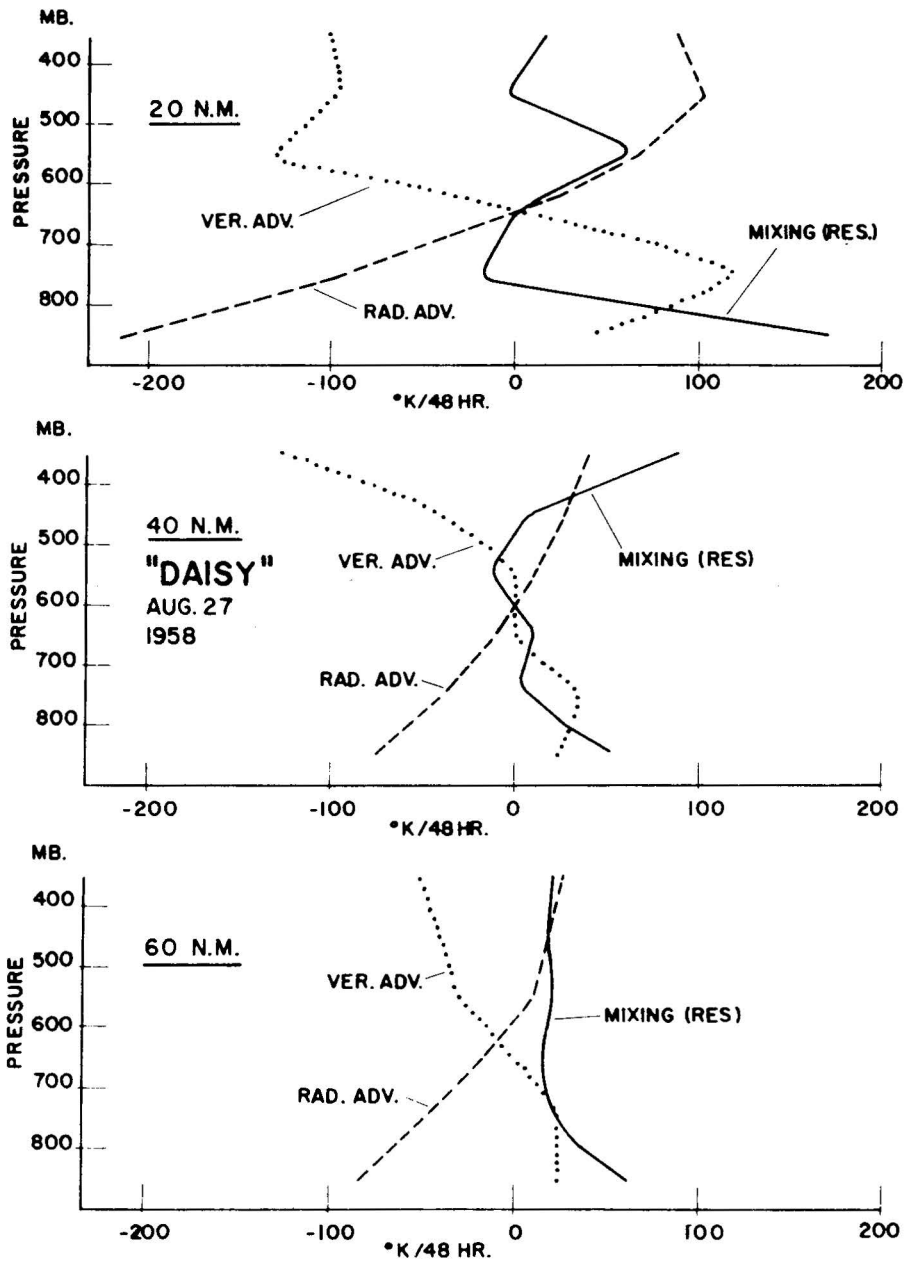


Figure 13. Advective and mixing term distributions of equivalent potential temperature for Hurricane Daisy, Aug. 27, 1958. Ver. Adv. and Rad. Adv. indicate vertical advection and radial advection, respectively. The mixing is obtained as a residual.

