*TRANSIENT ELECTROMAGNETIC RESPONSE OF A CONDUCTING CYLINDER IN A CONDUCTING MEDIUM: NUMERICAL RESULTS**

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RESUMEN

Se presentan resultados numéricos para la respuesta electromagnética transitoria de un cilindro conductor infinito inmerso en un medio conductor excitado por una fuente lineal infinita. La corriente de entrada es una función de Heaviside de tiempo. Las corrientes de desplazamiento se supone que son insignificantes en comparación con las corrientes conductoras. La contribución del término $n = o$ (corriente lineal inducida en el cilindro) aparece predominando sobre los multipolos superiores. Como regla aproximada el efecto de la conductividad exterior debe tomarse en consideración si es mayor de 10^{-2} veces que el valor de la conductividad del cilindro.

ABSTRACT

Numerical results for the transient electromagnetic response of a conducting infinite cylinder embedded in a conducting medium excited by an infinite line source are presented. The input current is a Heaviside function of time. The displacement currents are assumed to be negligible in comparison with the conduction currents. The contribution of $n = 0$ term (induced line current in the cylinder) is shown to predominate over the higher multipoles. As a rough rule the effect of the outer conductivity must be taken into consideration if it is more than 10^{-2} times the conductivity of the cylinder.

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INTRODUCTION

The electromagnetic (EM) response of a finite conducting cylinder buried in a half-space, excited by a finite line current source which lies on the surface, is obviously very useful in EM prospecting. Since the solution of the problem as posed above is difficult to obtain, various approximations are made. The cylinder and the line current source are assumed to be infinite in extent, with the line source lying parallel to the axis of the cylinder, thus reducing the problem to a two dimensional one. Usually at this stage conductivity of the half-space is neglected (cylinder is now lying in air) and a solution under quasi-static approximation is obtained. Many such solutions, both in the frequency as well as the time-domain, have been published. There are also some articles which provide improvement over these approximations. For example, Wait and Hill (1973) have attempted to evaluate the effect of finiteness of the current element and the cylinder in the frequency-domain. Wait (1972), Parry and Ward (1971), and Hohmann (1971) have considered the effect of finite conductivity of the half space on the EM response of a buried infinite cylinder, again in the frequency domain.

In this paper we present numerical results on the transient response of an infinite conducting cylinder embedded in conducting whole space excited by an infinite insulated line source. The theory is presented in an earlier paper (Singh, 1972). The model is admittedly unrealistic but it clearly demostrates the need to consider the effect of the conductivity of the medium outside the cylinder in the time-domain under certain conditions. Hopefully the results would also be useful in obtaining and/or checking solutions of more realistic models.

SOLUTION

The geometry of the problem is shown in Figure 1. An infinite cylinder of radius a and electrical properties ϵ_2 σ_2, μ_2 is embedded in a whole space of properties ϵ_1, σ_1, μ_1. An infinite line source lying parallel to the axis of the cylinder at S $(r_o, \phi_o; r_o > a)$ is excited by a Heaviside current. If we assume that the displacement current is negligible in comparison with the conduction current, the secondary magnetic field components outside the cylinder, dropping the static part of the solution, can be written (Singh, 1972).

$$H_r^s(t) = \frac{I}{2\pi r} \sum_{n=1}^{\infty} \frac{\sin n(\phi - \phi_o)}{(bd)^n} \left[h_{nr}^s(t) \right] \tag{1}$$

$$H_\phi^s(t) = -\frac{I}{2\pi r} \sum_{n=0}^{\infty} \frac{\cos n(\phi - \phi_o)}{(bd)^n} \left[h_{n\phi}^s(t) \right] \tag{2}$$

where,

$$h_{nr}^s(t) = -2n(bd)^n \left[\int_o^\infty \frac{p_n(u)\,[p_n(u)\,f_n(u) + g_n(u)\,q_n(u)]}{(p_n^2 + q_n^2)} \ldots \right.$$

$$\left. e^{-u^2 t/\beta^2}\, du \right] \tag{3}$$

$$h_{n\phi}^s(t) = \delta_n (bd)^n \left[\int_o^\infty \frac{Cb\, p_n(u)\,[p_n(u)\,f_n(u) + g_n(u)\,q_n(u)]}{(p_n^2 + q_n^2)} \ldots \right.$$

$$\left. e^{-u^2 t/\beta^2}\, du \right] \tag{4}$$

are the response functions.

