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ELECTROMAGNETIC RESPONSE OF A CONDUCTING CYLINDER OF FINITE LENGTH

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RESUMEN

Los campos electromagnéticos se calculan para un cilindro finito, excitado por una fuente circular y, para propósitos de comparación, mediante excitación de una fuente lineal. Estos resultados también se comparan con los del caso del cilindro infinito. Se encuentra que la importancia relativa de la componente con simetría acimutal de la corriente disminuye cuando el cilindro es finito. Este artículo ha sido motivado por las convincentes sugerencias de S. K. Singh relativas a la posible importancia de la corriente con simetría acimutal inducida en cuerpos alargados de minerales, por algunas configuraciones de fuentes electromagnéticas utilizadas en la exploración geofísica.

ABSTRACT

The electromagnetic fields are calculated for a truncated cylinder excited by a loop source and, for comparision, for excitation by a line source. These results are also compared with the infinite cylinder case. It is found that the relative importance of the azimuthally symmetric component of the current is diminished when the cylinder is truncated. This paper is prompted by the cogent suggestions of S. K. Singh concerning the possible importance of the symmetrical current induced in elongated ore bodies by certain electromagnetic source configurations used in geophysical exploration.

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INTRODUCTION

In a previous analysis (Wait and Hill 1973), we formulated the boundary value problem of a truncated cylindrical region excited by a specified distribution of electric current over a concentric cylindrical surface. Preliminary numerical results were given for excitation by an electric line source of finite length. Here we consider the extension for excitation by a long slender loop. Also we examine the importance of the azimuthally symmetric component of the current induced in the cylinder. The effect of truncation and type of excitation on the azimuthally symmetric current component provides insight to the scattering mechanism for scatterers of finite length.

Quantitative information on this subject is important from the standpoint of determining the performance of electromagnetic source location schemes in mine rescue operations. For example, we can anticipate major distortions of the surface fields for a buried source if metallic conductors, such as pipes and rails, are in the vicinity. It is also possible that the truncated cylinder could serve as a convenient model for an elongated sulphide metallic body that is to be detected by a grounded or non-grounded e.m. exploration technique.

FORMULATION

For the sake of simplicity, we consider a perfectly conducting cylinder excited by a finite line source as shown in Figure 1. The cylinder occupies the space $\rho < a$ and $|z| < s$, and the region external to the cylinder ($\rho > a$) has conductivity σ, and magnetic permeability μ_0. The problem is made tractable by requiring that the normal current density be zero at the two horizontal surfaces where $|z| = s$. This condition requires that the normal electric field is also zero at $|z| = s$. Because of this condition, we are not actually treating an isolated finite cylinder, but we expect the solution to tell us something about the effects of truncation.

If we excite the cylinder with a z-directed line current, $I \exp(i\omega t)$, from $z = -\ell$ to $z = +\ell$, then the solution is even about $z = 0$. The primary magnetic field components, H_ρ^{pr} and H_ϕ^{pr}, are given by (Wait and Hill, 1973).

$$H_\rho^{pr} = \frac{I\rho_o \sin(\phi_o - \phi)}{\pi\rho_d} \sum_{n=0}^{\infty} K_1(u_n\rho_d) \frac{\sin\lambda_n \ell}{\lambda_n s} u_n \cos\lambda_n z, \qquad (1)$$

$$H_\phi^{pr} = \frac{I[\rho - \rho_o\cos(\phi_o - \phi)]}{\pi\rho_d} \sum_{n=0}^{\infty} K_1(u_n\rho_d) \frac{\sin\lambda_n \ell}{\lambda_n s} u_n \cos\lambda_n z$$

Where $\rho_d = [\rho^2 + \rho_o^2 - 2\rho\rho_o\cos(\phi_o - \phi)]^{1/2}$,

$$\lambda_n = (2n+1)\pi/(2s), \quad u_n = [\lambda_n^2 + \gamma_o^2]^{1/2},$$

$$\gamma_o^2 = i\omega\mu_o\sigma,$$

and K_m is a modified Bessel function of the second kind. The secondary magnetic fields, H_ρ^{sec} and H_ϕ^{sec}, are given by

$$H_\rho^{sec} = \frac{I}{\pi\rho} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{I_m(u_n a)}{K_m(u_n a)} K_m(u_n\rho_o) K_m(u_n\rho) im \ldots$$

$$\exp[im(\phi_o - \phi)] \frac{\sin\lambda_n \ell}{\lambda_n s} \cos\lambda_n z, \qquad (2)$$

$$H_{\phi}^{sec} = \frac{I}{\pi} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{I_m(u_n a)}{K_m(u_n a)} K_m(u_n \rho_o) u_n K_m'(u_n \rho) \ldots$$

$$\exp[im(\phi_o - \phi)] \frac{\sin \lambda_n \ell}{\lambda_n s} \cos \lambda_n z,$$

where I_m is a modified Bessel function of the first kind. The general expressions for the secondary fields of a cylinder of finite conductivity are given by Wait and Hill (1973).

If we consider two oppositely directed line sources to approximate a long slender loop source, then the line source solutions for the primary and secondary fields are directly applicable. If the positive line source is located at (ρ_o^+, ϕ_o^+) and the negative line source is located at (ρ_o^-, ϕ_o^-), then the secondary fields for loop excitation, $H_\rho^{\ell,sec}$, are given by

$$H_\rho^{\ell,sec} = H_\rho^{sec}\Big|_{\substack{\rho_o = \rho_o^+ \\ \phi_o = \phi_o^+}} - H_\rho^{sec}\Big|_{\substack{\rho_o = \rho_o^- \\ \phi = \phi_o^-}}$$

$$H_\phi^{\ell,sec} = H_\phi^{sec}\Big|_{\substack{\rho_o = \rho_o^+ \\ \phi_o = \phi_o^+}} - H_\phi^{sec}\Big|_{\substack{\rho_o = \rho_o^- \\ \phi_o = \phi_o^-}}$$

where H_ρ^{sec} and H_ϕ^{sec} are given by (2).

