A METHOD FOR EPICENTER DETERMINATION

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RESUMEN

El método propuesto corrige la inhomogeneidad en la distribución azimutal de estaciones, adaptando los residuales a una función coseno del azimut. A los arribos tempranos se les asigna mayor peso que a los arribos tardíos. Los pesos son optimizados experimentalmente mediante el uso de datos de grandes explosiones. En lugar de tabular tiempos de viaje se utilizan aproximaciones polinomiales; por lo tanto, los nuevos programas requieren considerablemente menos tiempo de computación y de almacenamiento de datos. La explosión Longshot del 20 de Octubre de 1965 ha sido relocalizada. El desplazamiento epicentral de Longshot es normal al eje de la trinchera y mayor que el reportado anteriormente. Esto significa que el término de fuente de Longshot no puede ser adecuadamente explicado por diferencias de temperatura entre la placa hundida y el manto que la rodea.

ABSTRACT

The proposed method corrects for inhomogeneity in the azimuthal distribution of stations, by fitting the residuals to a cosine function of the azimuth. Early arrivals are assigned greater weights than late arrivals. The weights are optimized experimentally by using data from large explosions. Instead of tabulated travel times, polynomial approximations are utilized; hence the new programs require considerably less computer time and storage. The Longshot explosion of 20 October 1965 is relocated. The bias-free epicentral shift of Longshot is normal to the axis of the trench, and is larger than previously reported. This means that the Longshot source term cannot be adequately accounted for by temperature differences between the sinking plate and the surrounding mantle.

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1. THEORY

A bias in seismic travel-time tables is due to the assumption that travel-time residuals are normally distributed with zero mean (Lomnitz, 1971). This assumption leads to a systematic mislocation of epicenters, particularly when a majority of recording stations is concentrated in one quadrant.

Most location procedures in current use do not take into account the azimuthal distribution of stations (Jeffreys, 1936; Tucker, Herrin and Freedman, 1968). When travel-time errors are systematically biased towards late arrivals, the errors in locations or in origin times may be fed back into future revisions of the tables. Such errors are difficult to detect, because epicentral solutions are systematically pulled towards, or pushed away from, the center of gravity of recording stations, until the data are forced into agreement with the tables. In regions such as South America and Mexico, this may lead to location errors in excess of the least-squares probable error. Furthermore, often spurious regional residuals tend to be interpreted as anomalies of crustal and subcrustal structure. In the present paper we compute the systematic location shift introduced by travel-time bias, and we propose a method of epicenter determination which tends to eliminate systematic location shifts in the presence of inhomogeneities in the azimuthal distribution of stations.

Let 0 be the true epicenter of a zero-depth seismic event and let 0' be the location estimate obtained by current least-squares methods. Consider the travel-time residual ρ at an arbitrary station A (Fig. 1):

$$\rho = t(\Delta) - t(\Delta') + \epsilon \tag{1}$$

where $t(\Delta)$ is the travel time from 0 to A, and $t(\Delta')$ is the travel time from 0' to A. The travel-time error ϵ was assumed by earlier authors to have a zero mean; even though this assumption is incorrect we shall accept it provisionally in the first section of this paper.

In the triangle A00' (Fig. 1) we have

$$C = \Delta' - \Delta \tag{2}$$

where C is the epicentral shift vector, i.e. the error in the location procedure. Let us find out if this error can be estimated and corrected. Introducing into (1):
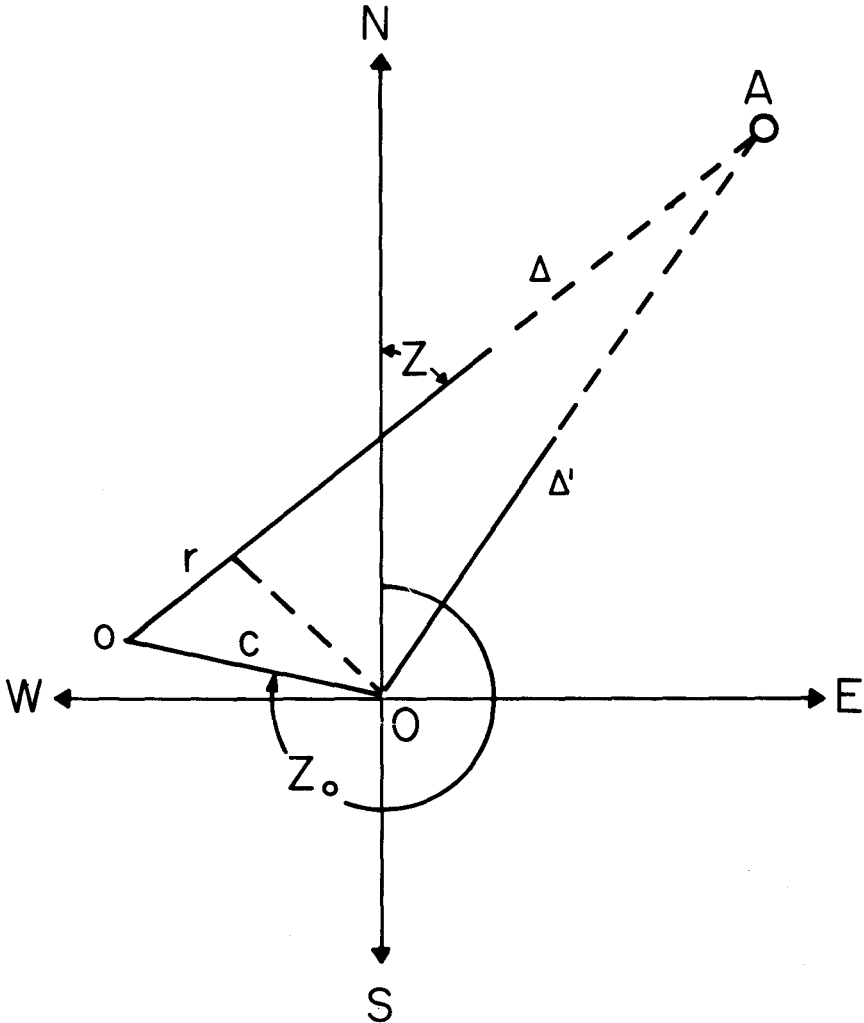
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Fig. 1