In equations (3) and (4)

$$p_n(u) = J_n(Cu)\, J'_n(u) - CK\, J'_n(Cu)\, J_n(u) \tag{5a}$$

$$q_n(u) = Y_n(Cu)\, J'_n(u) - CK\, Y'_n(Cu)\, J_n(u) \tag{5b}$$

$$f_n(u) = J_n(Cdu)\, J_n(Cbu) - Y_n(Cdu)\, Y_n(Cbu) \tag{5c}$$

$$f'_n(u) = J_n(Cdu)\, J'_n(Cbu) - Y_n(Cdu)\, Y'_n(Cbu) \tag{5d}$$

$$g_n(u) = J_n(Cdu)\, Y_n(Cbu) + Y_n(Cdu)\, J_n(Cbu) \tag{5e}$$

$$g'_n(u) = J_n(Cdu)\, Y'_n(Cbu) + Y_n(Cdu)\, J'_n(Cbu) \tag{5f}$$

$$C^2 = (\sigma_1 \mu_1 / \sigma_2 \mu_2) \tag{6}$$

$$K = (\mu_2 / \mu_1) \tag{7}$$

$$b = {}^{r}/a \tag{8}$$

$$d = r_o/a \tag{9}$$

$$\delta_n = \begin{cases} 1 , & n = 0 \\ 2 , & n \geqslant 1 \end{cases} \tag{10}$$

$$\beta^2 = \sigma_2 \mu_2 a^2 \tag{11}$$

Since we wish to compare our results with the results obtained under quasi-static approximation, a few words about the quasi-static approximation are in order. Under quasi-static approximation, the response functions, except for $h^s_{o\,\phi}(t)$ simplify considerably. $h^s_{nr}(t)$ and $h^s_{n\,\phi}(t)$ in equations (1) and (2) for $n \geqslant 1$ should be replaced by $R_n(t)$ which is defined below. Computation of $h^s_{o\,\phi}(t)$ must be done by equation (4). It has been shown by Singh (1972) that for a non-permeable cylinder (i.e., K = 1)

$$R_n(t) = 4n \sum_{j=1}^{\infty} \frac{e^{-y^2_{n-1,j}\, t/\beta^2}}{y^2_{n-1,j}} \tag{12}$$

where $\pm i y_{n-1,j}$ (j = 1,2,3,. . .) are zeros of $I_{n-1}(z)$ and for a permeable cylinder (i.e., $K \neq 1$), neglecting static part,

$$R_n(t) = 4n^2 K(K-1) \sum_{j=1}^{\infty} \frac{J_n(y_{n,j})\; e^{-y^2_{n,j}\, t/\beta^2}}{y_{n,j}\, J_{n-1}(y_{n,j})\,[n^2(1-K^2) - y^2_{n,j}]} \tag{13}$$

where $\pm i y_{n,j}$ (j = 1,2,3,. . .) are zero of $zI'_n(z) + n\,K\,I_n(z)$. Verma (1973) has rederived equation (12). However, he omits the $h^s_{o\phi}(t)$ term in the total response. Singh (1973a) has shown in the frequency domain that this term must be included for this case of a purely two dimensional problem. Wait (1973) claims that in a real case where both the cylinder and the line source are finite, this term would not be important. His recent work (Wait and Hill, 1973) indicates that there are quantitative differences in the response of finite cylinder and an infinite one (i.e., the two dimensional case) but one can easily show that if the n = 0 term is excluded in the two dimensional case, the difference is much larger. Thus, in as for as two dimensional models are used in interpretation, the n = 0 term must be included to minimize error. The same holds good in the time-domain. It follows that in Verma's (1973) result, obtained under quasi-static approximation, the $h^s_{o\phi}$ (t) term must be added. For the effect of n = 0 term in the plane wave excitation of an infinite cylinder we refer to another paper (Singh, 1973b).