Since we are particularly interested in the behavior of the azimuthally symmetric (m = 0) term of the secondary field, it is useful to examine the behavior of this term analytically as the loop width w goes to zero. For the case of broadside excitation ($\rho_o^+ = \rho_o^-$) the loop width w is equal to $\rho\,\Delta$, where Δ is the angular separation of the two line sources. We examine only the ϕ component, since the ρ component of the secondary field has no m = 0 term. The limit of $H_\phi^{\ell,sec}$ as w (or Δ) goes to zero is

$$H_\phi^{\ell,\,sec} = H_\phi^{sec}\Big|_{\phi_o = \phi_o + \frac{\Delta}{2}} - H_\phi^{sec}\Big|_{\phi_o = \phi_o - \frac{\Delta}{2}} \simeq \frac{w}{\rho_o}\,\frac{\partial H_\phi^{sec}}{\partial \phi_o} \qquad (4)$$

By noting that $\partial/\partial\phi_o$ introduces a factor im in (2), we conclude that the m = 0 term is not excited by broadside excitation.

For the case of end-on excitation ($\phi_o^+ = \phi_o^-$), we again examine the result as w goes to zero. In this case, the limit of $H_\phi^{\ell,sec}$ is given by

$$H_\phi^{\ell',sec} = H_\phi^{sec}\Big|_{\rho_o = \rho_o + \frac{w}{2}} - H_\phi^{sec}\Big|_{\rho_o = \rho_o - \frac{w}{2}} \simeq w\,\frac{\partial H_\phi^{sec}}{\partial \rho_o} \qquad (5)$$

By Examing (5) and (2), we note that the m = 0 term is excited by end-on excitation.

Since the effect of truncation on the secondary fields is of interest, it is useful to examine the limit of an infinite cylinder. If we let $\ell = s$ and let s approach ∞, then u_m approaches γ_o and λ_n approaches zero. Consequently, the n summation in (2) reduces to

$$\sum_{n=0}^{\infty} \frac{\sin \lambda_n s}{\lambda_n s} \rightarrow \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = \frac{1}{2}, \tag{6}$$

where a summation formula by Wheelon (1968) has been used. By applying (6) to (12), the secondary fields for an infinite cylinder, $H_\rho^{sec,\infty}$ and $H_\phi^{sec,\infty}$, are found to be

$$H_\rho^{sec,\infty} = \frac{I}{2\pi\rho} \sum_{m=-\infty}^{\infty} \frac{I_m(\gamma_o a)}{K_m(\gamma_o a)} K_m(\gamma_o \rho_o) K_m(\gamma_o \rho)\, im \ldots$$

$$\exp[im(\phi_o - \phi)], \tag{7}$$

$$H_\phi^{sec,\infty} = \frac{I}{2\pi} \sum_{m=-\infty}^{\infty} \frac{I_m(\gamma_o a)}{K_m(\gamma_o a)} K_m(\gamma_o \rho_o)\, \gamma_o K_m'(\gamma_o \rho) \ldots$$

$$\exp[im(\phi_o - \phi)]$$

The results in (7) agree with the known two dimensional result (Wait, 1952; 1959).

NUMERICAL RESULTS FOR LINE SOURCE EXCITATION

It is useful to examine some numerical results for line source excitation before examining loop excitation. The special geometry for numerical results is shown in Figure 2 with the line source at the origin of a cartesian coordinate system x, y, z. The magnetic field components H_x and H_y are obtained from

$$H_x = H_\rho \cos\phi - H_\phi \sin\phi \tag{8}$$

$$H_y = H_\rho \sin\phi + H_\phi \cos\phi$$

All results are computed for an observer on the x-axis. Consequently, the situation could involve scanning the vertical and horizontal magnetic fields (H_y and H_x) at the earth's surface ($y = 0$) for a buried cylinder centered at $x = d$ and $y = -h$.

In Figure 3, we show the magnitude of the normalized vertical secondary magnetic field $2\pi h |H_y^{sec}|/I$ as well as the $m = 0$ term for an infinite cylinder. For all cases in this section, we choose $d = 0$, $a/h = 0.5$, and $|\gamma_0 h| = 0.5$. Since $d = 0$, the curves are symmetrical about $x = 0$. In Figure 4, we show the magnitude of the normalized horizontal magnetic field $2\pi h|H_x^{sec}|/I$ as well as the $m = 0$ term for an infinite cylinder. Results in Figures 3 and 4 are obtained from (7) and (8).

In Figure 5, we show the case similar to that of Figure 3, but for a truncated cylinder with $s/h = 2.0$ and $\ell/s = 0.5$. Note that the relative importance of the $m = 0$ terms is somewhat reduced here due to the truncation of the cylinder. In Figure 6, we show the case similar to that in Figure 4, but again for a truncated cylinder with $s/h = 2.0$ and $\ell/s = 0.5$. Again the relative magnitude of the $m = 0$ term is somewhat reduced due to the truncation. Results in Figures 5 and 6 are obtained from (2) and (8).

NUMERICAL RESULTS FOR LOOP EXCITATION

The geometry for loop excitation is also shown in Figure 2. The positive line source is located on the x-axis at $x = w/2$ and the negative line source is on the x-axis at $x = - w/2$. The required values of ρ_o^+, ρ_o^-, ϕ_o^+, and ϕ_o^- are given by:

$$\rho_o^+ = [h^2 + (d - w/2)^2]^{1/2} \;, \; \rho_o^- = [h^2 + (d + w/2)^2]^{1/2} \;, \ldots$$

$$\phi_o^+ = \tan^{-1}[h/(-d + w/2)], \phi_o^- = \tan^{-1}[h/(-d - w/2)]. \quad (9)$$

The results in this section are obtained from (3) and (8). We again can visualize the situation as scanning the vertical and horizontal magnetic field components at the surface ($y = 0$) for a buried cylinder with a loop excitation. One possible reason for using a loop is that no grounding of the current-carrying cable is required.

In Figures 7 and 8, we show the magnitudes of the vertical and horizontal secondary normalized components, $2\pi h^2 |H_y^{\ell,sec}|/(w\ I)$ and $2\pi\ h^2 |H_x^{\ell,sec}|/(w\ I)$, for an infinite cylinder. Again we have chosen $a/h = 0.5$ and $|\gamma_o h| = 0.5$, but we have taken d/h equal to 0.5 so that the $m = 0$ term will be nonzero. We can see from (4) that the $m = 0$ term would be zero for $d = 0$ because ρ_o^+ would equal ρ_o^-. Also, while the $m = 0$ terms are symmetric about $x/h = 0.5$, the total secondary fields are not.

