OCEANIC RIDGES, MAGMA FILLED CRACKS, AND MANTLE PLUMES

JOHANNES WEERTMAN*

RESUMEN

Se propone una modificación del modelo del autor, que utiliza la inyección de grietas rellenas con magma como mecanismo motor de una placa oceánica. En el modelo original se suponía que toda la placa oceánica era fría y elástica; el modelo modificado supone estas propiedades en la parte superior de la placa solamente. La parte inferior de la placa se toma como caliente y plástica, y se supone deformable a la fluencia de alta temperatura. Las grietas rellenas con magma se congelan únicamente en la parte superior y fría de la placa. Cuando una cresta es de extrusión relativamente rápida, podrá formarse una segunda cámara magmática por debajo de la cresta oceánica, en la frontera entre las zonas elástica y plástica de la placa oceánica.

ABSTRACT

The author's model of the spreading of an oceanic ridge by injection of magma filled cracks into an oceanic plate is modified. Whereas in the original model the entire oceanic plate was considered to be cold and elastic now only the upper portion is assumed to have these properties. The lower part of the plate is taken to be hot and plastic, deformable by high temperature creep. Magma filled cracks freeze in place only in the cold upper part of the plate. If the spreading rate of a ridge is relatively fast a second magma chamber is likely to form under an oceanic ridge at the boundary between the elastic and the plastic portion of the oceanic plates.

* *Northwestern University, Evanston.*

INTRODUCTION

In previous papers (Weertman, 1971a; 1971b) I considered the problem of the vertical transport of magma under oceanic ridges*. Magma was assumed to originate somehow at a depth underneath oceanic ridges. My model of magma transport is shown in Fig. 1. A small crack is nucleated above the magma source pool. Magma enters the crack and enlarges this crack as it does so. When the crack reaches a critical length (determined by how quickly the pressure drops within the magma source pool) its lower end pinches shut. The magma-filled crack then rises upwards by fracturing rock ahead of its upper tip while at the same time healing itself shut at its lower tip. The magma-filled crack stops either near the earth's surface or at deeper depths if forced to do so by the stress fields of previous cracks. The magma of the stopped crack then freezes in place. The width of a crack is very much smaller than its length and thus a solidified crack essentially is a parallel walled dike. The oceanic ridge is expanded by this dike injection. (Magma also can spill out on the earth's surface and form volcanoes if the amount of magma within a crack is relatively large or the tensile stress within the upper crust is relatively small).

In this paper I develop further my model. In particular, I bring into the analysis the effect of creep deformation underneath oceanic ridges. The previous analysis ignored the effect of creep deformation; it was assumed that oceanic ridges and the oceanic plates on either side of ridge obey only the elastic equations of deformation.

The new analysis leads to a qualitative understanding of why magma "chambers" may form at intermediate depths underneath volcanoes and oceanic ridges.

* Erratum: Equation (9) of Weertman (1971a) should read $B(y) = (2\,\alpha\sigma_1 / \pi\mu) \log \left| \left\{ (\angle^2 - y^2)^{1/2} + (\angle^2 - \angle'^2)^{1/2} \right\} / \left\{ (\angle^2 - y^2)^{1/2} - (\angle^2 - \angle'^2)^{1/2} \right\} \right|$

THEORY

Tensile Stress in Oceanic Ridges

Consider the oceanic ridge shown in profile in Fig. 2. It is reasonable to expect that a tensile stress T is set up across the ridge. (This tensile stress has its axis aligned normal to the ridge axis and parallel to the earth's surface). A tensile stress is produced if convective flow of rock in the low strength zone beneath the oceanic plates diverges away from the oceanic ridge as shown in Fig. 2. This divergent motion beneath the plates places a shear stress σ at the lower plate boundary. This shear stress in turn produces the tensile stress T within the oceanic plate.

A tensile stress also is produced in the oceanic ridge even if the shear stress σ produced by motion in the low strength zone beneath the plates is so small it can be neglected. This tensile stress has its origin in the same set of circumstances that leads to the existence of tensile stresses within floating ice shelves (Weertman, 1957; 1962). Both the oceanic ridge and the ice shelf are lighter density solid material "floating" in a sea of heavier density material. Tensile stresses are set up in the floating solid.

Let $T(y)$ represent the value of the tensile stress at a depth y below the earth's surface. The average value $< T >$ of $T(y)$ is given by

$$< T > = H^{-1} \int_0^H T(y)\, dy \tag{1}$$

where H is the thickness of an oceanic plate. The tensile stress $T(y)$ can produce appreciable creep deformation within an oceanic plate if its magnitude is sufficiently high.

Suppose that the deformation of rock in unaxial tension or compression obeys the creep equation (Weertman, 1970)

$$\dot{\epsilon} = \dot{\epsilon}_0 (T/\mu)^{n-1} (T\Omega/k\theta) \exp(-g\theta_m/\theta) \tag{2}$$

where $\dot{\epsilon}$ is the creep rate, μ is the shear modulus, θ is the temperature, θ_m is the melting temperature, Ω is the atomic volume, k is Boltzmann's constant, and $\dot{\epsilon}_0$, n, and g are constants ($n \approx 3$, $g \approx 18$, and $\dot{\epsilon}_0 \sim 2\text{x}\ 10^{10}\ \text{s}^{-1}$). Suppose further that the shear stress σ at the base of the oceanic plate is negligible small in magnitude. From the theory of deformation of floating ice shelves it is possible to find $T(y)$ and the creep rate $\dot{\epsilon}$ of the oceanic plate in a region away from a ridge (say, at B in Fig. 2) once $< T >$ and $\theta(y)$ and $\theta_m(y)$ are given. The creep rate $\dot{\epsilon}$ is a constant and does not depend on y. The creep rate is given by

$$\dot{\epsilon} = (\dot{\epsilon}_0 H^n < T >^n \Omega / k\mu^{n-1}) \left[\int_0^H \theta^{1/n} exp(g\theta_m/n\theta)dy\right]^{-n} \quad (3)$$

and $T(y)$ by

$$T(y) = \left[< T > H\theta^{1/n} exp(g\theta_m/n\theta)\right] \left[\int_0^H \theta^{1/n} exp(g\theta_m/n\theta)dy\right]^{-1} \quad (4)$$