NUMERICAL RESULTS AND DISCUSSION

Equations (3) and (4) do not lend themselves to further simplification except in the limiting cases. Thus, these integrals need to be numerically evaluated. The computations require a very close study of the integrands, since most of the contributions to the integrals come from small regions where the denominators become very small. Details of the numerical computation is very similar to the one discussed in an earlier paper (Singh, 1973c). Computation of R_n (t) is straight-forward.

In the following we present graphs of the response functions $h^s_{nr}(t)$ and $h^s_{n\phi}(t)$ upto n = 2. In all the cases considered, the cylinder is assumed to be non-permeable (K = 1) and $\frac{r}{a} = \frac{r_o}{a} = 2.0$. Figures 2, 3 (a) and 3(b) show plots of $h^s_{o\phi}(t)$, $h^s_{1r}(t)$ and $h^s_{1\phi}(t)$ respectively for various values of σ_1/σ_2. For increasing σ_1/σ_2 the response functions start at a smaller value, rise to a maximum and then decay. Whereas for n = 1 the curves for increasing σ_1/σ_2 have greater value for large $T = t / \sigma_2\mu_2 a^2$, the situation for n = 0, at least upto T = 10, is just the opposite. After T = 0.1, n = 0 term clearly dominates n = 1 term. $h^s_{1r}(t)$ deviates more from the quasi-static case ($\sigma_1/\sigma_2 = 0$) for a finite σ_1/σ_2 than does $h^s_{1\phi}(t)$. Higher multipoles decay faster but otherwise follow the same trend as n = 1. Figures 4a and 4b show plots of $h^s_{2r}(t)$ and $h^s_{2\phi}(t)$ respectively.

Figure 2 giving $h^s_{o\phi}(t)$ for $\sigma_1/\sigma_2 \geqslant 10^{-3}$ clearly shows the importance of the contribution of n = 0 term in the transient response under quasi-static approximation, previously neglected by other authors (e.g. Verma, 1973).

Results also demonstrate the invalidity of linear superposition of solutions of a cylinder and a halfspace to obtain solution of a buried cylinder since if it was true the response functions in our case would be the same as for the quasi-static case (there being no secondary field in an infinite space).

CONCLUSIONS

As expected, the finite outer conductivity changes the transient response of a conducting cylinder. As a rough rule this effect must be considered if $\sigma_1/\sigma_2 \geqslant 10^{-2}$. The term n = 0 must be included no matter what the value of σ_1/σ_2. Thus, the interpretation schemes given by Verma (1973) are meaningless since he neglects this term.

Since the theoretical model considered here is unrealistc, the results are only useful in warning against the blind use of quasi-static approximation. For this same reason, there appears no reason in presenting more extensive numerical results. The paper should however provide a check for more exact models which may be considered in future.

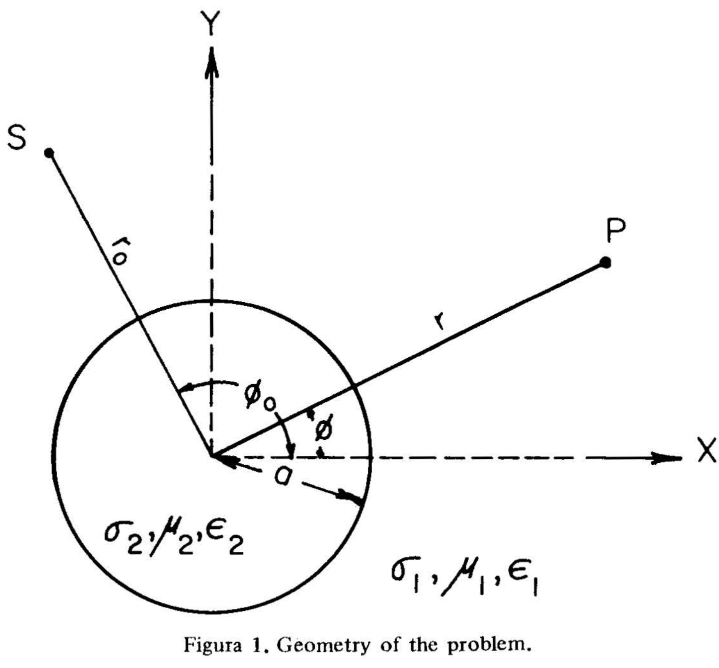

Figura 1. Geometry of the problem.