$$\rho - \epsilon = t(\Delta' - C) - t(\Delta') \approx -C \frac{dt}{d\Delta} \tag{3}$$

provided that $C \ll \Delta$ and that the travel-time function is continuously differenciable in the neighborhood of Δ. If ϵ has zero mean, the expected value of the residual is, from (3),

$$\hat{\rho} = -C \cos(Z - Z_0) \frac{dt}{d\Delta} \tag{4}$$

which shows that the residual is a function of station azimuth Z.

Clearly, it is possible to estimate the parameters (C, Z_0) of the epicentral shift vector simply by fitting the station residuals to eq (4). In other words, given an approximate location of the epicenter it is possible to use (4) in order to refine the epicentral location. This method is not mathematically distinct from the traditional least-squares method, which does not involve the azimuth Z as a regression variable. Methods which make use of equations similar to (4) have been known as "relative" location methods. They are employed for determining the location of two neighboring earthquakes relative to each other (Gutenberg and Richter, 1937; Wyss and Brune, 1967). They have not been used as a general method of epicenter determination.

The remainder of this article is devoted to an analysis of travel-time errors, and a discussion of how the proposed method of epicenter determination can be used to minimize these errors. First, however, we shall propose a change of variable which will simplify the problem of epicenter determination in general.

Let us define a new kind of residual, to be called the *distance residual*:

$$r = \Delta - \Delta' \tag{5}$$

The distance residual is the difference between the observed and computed epicentral distances to a station. For historical reasons it was found necessary to use the travel-time residual ρ in earlier location procedures; this is awkward since the statistical process of epicenter determination occurs in the space domain. The variable to be minimized should be a distance, not a time. The distance residual r is related to the travel-time residual ρ by

$$r = \rho \, / \, \frac{dt}{d\Delta} \quad , \tag{6}$$

which can be easily derived from Fig. 1. Hence, introducing into (4),

$$\hat{r} = - C \cos (Z - Z_0) \quad , \tag{7}$$

which shows that the mean distance residual $\hat{r}$ is dependent only on azimuth and not on distance. Therefore the regression equation (7) is simpler to use than (4). Once the regression has been performed the problem is solved, since the amplitude of (7) represents a maximum-likelihood estimate of the scalar shift $\overset{.}{C}$, while the phase of (7) represents a maximum-likelihood estimate of the azimuth from 0 to 0'.

Baseline shifts in the regression equation may occur when there is also an error $H' - H \neq 0$ in the origin time:

$$\hat{r} = C \cos (Z - Z_0) - (H - H') / \frac{dt}{d\Delta} \tag{8}$$

Because $dt/d\Delta$ is always positive, the baseline shift can be zeroed by trial and error, thus obtaining a corrected estimate of H by the same process which yields the epicentral location.

2. TRAVEL-TIME BIAS

In the preceding theory we have assumed that the epicentral location $0'$ obtained by ordinary least-square methods does not, in general, coincide with the true epicenter 0. However, Jeffreys (1936) has shown that ordinary least-squares methods do yield the correct epicenter, provided that the travel-time error is normally distributed with zero mean. This is a critical assumption, because "in a least-squares procedure of this kind non-normal errors in the data can lead to serious problems" (Herrin *et al.*, 1968). The difference between our proposed method and the older method appears when the travel-time error has a mean different from zero. In order to prove this result it will be sufficient to show that the epicentral shift C is non-zero for $\hat{\epsilon} \neq 0$.