The temperature $\theta(y)$ in an oceanic plate is appreciably smaller than $\theta_m/3$ in the upper part of the plate and may approach θ_m in value in the lower part of the plate. Because of the large value of the expression exp $(g\theta_m/\theta)$ when $\theta > \theta_m/3$, for all practical purposes the creep rate $\dot{\epsilon}$ given by Eq. (3) is equal to zero if the depth $y = H_\sigma$ at which $\theta(y) \approx \theta_m/3$ is an appreciable fraction (say, $\theta\ H_0/H \approx 0.2$) of the plate thickness. (We are assuming that the value of $\theta(y)$ increases monotonically with increasing depth y).

The tensile stress $T(y)$ given by Eq. (4) is approximately equal to

$$T(y) \approx (H/H_0) < T > \quad (5a)$$

for $0 < y < H_0$ and

$$T(y) \approx 0 \quad (5b)$$

for $H_0 < y < H$. In other words, the total force exerted across a cross section of an oceanic plate is supported almost entirely by the colder upper portion of the plate that creeps at exceeding small rates under any stress that does not exceed the theoretical strength of rock. Figure 3 shows a schematic plot of $T(y)$ versus y given by Eqs. (5). (In addition to the tensile stress $T(y)$ a hydrostatic pressure $P(y) = \rho g y$, where ρ is the density of rock and g is the gravitational acceleration, also exists in the plate).

Tensile Stress at Ridge Axis

The tensile stress considered in the preceeding section exists in an oceanic plate near but not at the oceanic ridge axis. What is the tensile stress that acts across the cross section at the very center of an oceanic ridge?

Oceanic ridges are presumed to spread apart at rates $\dot{s}$ of the order of 50 mm a^{-1}. In the deeper, hoter parts of a ridge this spreading can take place by creep deformation. But in the colder, upper parts of the ridge appreciable creep deformation cannot occur. In this region spreading can be accomplished by dike injection in the manner shown in Fig. 1. If the tensile stresses in the colder, upper portion of the ridge were extremely high, extension of the ridge by shear fracture and slip on sets of inclined planes, as shown in Fig. 4, could take place. We assume that the tensile stresses are not sufficiently high to produce appreciable spreading by shear fracture and slip.

Consider qualitatively the change in the tensile stress $T(y)$ produced by the dike injection mechanism. Suppose a volume V of magma is generated per unit time and per unit distance along an oceanic ridge. This magma freezes in cracks and dikes in the colder, upper portion of the ridge. If no magma spills out of volcanoes onto the earth's surface the average spreading rate $\dot{s}$ is equal to

$$\dot{s} \approx V/H_0 \qquad (6)$$

since the thickness H_0 of cold rock is approximately equal to the depth of the zone in which magma freezes. Of course H_0 is smaller in magnitude underneath a ridge than elsewhere because of the heating effect produced by the introduction of magma filled cracks.

Below the depth H_0 spreading of a ridge is produced by creep deformation. The freezing magma zone above $y = H_0$ can, on average, be considered to be an ever expanding crack (see Fig. 5). This expanding crack sets up (across the vertical cross section) tensile stresses T_c (y) beyond its "tip", that is, in the region $y > H_0$. This tensile stress decreases in magnitude as the distance below $y = H_0$ increases. Above $y = H_0$ the expanding crack sets up a compressive stress (that is, a negative tensile stress). Figure 6 shows a schematic plot of the tensile stress T_c (y) that acts across the vertical cross section through the center of an oceanic ridge that is produced by the freezing of magma in the colder, upper portion of the ridge. It should be noted that at a given depth y the value of T_c (y) fluctuates with time. Individual magma filled cracks periodically enter the freezing zone and alter the stress T_c (y). The value of T_c (y) shown in Fig. 6 is averaged over time.

The stress and velocity field of a crack that is opened up in a material whose constitutive equation in that of a Newtonian solid (the creep rate of the material is proportional to the first power of the stress) has been solved (see Green et al, 1969 and Weertman, 1969). The creep equation that describes the creep behavior of rock at a relatively high temperature is more likely (Weertman, 1970) to be a power law equation (creep rate proportional to stress raised to about a 3rd power). An exact solution of a crack opened up in a material obeying power law creep is not know. It is clear, however, that the solution is qualitatively similar to that of a crack in a Newtonian solid (Weertman, 1969). The chief difference in the solutions is that the stress field of an expanding crack in power law material falls off much less rapidly with distance than does the stress field of an expanding crack in a Newtonian solid. A rough estimate can be made of T_c (y) (through the use of Eqs. (24) to (30) in Weertman, 1969 for the analogous problem of the shear crack in a power law creep material). The estimated value of T_c (y) are

$$T_c(y) \sim -(\dot{s}/2\pi CH_o)^{1/n} \quad (7a)$$

for 0 y H_o,

$$T_c(y) \sim (s/4\pi C)/(y-H_o) \quad ^{1/n} \quad (7b)$$

for H_o y $2H_o$, and

$$T_c(y) \sim (nsH_o/2\pi C)/(y-h_o)^2 \quad ^{1/n} \quad (7c)$$

for $y > 2H_o$. In Eqs. (7) the constant C is equal to$(\epsilon_o\ \Omega\ /\mu^{n-1}\ K)$ $\exp(-g\theta_m/\theta)$.