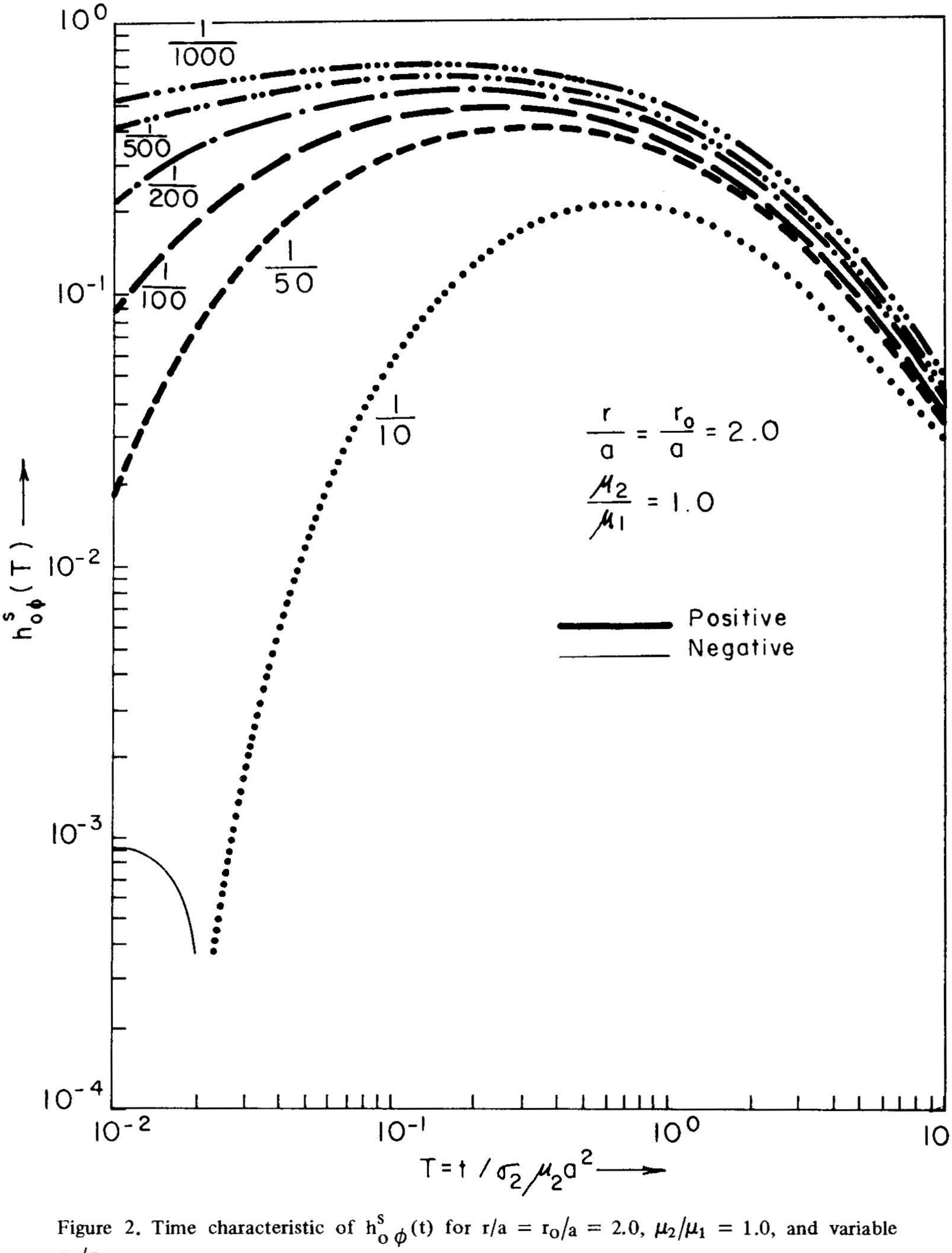

Figure 2. Time characteristic of $h^s_{o\phi}(t)$ for $r/a = r_o/a = 2.0$, $\mu_2/\mu_1 = 1.0$, and variable σ_1/σ_2.