In Figures 9 and 10, we show the same cases for a truncated cylinder with $s/h = 2.0$ and $\ell/s = 0.5$, and the results are quite similar to those in Figures 7 and 8. The total field which would be measured includes also a primary vertical component from the loop.

FINAL REMARKS

The results for loop excitation in Figures 7-10 are qualitatively similar to those for line source excitation in Figures 3-6, but they show a reduced importance of the azimuthally symmetric $m = 0$ term. Of course the primary fields of the loop are quite different and, as a result, the measured vertical component would differ considerably from that of a line source.

Also it has been demonstrated in Figures 5, 6, 9, and 10, that truncation of the cylinder results in a reduced magnitude of the $m = 0$ term. This is expected on physical reasoning since the symmetric current must be zero at the ends, but other currents could flow up one side of the cylinder and down the other. As pointed out by Singh (1973), it is important to consider the effect of the $m = 0$

term since it may be very significant. It's effect was ignored in an early paper (Wait 1952) on the subject, although the general theory given there contained it.

The numerical results shown in Figs. 3 to 10 inclusive are restricted to the value $|\gamma_0 h| = 0.5$. This is a typical value of the parameter when the surrounding medium is finitely conducting. As pointed out by Singh (1973), this parameter should not be assumed zero when modelling ore bodies immersed by country rock. However, the conclusions and general nature of our results are not affected by the precise value of $| \gamma_0 h |$. To illustrate this point, we illustrate the effect of finite γ_0 for the infinite cylinder case and for line source excitation only in Figs. 11 and 12 with the pertinent parameters indicated on the figure. Corresponding curves for a truncated cylinder are illustrated in Figs. 13 and 14. Here the case $\gamma_0 = 0$ corresponds to a purely static condition. The general theoretical results given by Wait and Hill (1973) also allow the effect of finite conductivity of the cylinder to be accounted for. We plan to carry out these calculations in the near future.

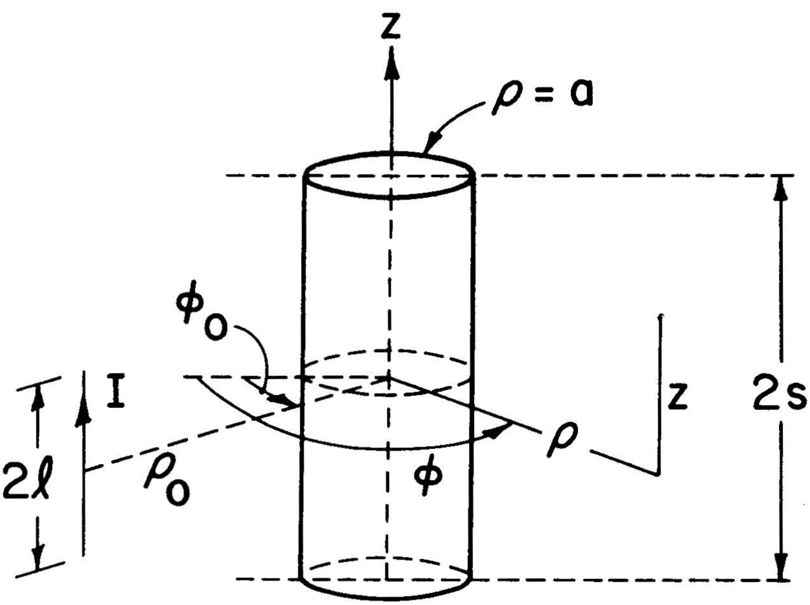

Figure 1. Geometry for a finite length cylinder and line source.

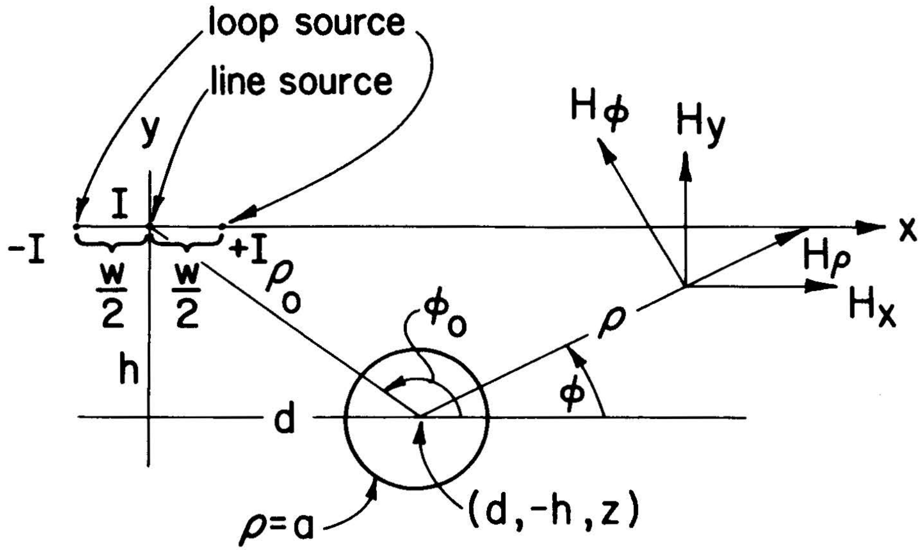

Figure 2. Special geometry for numerical results, both line source and loop excitation.

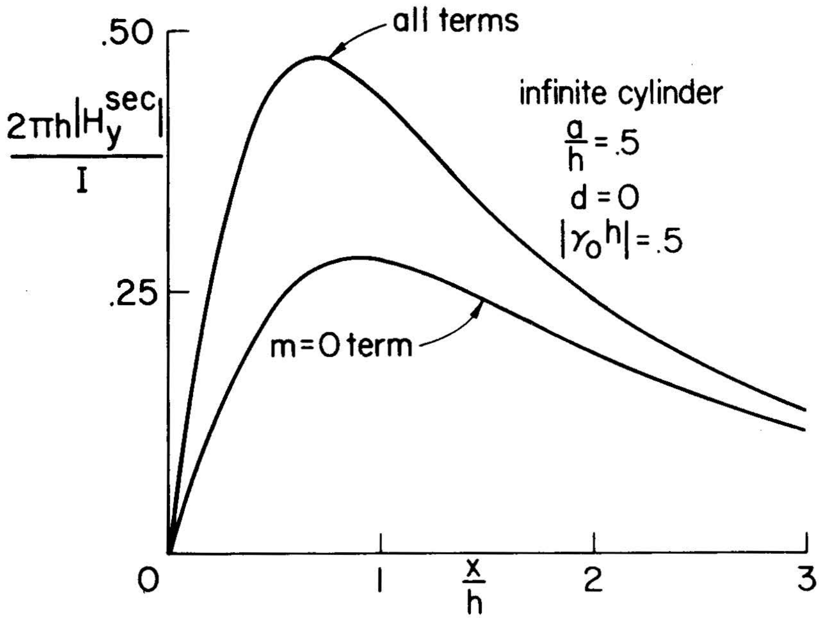

Figure 3. Normalized vertical secondary magnetic field of an infinite cylinder for line source excitation. All numerical results are for y and z equal zero.