The tensile stress that acts across the cross section through the center of an oceanic ridge is the sum of $T(y)$ given by Eqs. (5) and $T_c(y)$ given by Eqs. (7). Curve A in Fig. 2 shows a schematic plot of the sum of these stresses. The total tensile stress $T_t(y)$ that acts across this central cross section is the sum of $T(y)$, $T_c(y)$, and $-\rho gy$, the negative of the hydrostatic pressure. Figure 7 shows a schematic plot of $T_t(y)$ versus depth y. In general $T_t(y)$ decreases in value with increasing depth y. However in the region just below $y = H_o$ the stress $T_t(y)$ increases. This anomalous increase is larger the greater is the spreading rate $\dot{s}$. (It should be emphasized again that $T_t(y)$ shown in Fig. 7 is the time average of this stress at a given depth).

Peach-Koehler Force on a Liquid-Filled Crack

I pointed out earlier (Weertman, 1971a) that the force that drives a magma filled crack upwards towards the earth's surface in an elastic plate is not the Archimedian buoyancy force but rather is the Peach-Koehler force that acts on dislocations. For a magma filled crack of volume V_c per unit distance along the direction of the axis of an oceanic ridge the total upward force F on the crack is

$$F = -\rho_m g V_c - (dT_t/dy)V_c$$

$$= (\rho - \rho_m)gV_c - (dT/dy + dT_c/dy)V_c$$

where ρ_m is the density of magma. The term $\rho_m g V_c$ is the downward force on the crack produced by the weight of liquid within the crack. If $T + T_c$ is equal either to zero or to a constant the force on the crack is identical to that obtained from the Archimedian buoyancy equation. (The derivation of Eq. (8) requires that the derivative $d\,T_t/\,d\,y$ be a constant over the crack length. If this derivative is not constant over the crack length some correction terms must be added to the equation).

In general $\rho > \rho_m$ and $d\,(T + T_c)\,/\,dy > 0$. Thus a magma filled crack is driven towards the earth's surface. However in the region just below y = H_o in Figs. 2 and 7 the derivative $d\,(T + T_c)\,/\,dy\ _o\ 0$. If the value of the derivative is sufficiently high that

$$d(T + T_c)/dy \geqslant (\rho - \rho_m)g \tag{9}$$

either no force or a downwards force will act on the crack. A rising crack will be stopped in the anomalous region just below $y = H_o$. If the spreading rate $\dot{s}$ is made sufficiently great Eq. (9) always can be satisfied.

Equation (8) does not give the force on a crack that is very close to the earth's surface. Image force effects have to be considered in this situation. If the crack reaches the earth's surface its lenght L_c is equal to (Weertman, 1971a)

$$L_c = \pi T_t/2(\rho - \rho_m)g \tag{10}$$

if the crack remains filled with magma and if $d\,(T + T_c)\,/\,dy \approx 0$. If T_t is negative in value no magma filled crack can exist and all the magma must spill out onto the earth's surface.

Double Magma Chamber Model of a Spreading Oceanic Ridge

The results of the last section suggest that the model of a spreading oceanic ridge that is illustrated in Fig. 1 should be modified into the double magma chamber model shown in Fig. 8. In both the models of Figs. 1 and 8 a magma chamber exists at a depth equal to the thickness of an oceanic plate. Magma for this chamber presumably is brought to it by convection currents within the earth's mantle. Magma-filled cracks are nucleated at this chamber, they grow in length, pinch themselves off from the chamber, and move upwards. The motion of such a crack should occur rapidly (Weertman, 1971b). Consequently in both models the oceanic plates act as elastic plates for fast moving cracks although according to the model of Fig. 8 the lower part of an oceanic plate is not elastic in nature to stresses that act over long periods of time.

The stress field shown in Fig. 7 of the model of Fig. 8 insures that a rising crack from the lower magma chamber will be trapped in the region just below the depth H_0, the depth above which an oceanic plate is elastic in character to long term stresses and below which it is not. Magma collected at $y \approx H_0$ can form a second magma chamber there. At this depth rock can creep under long term stresses. Over short periods of time the stress field of a crack that has risen quickly from the lower magma chamber up to the depth $y \approx H_0$ resembles the stress field around a crack in elastic material. Creep deformation around this crack after it has stopped will blunt the crack and cause its elastic stress field to decay. Thus if no magma chamber were in existence at the depth $y \approx H_0$ the stopped crack and additional cracks that follow it could form a second magma chamber.

A second magma chamber at $y \approx H_0$ in turn can act as a source of magma for magma filled cracks that move upwards into the colder, upper portion of an oceanic ridge. The model of Fig. 1 now can be used to explain the spreading of the elastic portion of the oceanic ridge if in Fig. 1 the thickness H is considered to be the thickness H_0.

The second magma chamber at $y \approx H_0$ can act as a source of cracks that rise into the cold region above it once the vertical dimension of the chamber is of the order of or larger than the vertical distance over which the derivative $d(T+T_c) / dy \geqslant (\rho - \rho_m)g$ (the region A-A in Fig. 7). Once the chamber is of this size the stressed rock that contains the anomalously varying tensile stress is replaced with magma that can support only hydrostatic pressure. The variations in the anomalous zone thus is "smoothed" out and the Peach-Koehler force arising from this variation no longer acts as a barrier to cracks that propagate upwards. It is immaterial whether we consider a crack that comes from the lower chamber as passing through the upper chamber without stopping and moving into the upper part of the oceanic ridge or whether we consider this crack to be stopped in the second chamber and at a later date a new crack to be nucleated and escape from the second magma chamber.