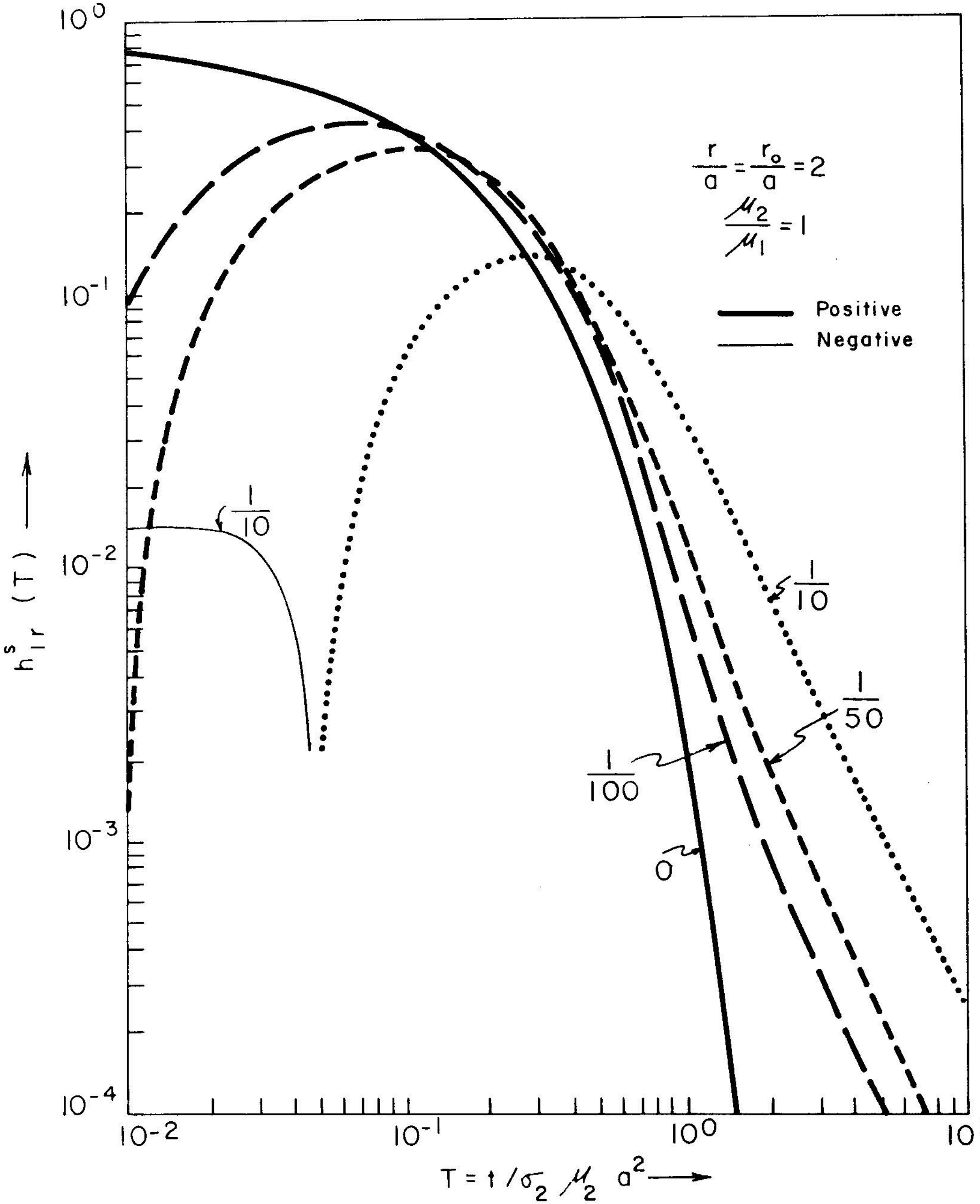

Figure 3a. Time characteristic of $h^s_{1r}(t)$ for $r/a = r_o/a = 2.0$, $\mu_2/\mu_1 = 1.0$, and variable σ_1/σ_2.