Over any set of n stations which record a given event, we may define a point C such that

$$\rho_G = \frac{1}{n} \sum_i \rho_i \tag{9}$$

and

$$\left| \frac{dt}{d\Delta} \right|_G = \frac{1}{n} \sum_i \cos (Z_i - Z_G) \frac{dt}{d\Delta_i} \tag{10}$$

We may call this point the "center of gravity of stations". The expected position of G coincides with $0'$ if the stations are distributed homogeneously about the epicenter, i.e. if the joint distribution of (Δ_i, Z_i) has its mean at $\Delta = 0$. Hence the excentricity of G is a measure of the inhomogeneity of the azimuthal distribution of stations.

Now, earlier least-squares methods operated on the sum of the squares of the travel-time residuals:

$$\sum_i \rho_i^2 = \sum_i [C \cos (Z_i - Z_0) \frac{dt}{d\Delta} + \epsilon_i]^2 \tag{11}$$

or, introducing (4) and neglecting all terms in ϵ^2 :

$$\sum_i \rho_i^2 = \sum_i \hat{\rho}_i^2 + 2 \sum_i \hat{\rho}_i \epsilon_i \quad . \tag{12}$$

The minimum of this expression may be obtained by differentiating with respect to $\hat{\rho}_i$ and equating to zero:

$$2 \sum_i \hat{\rho}_i + 2 \sum_i \epsilon_i = 0 \quad , \tag{13}$$

or, introducing (9),

$$\hat{\rho}_G = - \frac{1}{n} \sum_i \epsilon_i = - \hat{\epsilon}_i \tag{14}$$

Hence, if $\hat{\epsilon}_i \neq 0$ as was assumed at the outset, we must have $\hat{\rho}_g \neq 0$ as well. Therefore $C \neq 0$, which completes the proof. We have shown that, if the travel times are biased, the least-squares residual at the center of gravity of stations is different from zero. Unless G happens to coincide with the epicenter there will always be an epicentral shift, i.e. minimizing the sum of squares of station residuals does not generally yield the true location of an epicenter.

Let us now estimate the amount C and direction Z_0 of the epicentral shift. We may write

$$\rho_G = - C \cos (Z_G - Z_0) \mid \frac{dt}{d\Delta} \mid_G \tag{15}$$

Introducing (9) and (10) we obtain

$$\sum_i \rho_i = - C \cos (Z_G - Z_0) \sum_i \cos (Z_i - Z_G) \frac{dt}{d\Delta_i} \tag{16}$$

In order to make this an identity we require

$$Z_G = Z_0 \pm \pi \quad , \tag{17}$$

showing that the epicentral shift occurs in the direction of G, either towards or away from the center of gravity of stations. When most stations are located in the first quadrant, for example, the epicentral shift will be towards, or away from, the first quadrant.

The amount C of the epicentral shift is readily obtained from (14), (15), and (17):

$$C = \hat{e}_i / \frac{dt}{d\Delta}_G \tag{18}$$

Hence, the error in least-squares epicenter determination is proportional to the bias in the travel-time measurements.

3. COMPENSATION OF TRAVEL-TIME BIAS

Freedman (1966) experimentally determined the human error involved in reading first arrivals of seismic signals, and found it to be normally distributed. She obtained this result by having different professional seismogram readers read the same seismogram. In this manner she was unable to determine the bias at the station, since the true arrival time was unknown. This bias is not due to human error but to the presence of background noise on the record, as follows:

a. In the presence of noise the initial part of the signal is more likely to be missed; therefore, a *low* signal-noise ratio is more likely associated with *late* readings;

b. In the absence of noise the identification of the initial pulse of the signal can be normally performed; hence, a *high* signal-noise ratio is associated with *accurate* (not early) readings (Fig. 2).

Since the signal-noise ratio is always positive and finite it follows that travel-time readings tend to be consistently late. This is illustrated in Fig. 2 *b* and *c* where the first arrival is obscured by noise and some later, stronger, arrival may be picked as first arrival, i.e. $\hat{e}_i > 0$ in general. This positive bias contaminates the travel-time residuals, and may introduce considerable systematic error in the determination of epicentral locations.