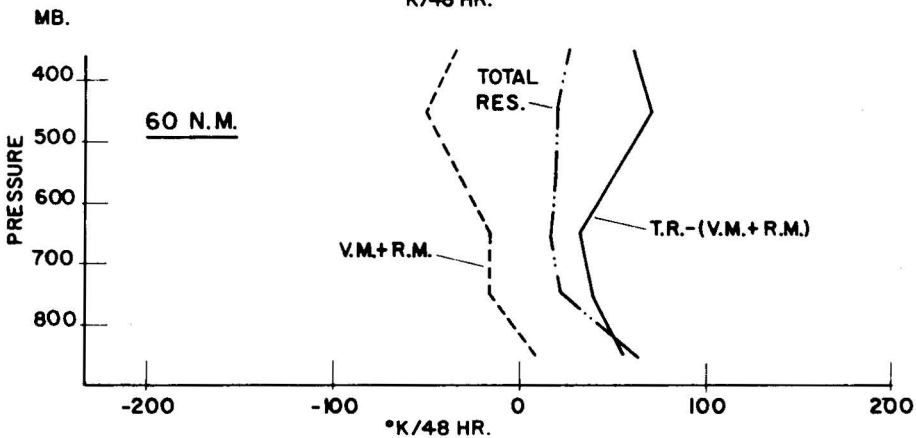
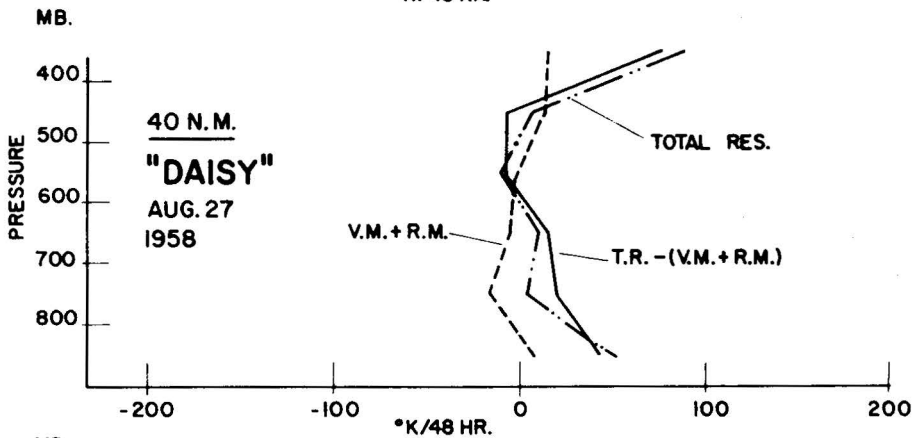
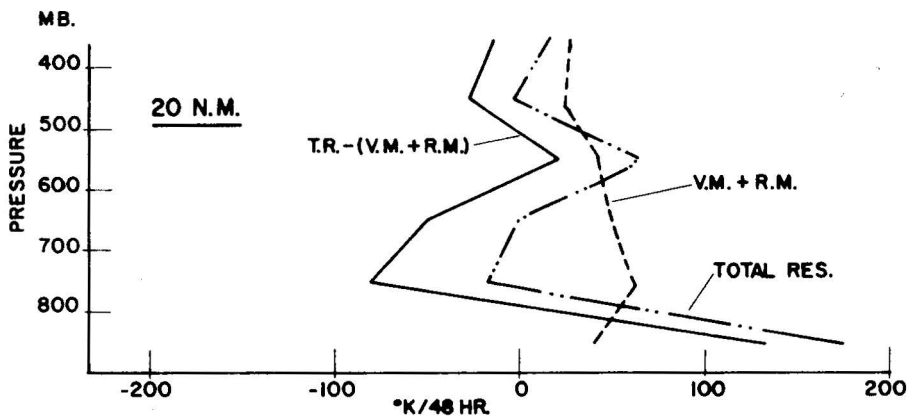


Figure 14. Total mixing, molecular-type mixing and cumulus-scale mixing distributions of equivalent potential temperature for Hurricane Daisy, Aug. 27, 1958. The mixing in Figure 13 is denoted here as Total Res. (T. R.). Molecular-type vertical and radial mixings are denoted by V. M. and R. M., respectively.

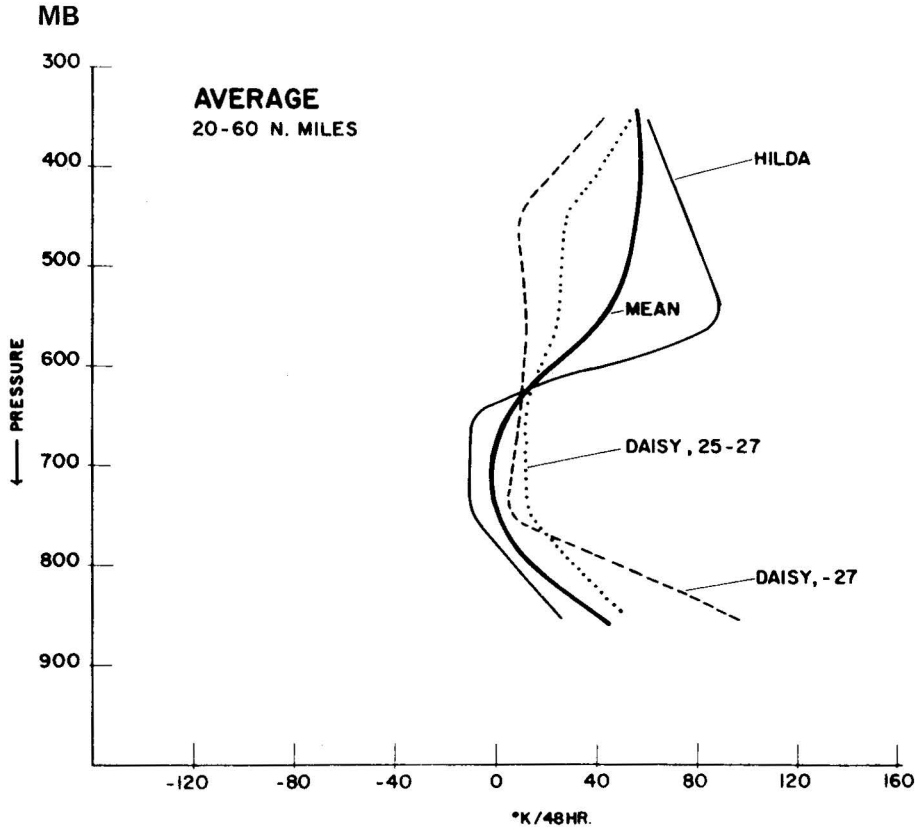


Figure 15. Average mixing of equivalent potential temperature for all radii and for both hurricanes Hilda, 1964 and Daisy, 1958.

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