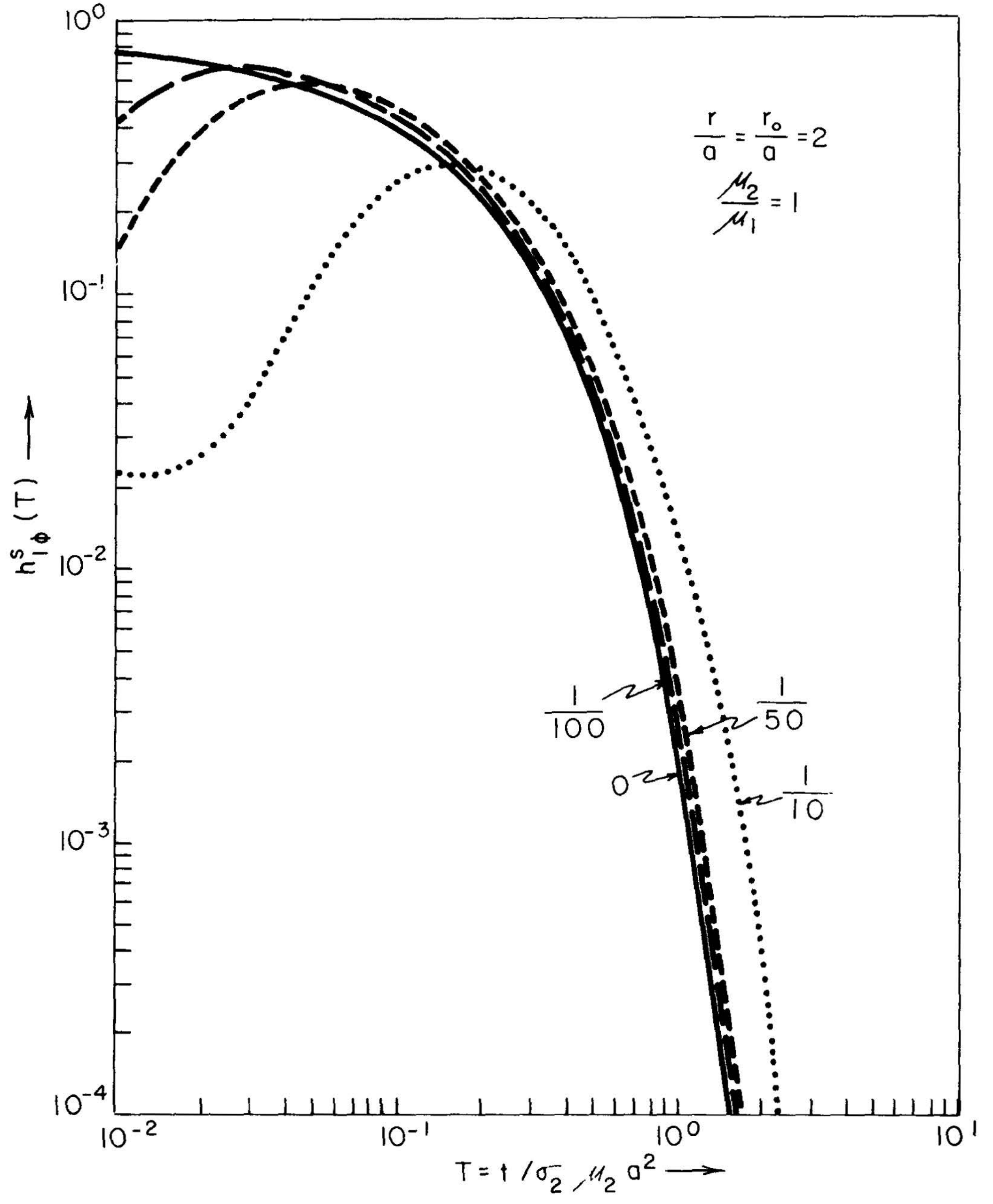

Figure 3b. Time characteristic of $h^s_{1\phi}(t)$ for $r/a = r_0/a = 2.0$, $\mu_2/\mu_1 = 1.0$, and variable σ_1/σ_2.