There are no general statistical estimation procedures for variables affected by systematic errors. We must first transform the biased distribution of residuals into an unbiased zero mean destribution, by assigning a weight to each residual as a function of its signal-noise ratio. This is achieved as follows: Let K be a function of the signal-noise ratio, presently to be determined. We assign a weight W to every residual, such that

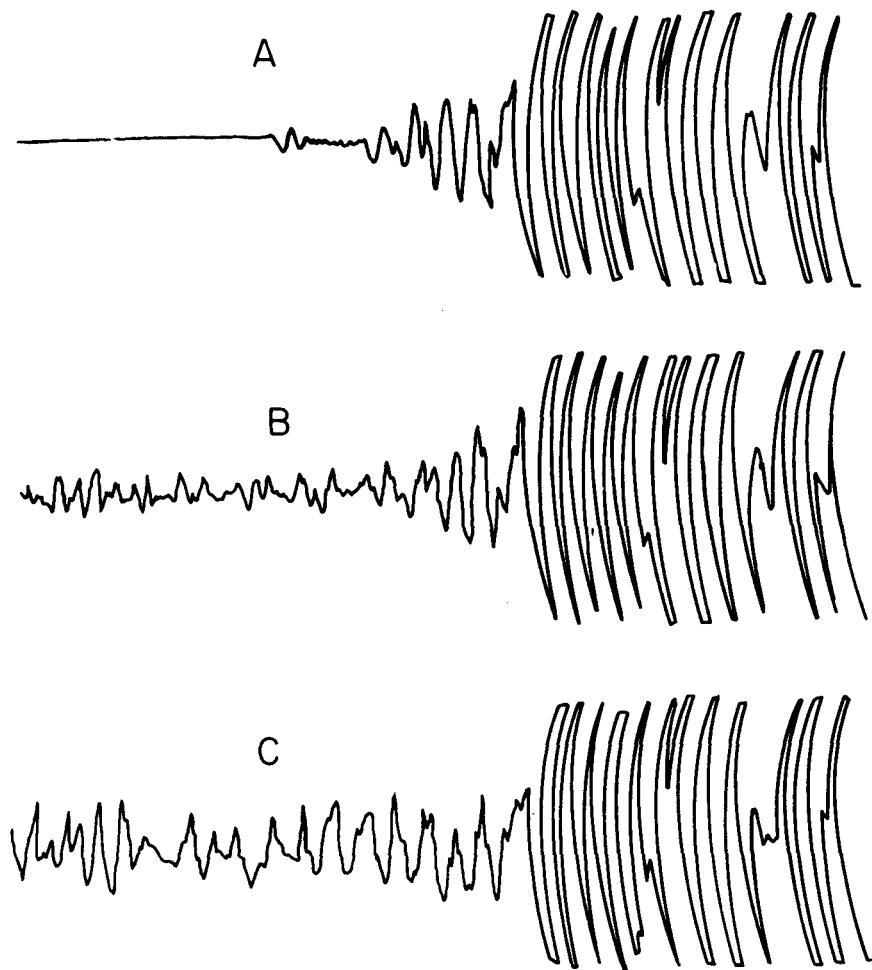$$W = \text{Integer } [K (r_{90} - \hat{r})] \quad , \tag{19}$$

Fig. 2

where r_{90} is the upper 90 % confidence bound of the distance residual r. This procedure gives an increasingly large weight to *early* arrivals. In order to eliminate coarse errors we truncate the data, by discarding all residuals which fall outside the $|r_{90}, r_{-90}|$ range.

For a given noise level the signal-noise ratio depends on the amplitude of the signal. Since the noise level at the station is independent of the signal, and is not generally reported, we assume that K is a pure amplitude function:

$$K = (3.07115 - 0.003141 \, e^M) \, F(\Delta) \qquad\qquad (20)$$

where M is the magnitude of the seismic event and Δ is the epicentral distance. The constants have been obtained from the definition of the Richter magnitude, and the function $F(\Delta)$ was taken from an empirical curve by Nuttli (1972). For a given magnitude, the weighting function K is shaped as in Fig. 3 and is taken to represent the expected variation of signal-noise ratio with distance.
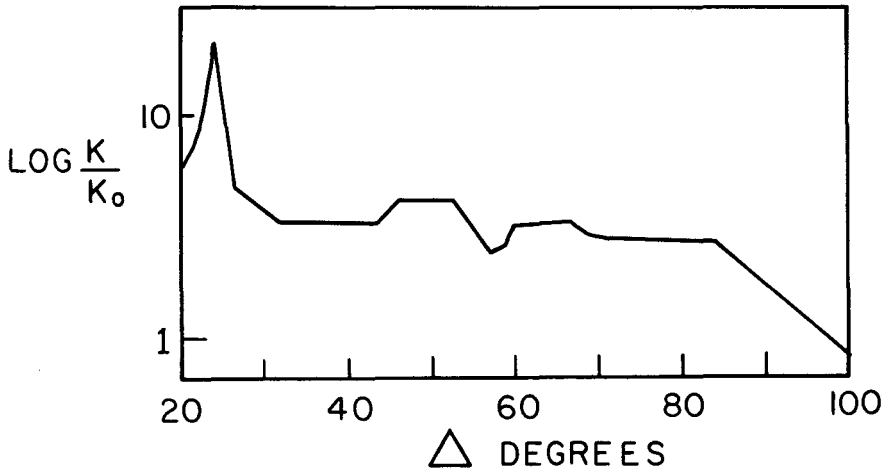

Fig. 3

4. CALIBRATING THE EPICENTER PROGRAM

The following steps of epicenter determination are proposed:

1. Compute the array of distance residuals for a trial epicenter.

2. Fit a cosine curve $\hat{f}(Z)$ by least squares to the set of points (r_i, Z_i). Find the corrections C, Z_0 and $(H' - H)$.