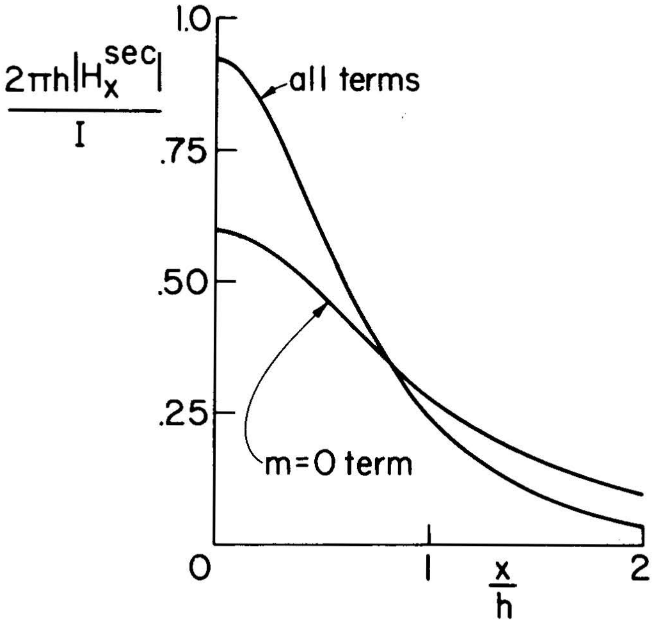

Figure 4. Normalized horizontal secondary magnetic field of an infinite cylinder for line source excitation.

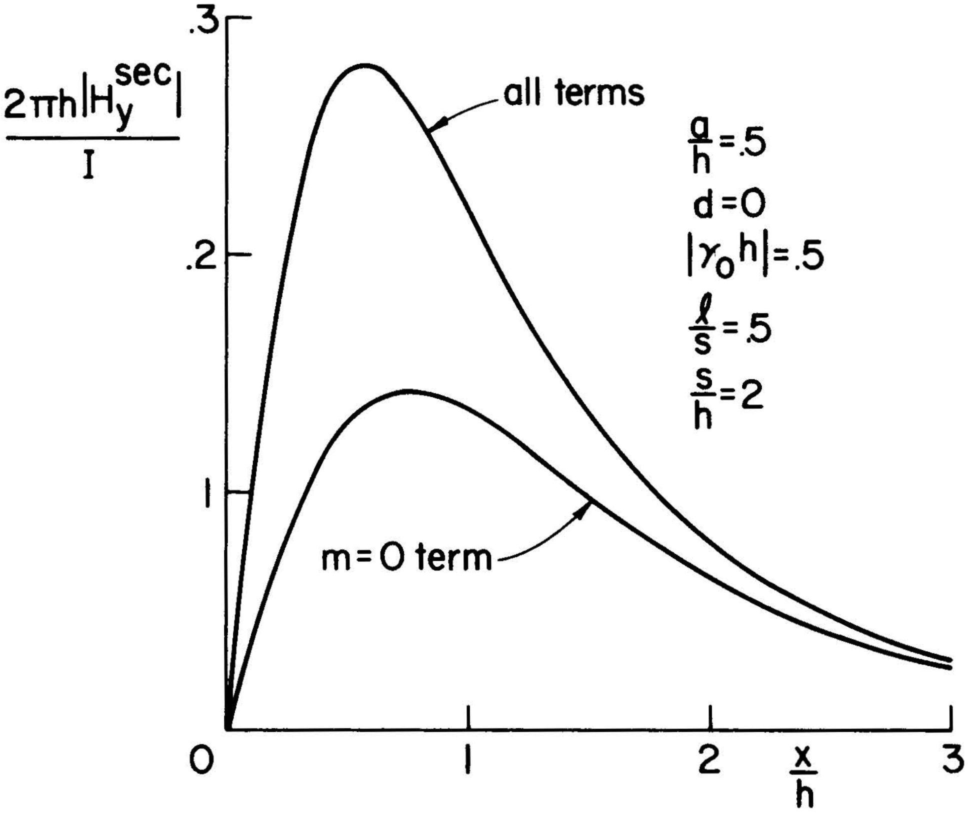

Figure 5. Normalized vertical secondary magnetic field of a truncated cylinder for line source excitation.