If the spreading rate $\dot{s}$ of a ridge is small then of course the stress $T_c(y)$ is small and a crack capture zone under the depth $y \approx H_0$ may not exist. In this situation a second magma chamber will not come into existence.

Mantle Plumes

Mantle plumes at particular spots under oceanic ridges, such as at Iceland in the mid-Atlantic Ridge, are assumed to bring up anomalously large amounts of magma from the mantle. This extra supply of magma cannot cause the spreading rate of the ridge to increase materially in the region just over the plume. Were the spreading rate $\dot{s}_m$ under the plume larger than the spreading rate elsewhere (see Fig. 9) a plug of material would be inserted into the oceanic plates that would be subjected to higher and higher compressive stresses. The stress $T_t(y)$ would become compressive up to the earth's surface ($y = 0$). But if $T_t(y)$ is compressive (see Weertman, 1971a) and Eq. (10) magma filled cracks move right up to the earth's surface and reduce their length to zero by dumping all their magma onto the earth's surface. Ejected magma cannot be used to spreading the oceanic ridge. It will only form volcanoes and islands on top of the oceanic ridge.

Thus a self regulating mechanism exists above mantle plumes that prevents an oceanic ridge from spreading at a greater than normal rate and which insures that the excess magma is cast upon the earth's outer surface.

CONCLUSION

The model of the spreading of an oceanic ridge by injection of magma filled cracks and dikes into cold, elastic oceanic plates can be modified to take into account the fact that the lower portion of the oceanic plates are hot and will deform by creep. It is not necessary that magma filled cracks freeze in place in the lower portion of an oceanic plate in order to produce spreading. If the spreading rate of a ridge is relatively fast it is likely that a second magma chamber will form within the interior of the plates in the region that divides the cold, elastic portion of the ridge from the hot, plastic portion. The formation of the second magma chamber eliminates an anomalous stress zone that would otherwise trap ascending magma filled cracks.