3. Compute the signal-noise function (21) and assign weights Wi to all residuals.

4. Recompute the cosine curve $\hat{f}(Z)$ by least squares, using weighted residuals this time. Find the new corrections C, Z_0 and $(H' - H)$.

5. Go to 1 and repeat until convergence.

Parabolic travel-time functions are used throughout, instead of tables stored in memory. These functions are selected to stay well within ± 0.5 seg, which is less than the uncertainty caused by crustal and deep-seated inhomogeneities. The basic zero-depth P function selected was the Lomnitz-Vargas (1970) approximation

$$t(\Delta) = 67.35 + 11.14 \Delta - 0.0356 \Delta^2 \tag{21}$$

which fits the 1968 travel-time in the range 30° - 80°. Small systematic errors in such approximations do not seriously affect the epicentral shift, as they would in the methods which neglect the azimuthal term in the least-squares determination. Considerable economy in computer storage and execution time can be achieved in this way.

The signal-noise function $K(M, \Delta)$ contains a constant (in the amplitude term F) which determines the steepness of the weighting function of station residuals. If the weighting function is too steep it eliminates all but the earliest arrivals; if it is too flat it assigns nearly equal weights to early and late arrivals, irrespective of magnitude or distance. Hence it is necessary to optimize the weighting function by calibration.

This was done by recomputing the epicentral location of two explosions: SALMON and CHASE V. The epicentral error was plotted against the mean weights and a well-defined value of the mean weights which minimizes the epicentral error was found.

SALMON was an explosion in Mississippi, and CHASE V was an underwater shot off the coast of Northern California. Apparently, neither location was affected by major near-source inhomogeneities. The effect of near-station inhomogeneities cannot be removed by any simple method of location. Explosions in Nevada or in the Aleutian Arc have shown strong azimuthal effects and hence were not used for purposes of calibration. The results of the calibration experiment are given in Table 1.

The error for SALMON was fairly large although it was smaller than the error of ordinary least-squares location methods (Fig. 4). This error is attributed to the fact that SALMON is located almost exactly on the zero-line for local travel-time anomalies within the United States (Herrin and Taggart, 1968) so that the azimuthal effects of these anomalies are at a maximum. This is not the case for CHASE V because U. S. stations are all in the same quadrant (Fig. 5). The error for CHASE V was within the range of uncertainty of the location of the shot (Lomnitz and Bolt, 1967).

Table 1 shows the location improvement obtained by the new epicenter program as compared to published results. In the case of SALMON the error was reduced by 52 % and in the case of CHASE V the error was reduced by 60 %.

TABLE 1

Locations for intra-plate events

Explosion	True location	Earlier estimate	Our estimate	Our error
SALMON				
latitude	31.14	30.52 [1]	31.31	27 km
longitude	−89.57	−89.13	−89.39	
origin				
time	16:00:00.0	15:59:56.3	16:00:02.3	+2.3 sec
CHASE V				
latitude	39.47	39.5 [2]	39.56	13 km
longitude	−125.80	−125.5	−125.72	
origin				
time	05:49:06.8	05:49:06.3	05:49:09.5	+2.7 sec

[1] Location from Herrin *et al.* (1968)

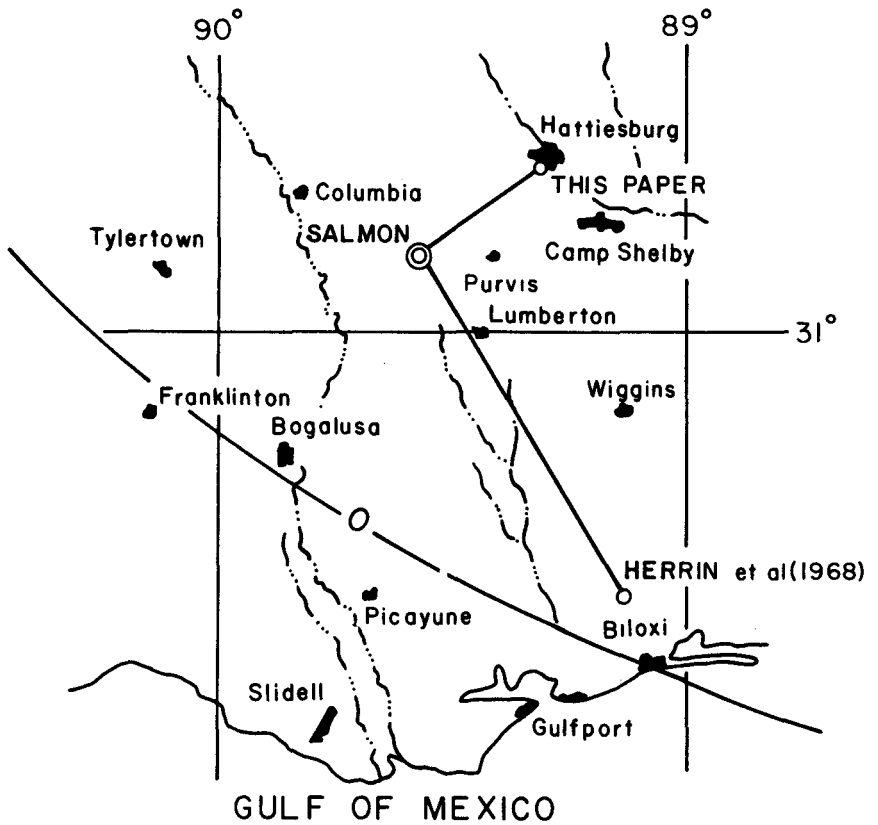[2] Location from USCGS, quoted in Lomnitz and Bolt (1967).