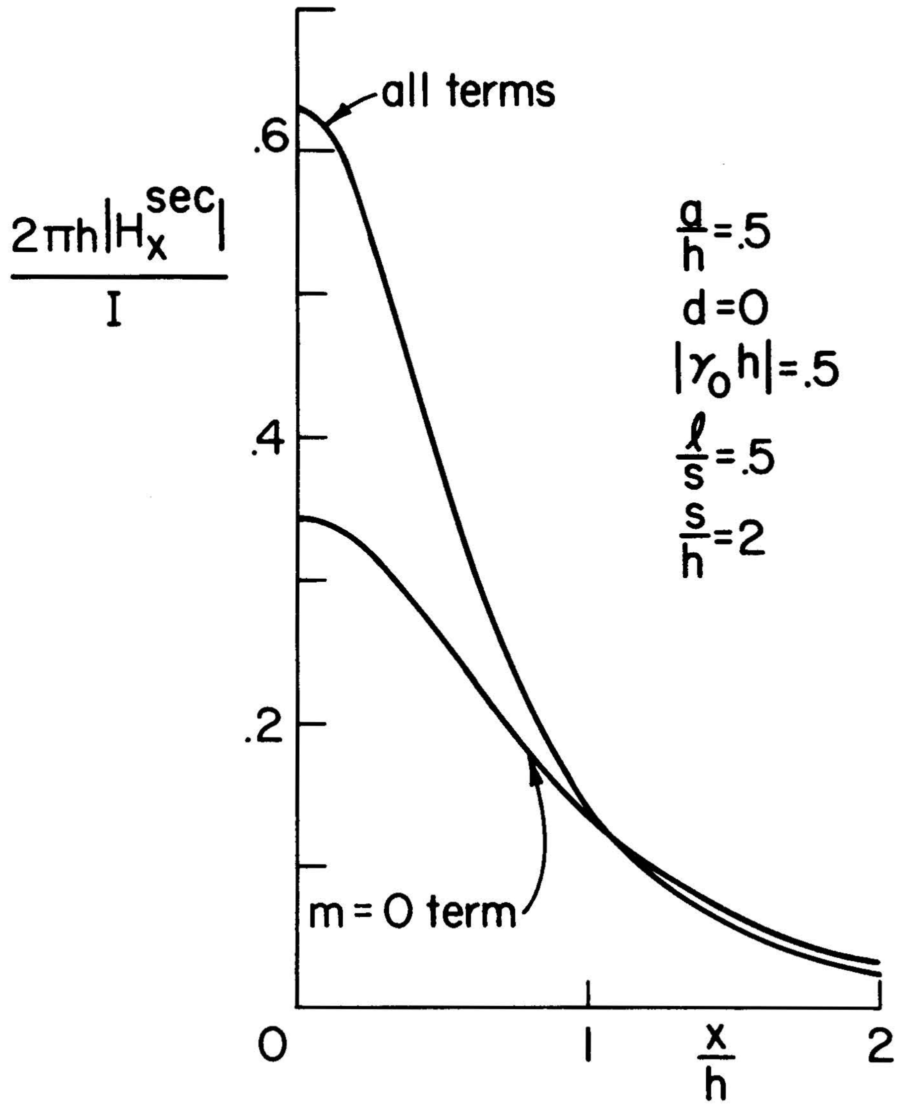

Figure 6. Normalized horizontal secondary magnetic field of a truncated cylinder for line source excitation.

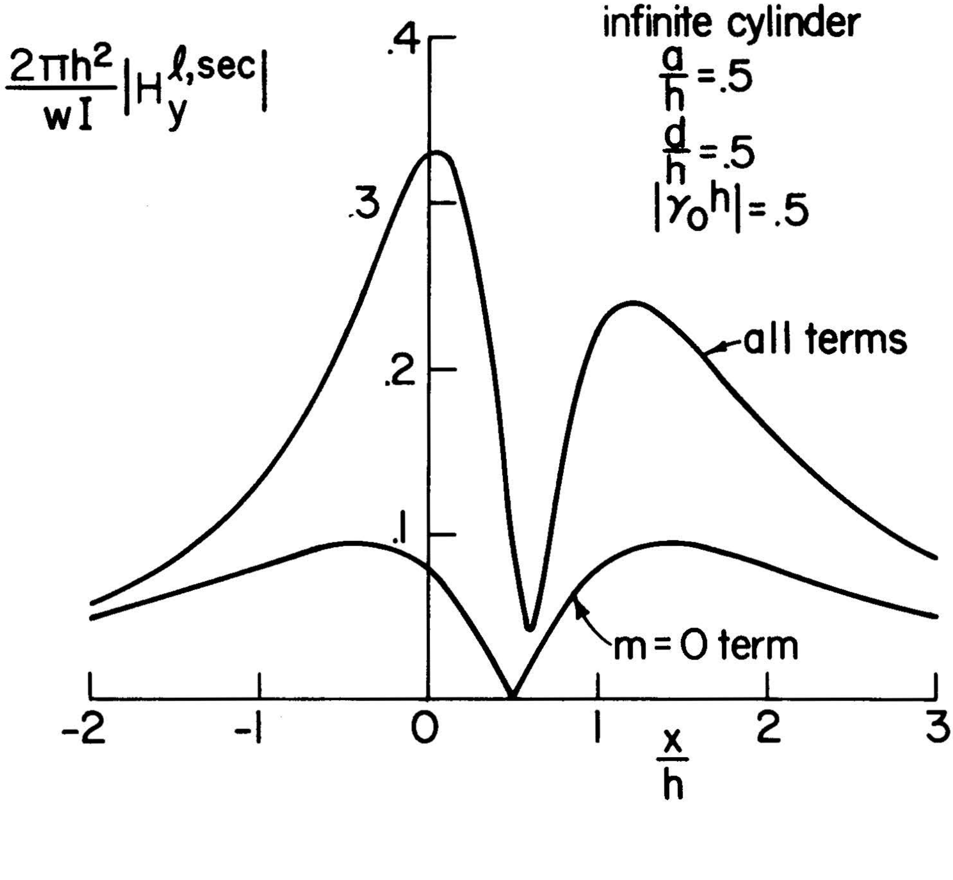

Figura 7. Normalized vertical secondary magnetic field of an infinite cylinder for loop excitation.