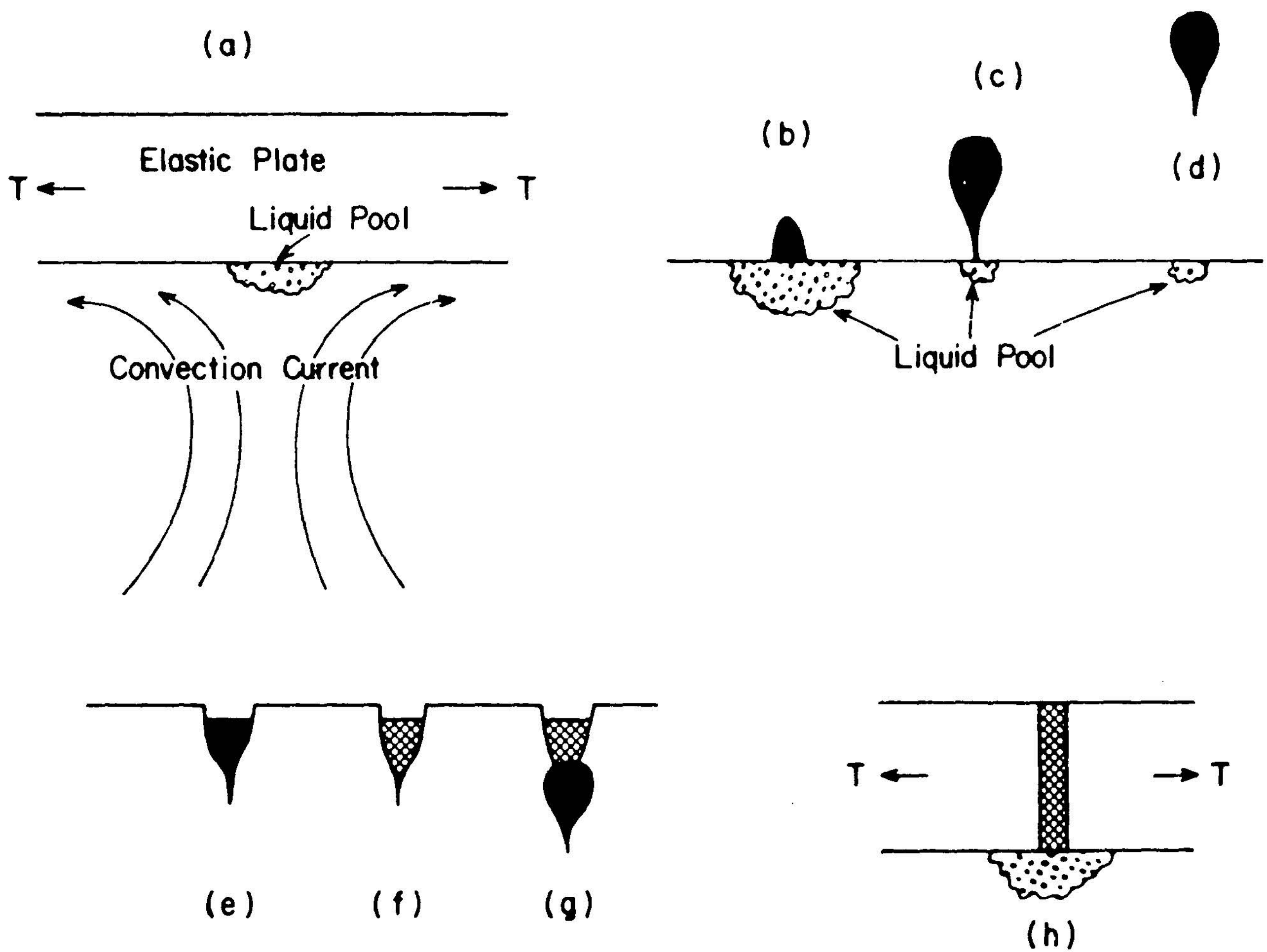

Figure 1. (a) Magma pool beneath an elastic plate created by convective motion in upper mantle. (b), (c) and (d) Magma filled crack nucleates, grows in length, pinches shut at bottom and rises in plate. (3) and (f) Crack reaches top surface and magma in it freezes. (g) New crack is trapped below old crack by stress field of old crack. (h) Frozen rock zone has extended to bottom of plate by repetitions of (g) and cycle can start again. (Taken from Weertman (1971a).)

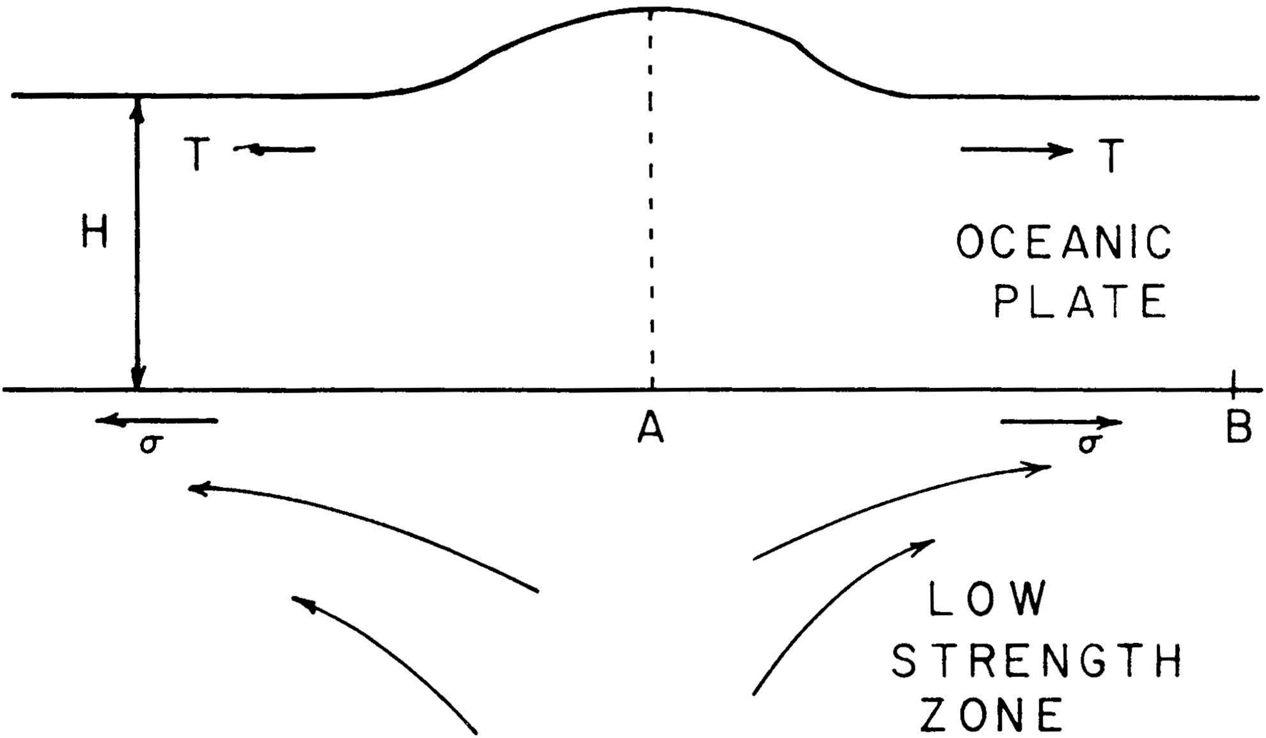

Figure 2. Cross section of oceanic ridge subjected to tensile stress.