90°   89°

Hattiesburg
●Columbia   THIS PAPER
SALMON   ●
Tylertown   Camp Shelby
Purvis
Lumberton   31°
Franklinton
Wiggins
Bogalusa

O

HERRIN et al(1968)
Picayune   Biloxi
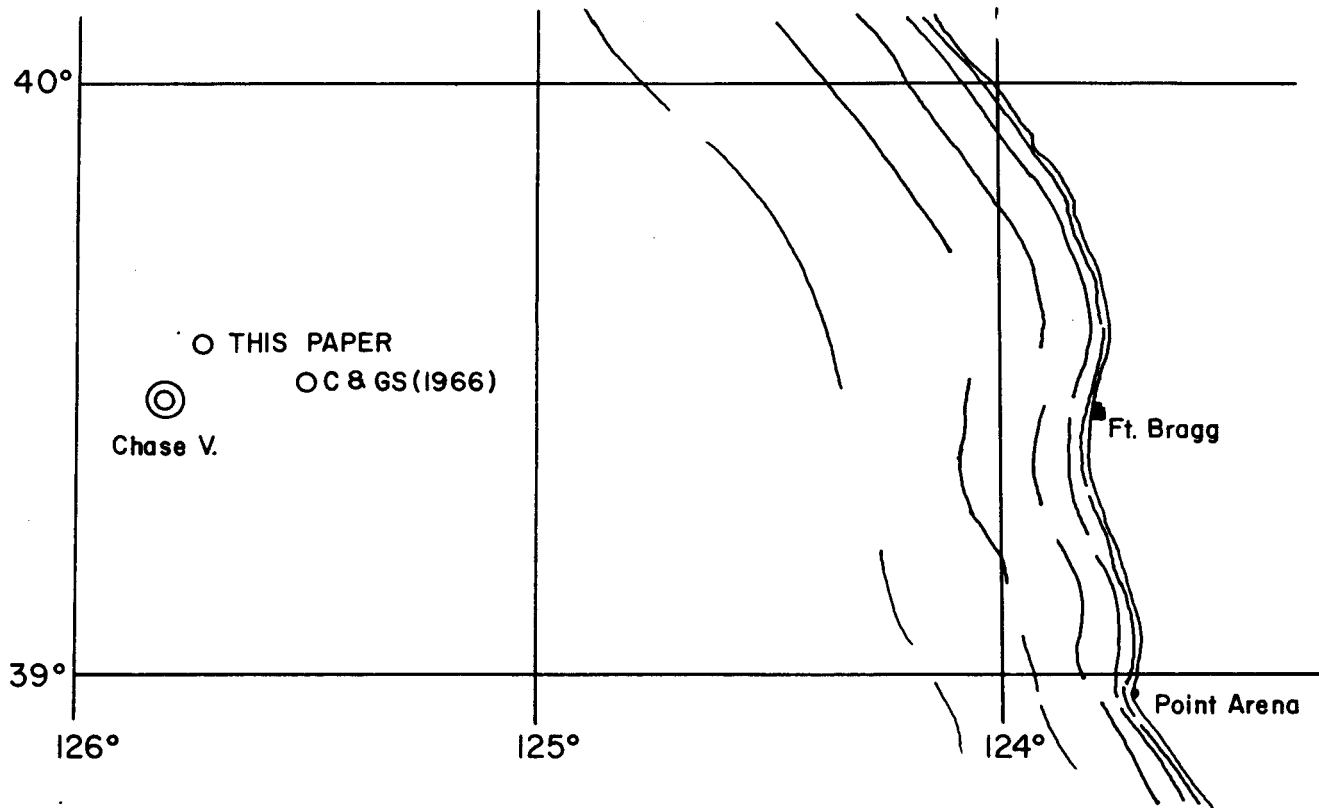
Slidell
Gulfport