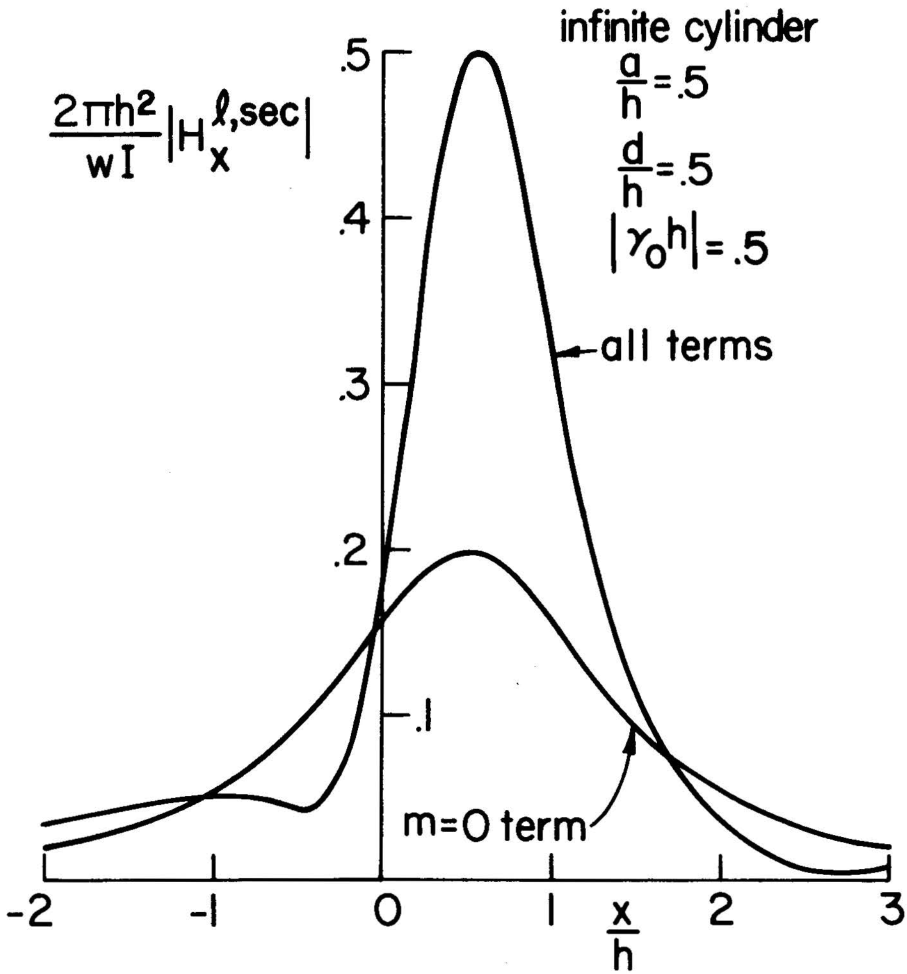

Figure 8. Normalized horizontal secondary magnetic field of an infinite cylinder for loop excitation.

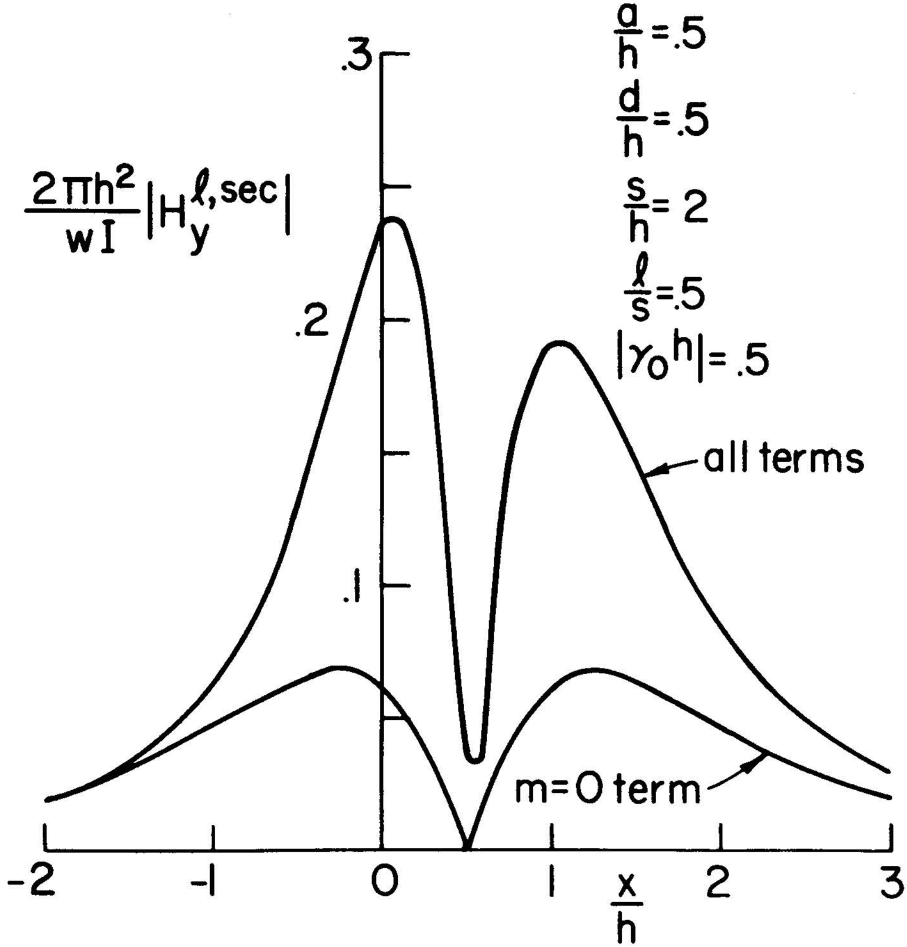

Figure 9. Normalized vertical secondary magnetic field of a truncated cylinder for loop excitation.