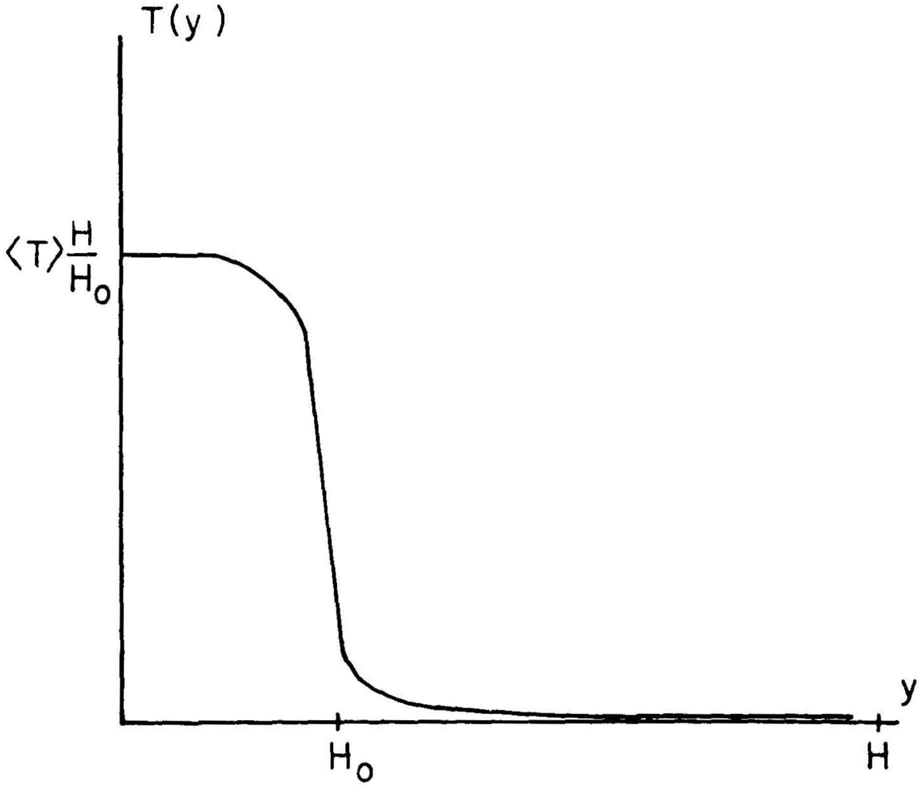

Figure 3. Schematic plot of tensile stress T(y) versus depth y.

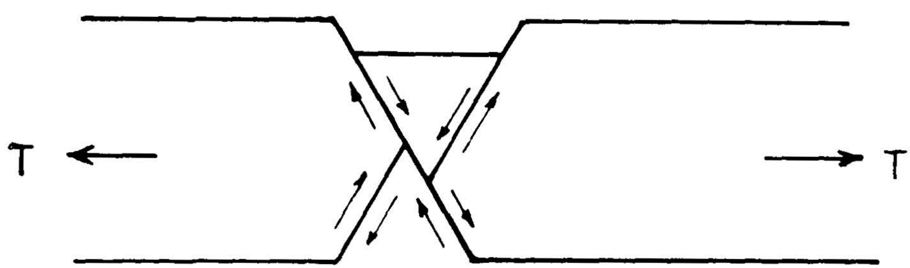

Figure 4. Spreading at a ridge axis accomplished by slip on shear fracture planes.

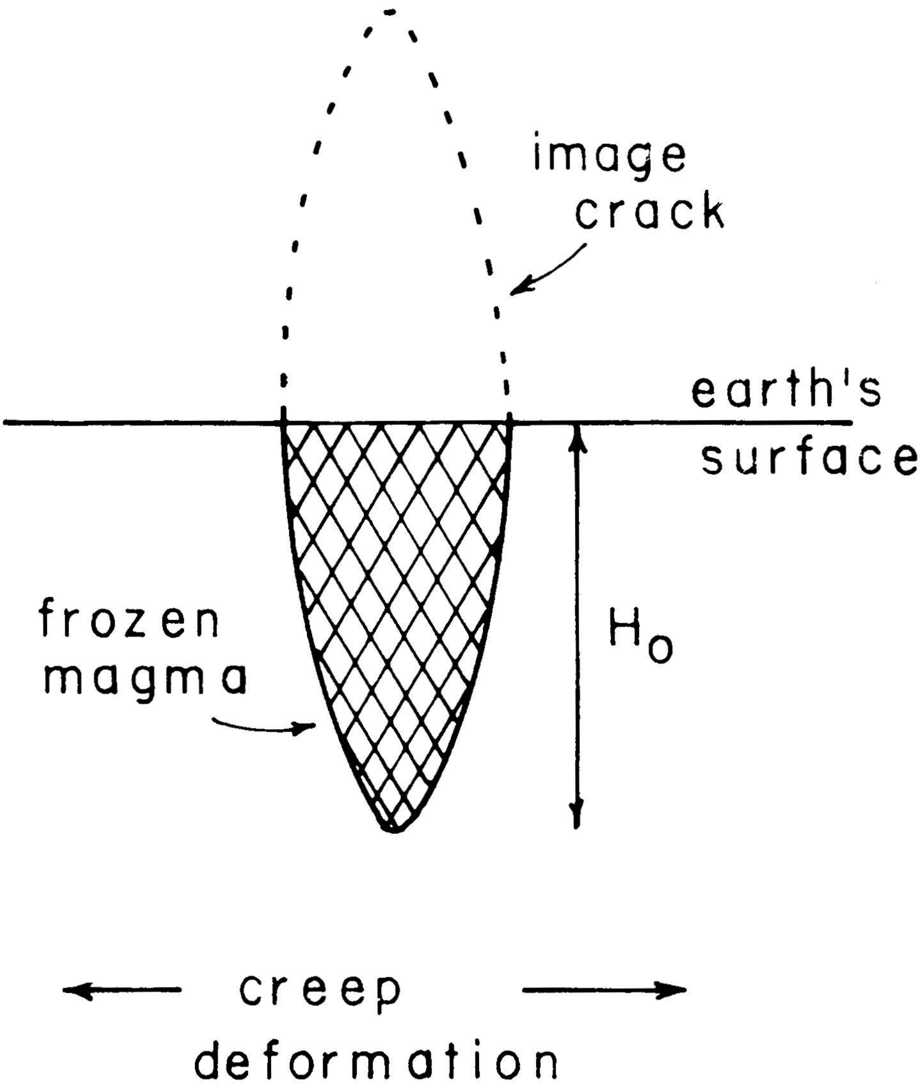

Figure 5. Freezing magma zone considered as an ever expanding crack. (An image crack exists above the earth's surface.)