GULF OF MEXICO

Fig. 4

Fig. 5

5. THE LONGSHOT SOURCE TERM

Longshot was an 80-kt nuclear explosion on Amchitka Island. Travel-time resid-
uals from this event show a severe negative bias in a general northerly direction,
which has been discussed under a variety of tectonic interpretations (Clark, 1955;
Lambert *et al.,* 1969; Carder *et al.,* 1967; Chinnery and Toksoz, 1967; Cleary,
1967; Kanamori, 1968; Douglas and Lilwal, 1968; Davies and McKenzie, 1969;
Lomnitz, 1971; Espinosa, 1971; Abe, 1972). Blind determinations of the epicen-
ter yield errors of 20-50 km. Table 2 summarizes the results obtained with the pre-
sent program, as compared to a published least-squares solution utilizing station
corrections (Herrin *et al.,* 1968).

TABLE 2

Relocation of LONG SHOT

	Latitude N	Longitude E	Origin Time
True location	51.44	179.18	21:00:00.1
Herrin *et al.* (1968)	51.67	179.15	20:59:59.3
epicentral shift	0.23	0.03	0.8
Our location:			
unweighted residuals	51.62	179.06	20:59:59.1
weighted residuals	51.73	179.19	20:59:59.5
epicentral shift	0.29	0.01	-0.6

The source anomaly points almost due north, normal to the strike of the Aleu-
tian Trench. This result differs from the estimated azimuth of N 15° E found by
Davies and McKenzie. The epicentral shift obtained by the present method is larger
(33 km) than that found by Herrin *et al.* (1968) (26 km). The difference is prob-
ably due to the fact that station corrections used by Herrin *et al.,* were contami-
nated by crust and mantle inhomogeneities at large distances (Lomnitz, 1971).
Jacob (1972) has observed that "the zones of large negative source terms (for Long-
shot) incidentally coincide often with regions of large negative station residuals".
This correlation suggests statistical contamination by bundles of fast seismic rays
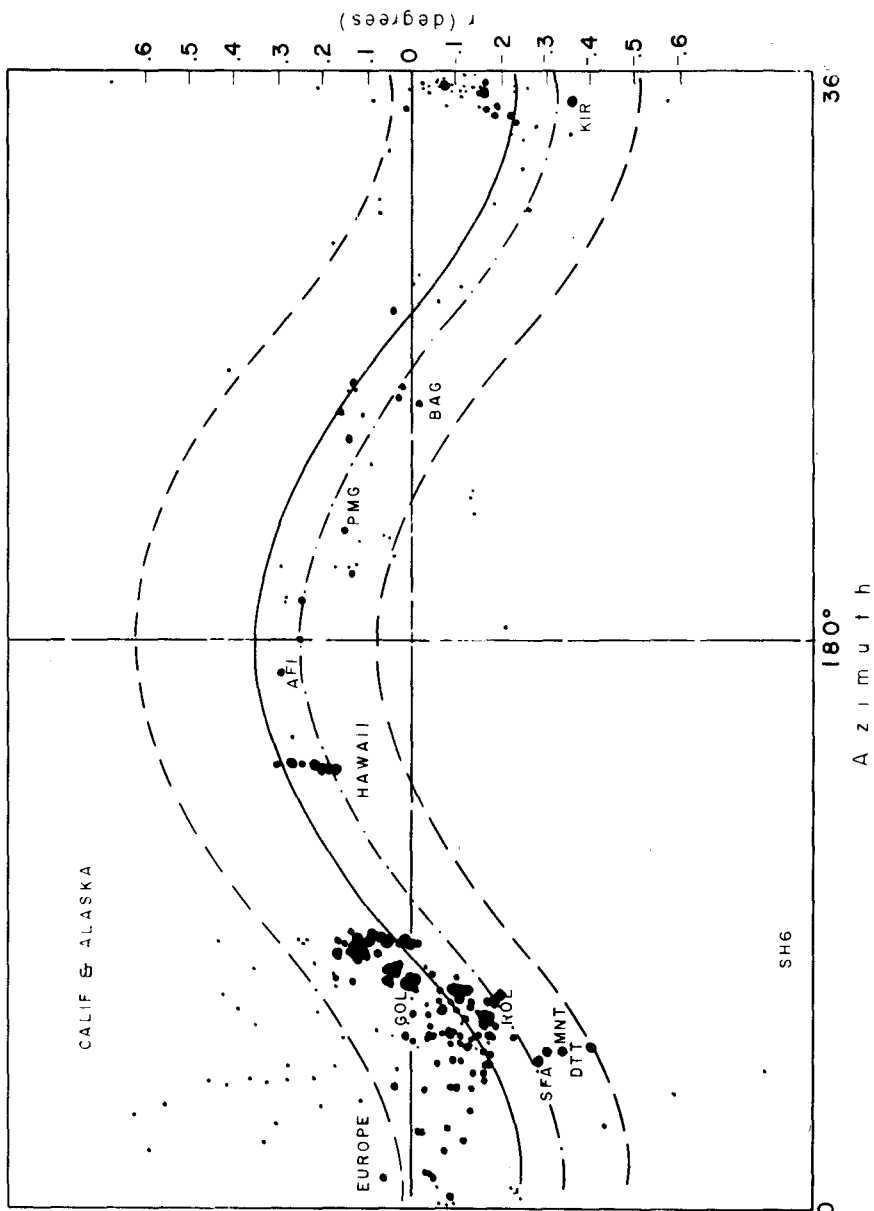
Fig. 6