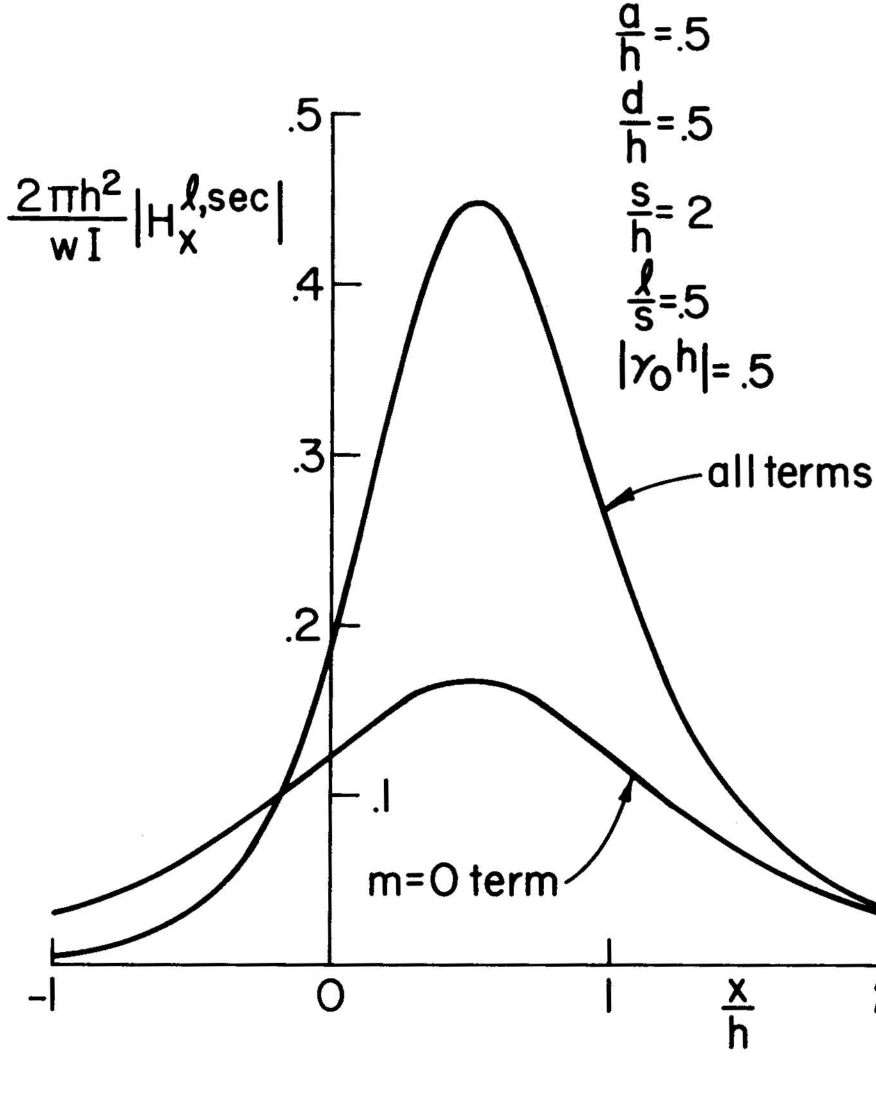

Figure 10. Normalized horizontal secondary magnetic field of a truncated cylinder for loop excitation.

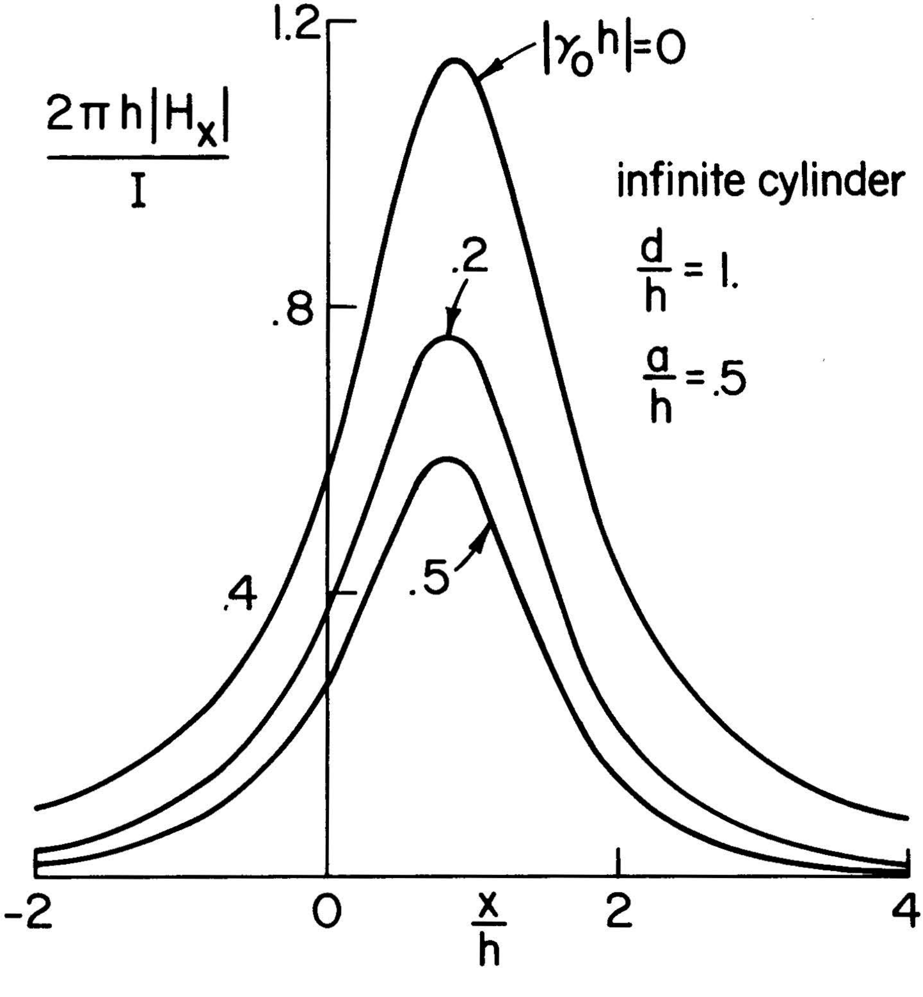

Figure 11. Effect of γ_0 on the horizontal magnetic field for the infinite cylinder case.

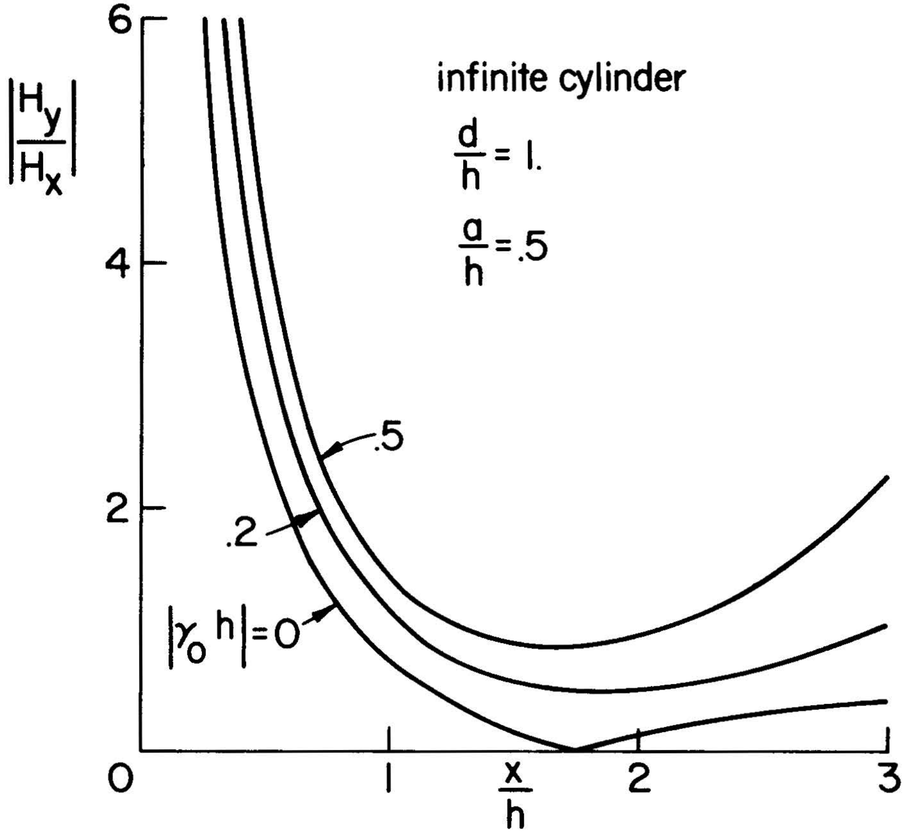

Figure 12. Effect of γ_0 on the magnitude of the ratio of vertical to horizontal magnetic field (infinite cylinder case).