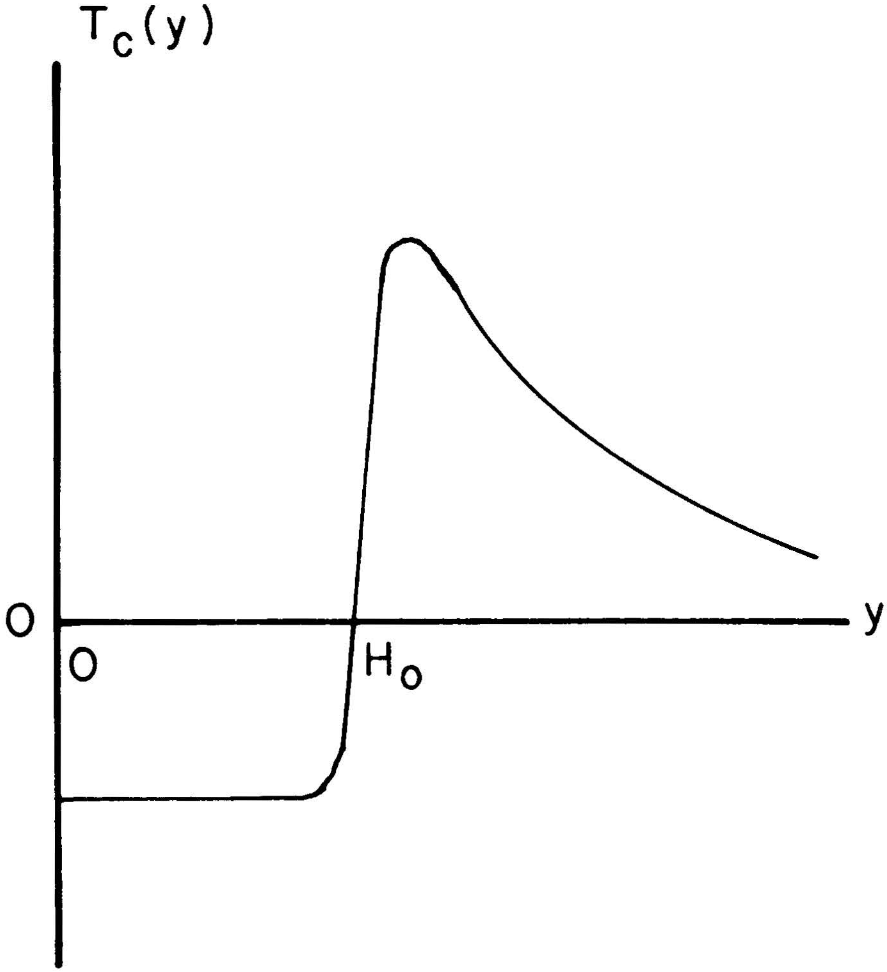

Figure 6. Schematic plot of stress produced by presence of crack shown in Figure 5.

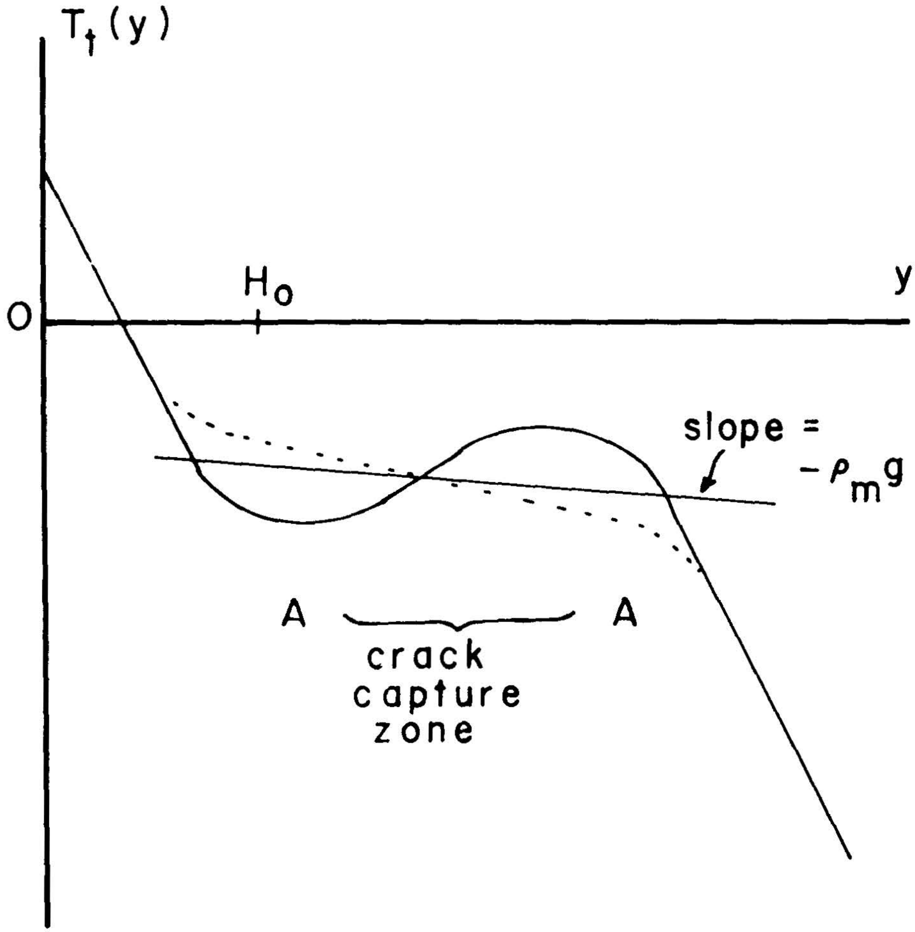

Figure 7. Schematic plot of total tensile stress $T_t\ (y) = T(y) + T_c(y) - \rho gy$ versus y. Crack capture can occur in region A-A where $dT_t/dy > -\rho_m g$. No crack capture can occur in the region A-A if the stress $T_t(y)$ is given by the dotted curve.