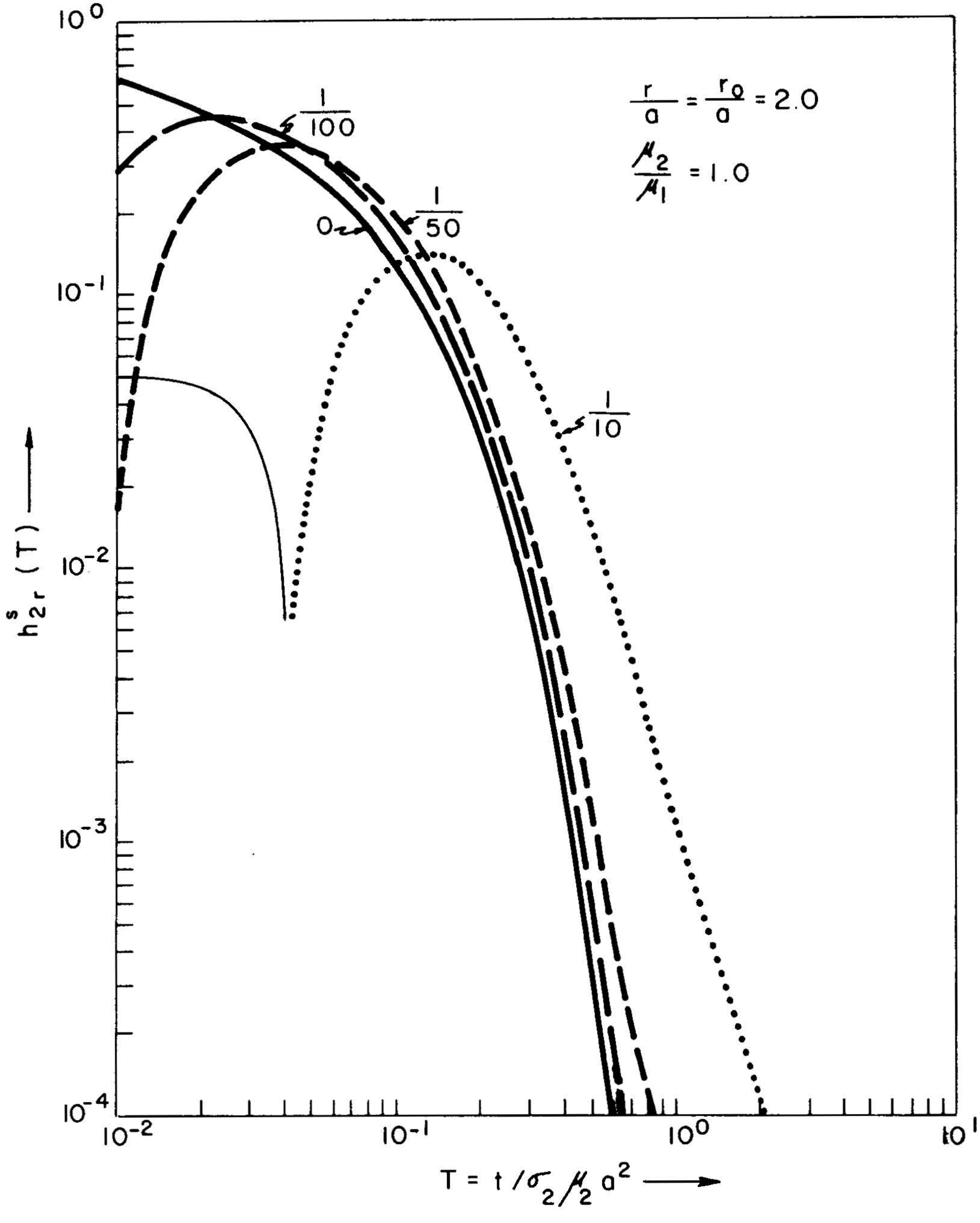

Figure 4a. Time characteristic of $h^S_{2r}(t)$ for $r/a = r_O/a = 2.0$, $\mu_2/\mu_1 = 1.0$, and variable σ_1/σ_2.