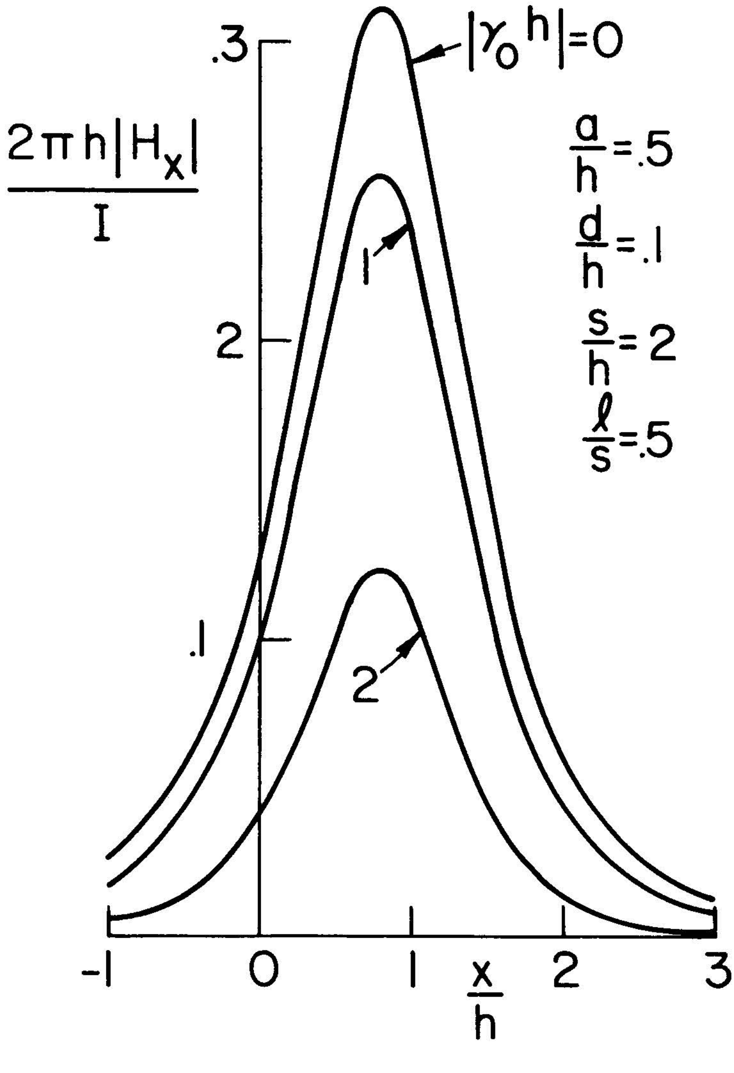

Figure 13. Effect of γ_0 on the horizontal magnetic field for the truncated cylinder case.

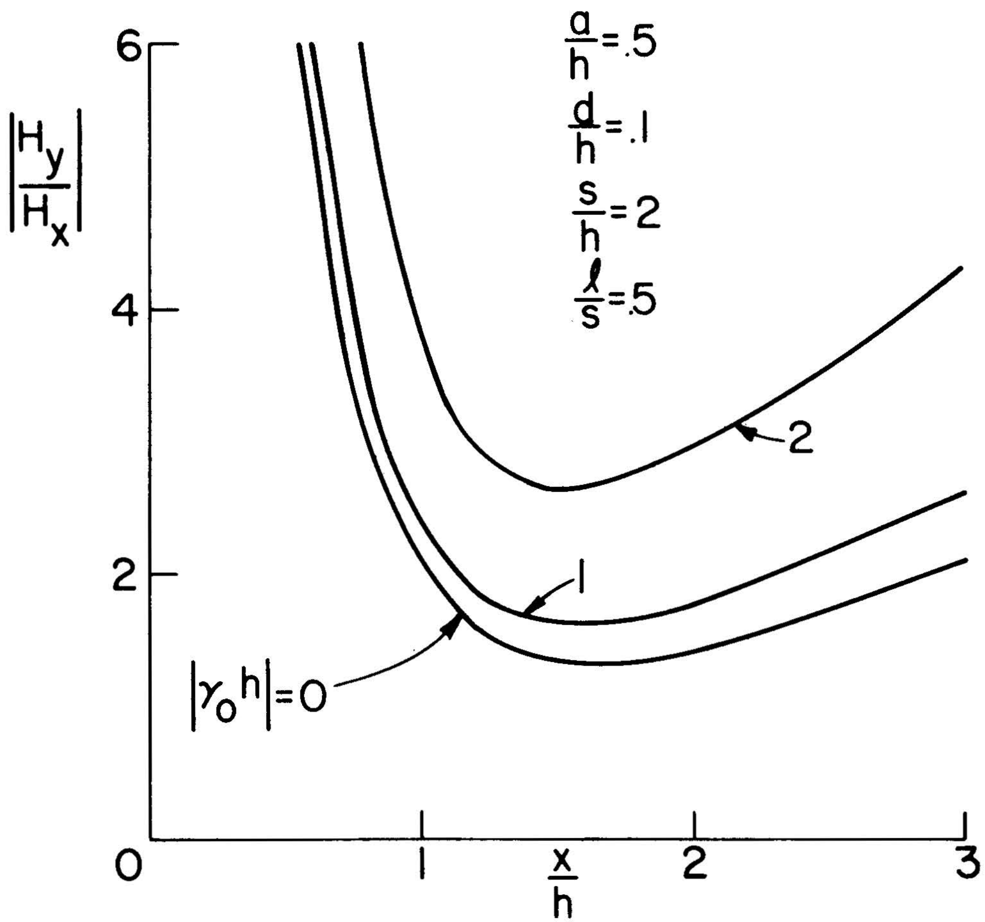

Figure 14. Effect of γ_0 on the magnitude of the ratio of vertical to horizontal magnetic field (truncated cylinder case).

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