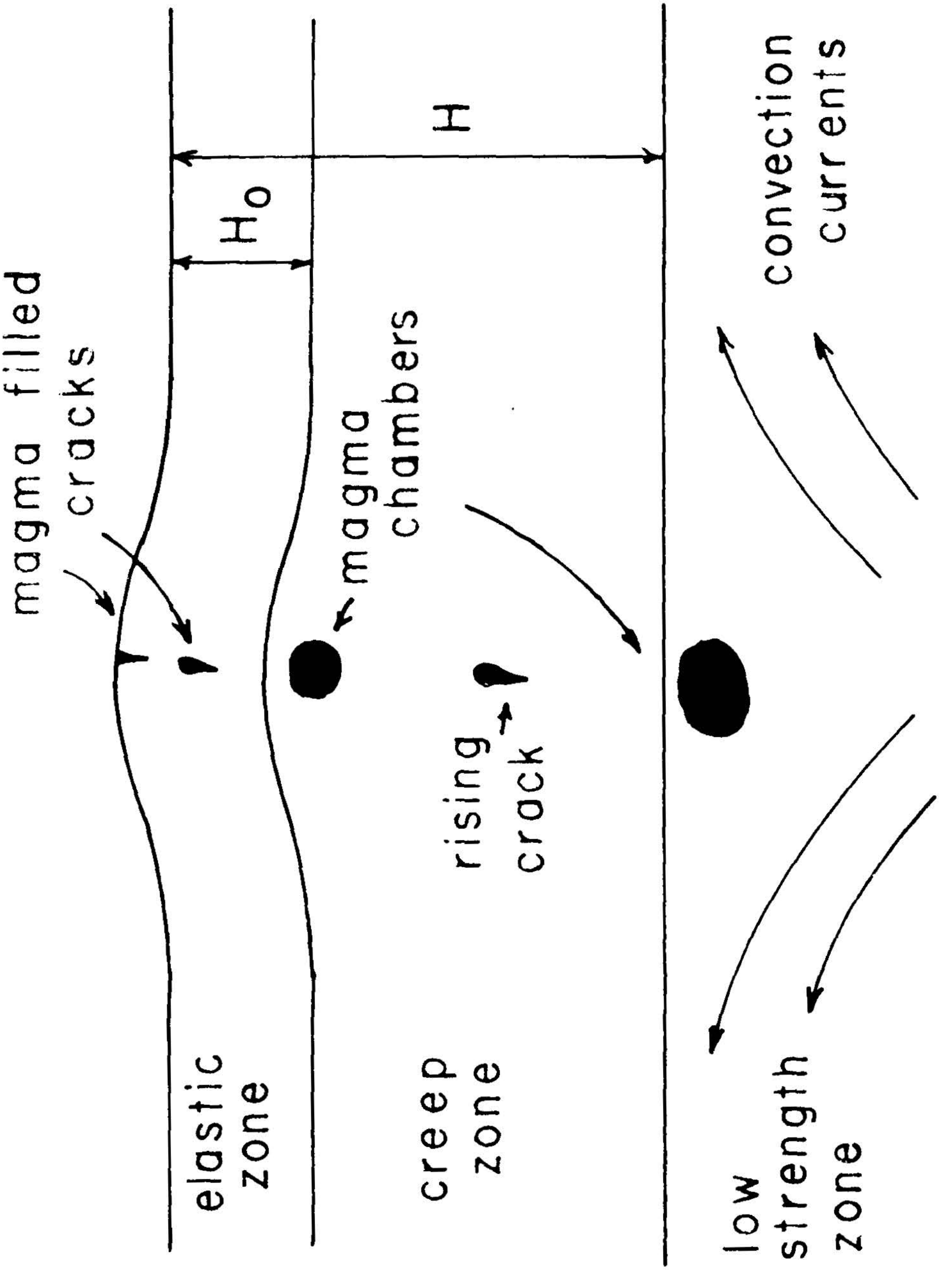

Figure 8. Model of a spreading oceanic ridge between two oceanic plates that are cold only in their upper portions.

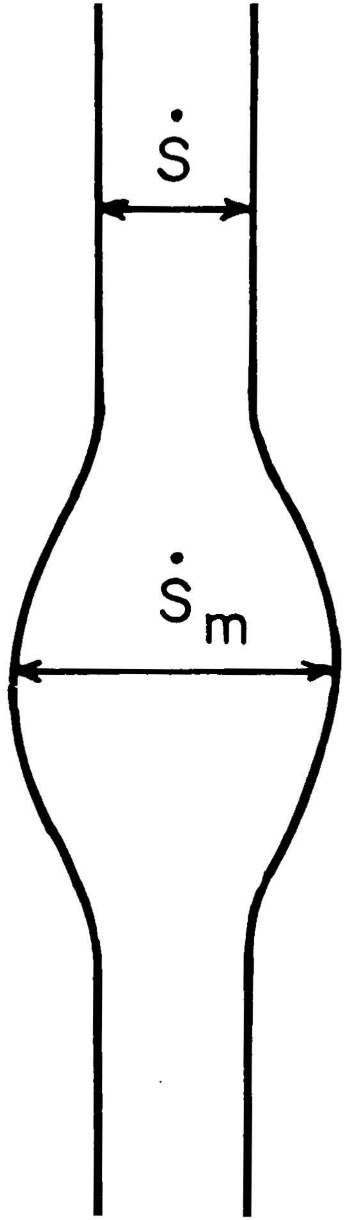

Figure 9. Spreading rate of oceanic ridge near mantle plume if all the mamga generate by the plume produced spreading.

ACKNOWLEDGMENT

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