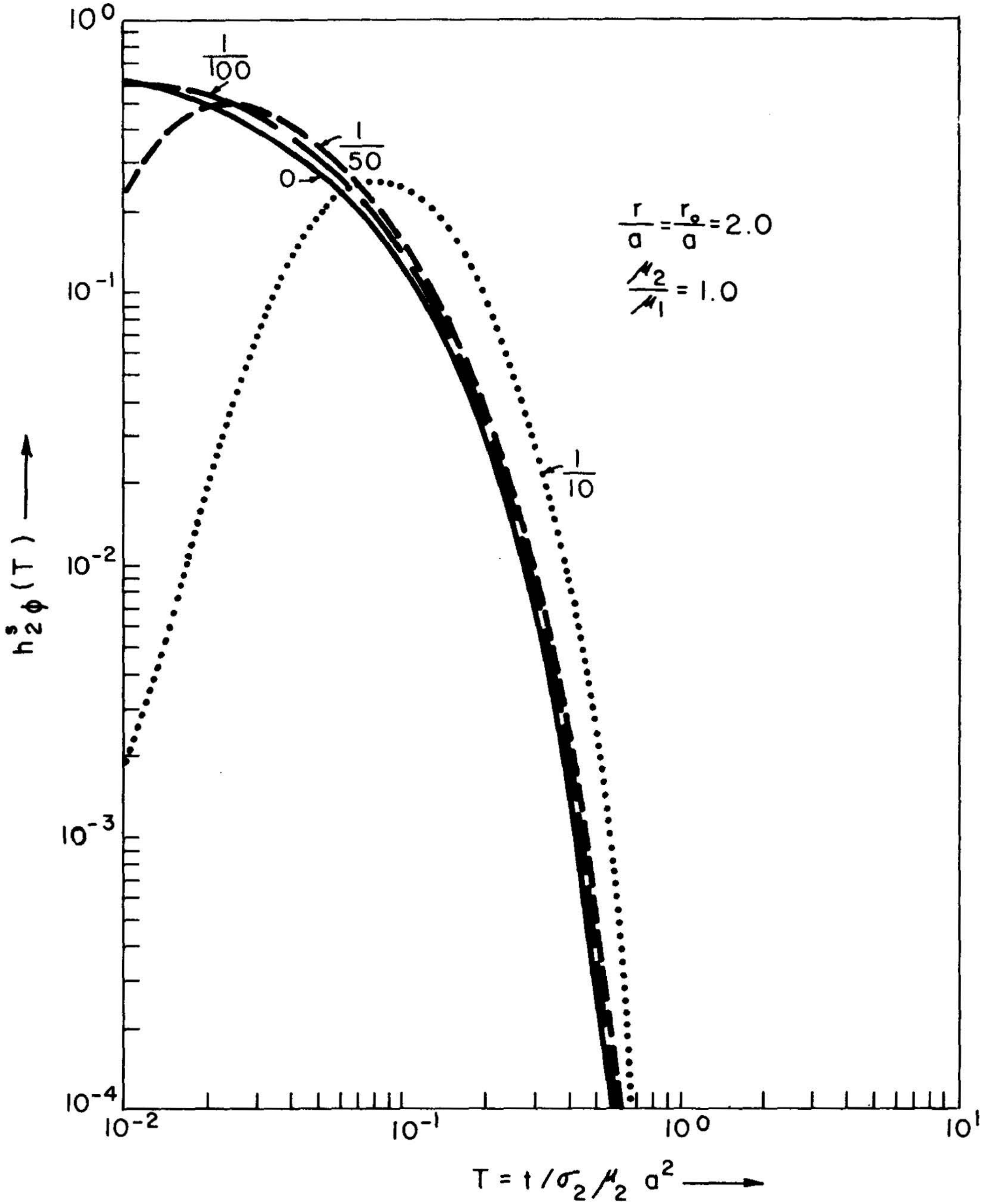

Figure 4b. Time characteristic of $h^s_{2\phi}(t)$ for $r/a = r_o/a = 2.0$, $\mu_2/\mu_1 = 1.0$, and variable σ_1/σ_2.

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