guided by subduction zones which point away from the center of the Pacific Ocean. In other words, the use of station corrections tends to smooth out the true Long-shot source anomaly, which causes negative residuals in the same general region of negative station corrections. Another possible cause is the combination of true fast corrections beneath Eastern U. S. stations with slow corrections beneath Western U.S. stations.

A detailed analysis of the Longshot source term will be carried out elsewhere. Considering a descending plate with a mean length of 250-300 km, the required ve-locity contrast with the surrounding mantle is unlikely to be caused by temperature differences alone, since this requires that the slab should be about 1000° C colder than the mantle (Davis and McKenzie, 1969; Jacob, 1972). The present data tend to reinforce this conclusion, because a velocity contrast of 7-10 % as proposed by Jacob is insufficient to explain the true source term as obtained by our method. If the velocity contrast is as high as 15-20 % , as our results suggest, a major part of the Longshot source anomaly must be accounted for by mechanisms other than temperature differences between the sinking plate and the normal mantle.
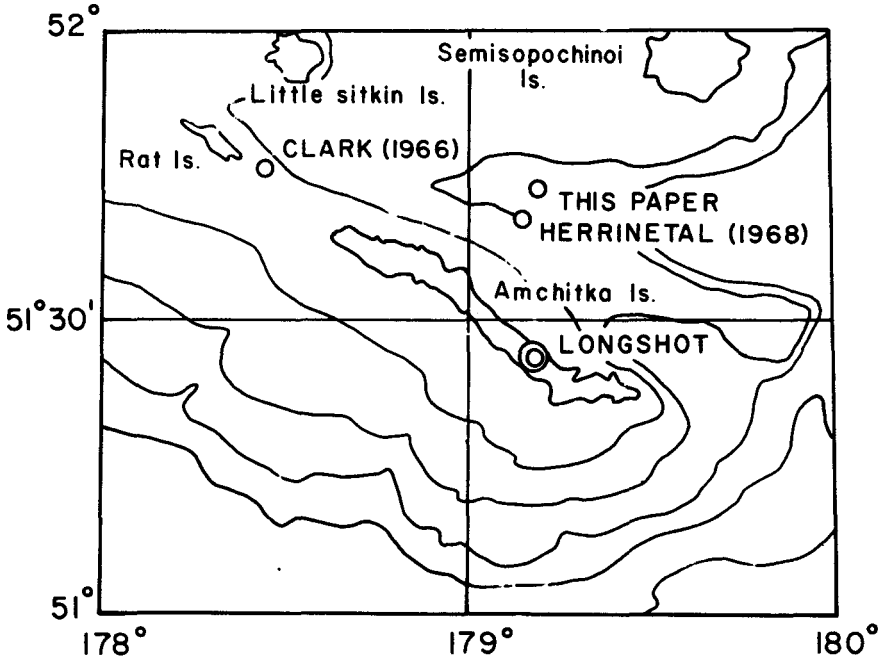

Fig. 7

Figure 6 shows the distribution of station residuals for Longshot. The solution obtained by the present method corrects simultaneously for azimuthal bias and for station terms, because early arrivals irrespective of azimuth are given greater weight than late arrivals. The superiority of this location method depends on the existence of at least one or two *early arrivals* in each quadrant; otherwise, no effective improvement over older methods is obtained. It is recognized that observations in regions of anomalous high mantle velocity may be assigned excessive weight.

In the Longshot case, our solution is intermediate between least-squares solutions with and without station corrections. This appears to be due to the peculiar location of Longshot, where the two northern quadrants contain most of the negative anomalies caused by Pacific subduction zones. A solution which makes no use of station corrections will overshoot the unbiased apparent location, whereas a solution with station corrections will over-correct by absorbing a part of the anomalies into the corrections (Fig. 7).

In order to make consistent interpretations of travel-time anomalies at plate boundaries it is essential to separate the errors due to unrealistic assumptions made about travel times from the effects of true anisotropy in the earth. If the mean values of $dt/d\Delta$ in the earth is about 5 sec/degree and if the precision of travel-time measurements is ±0.1 second, the mean error in distance estimates ought not to exceed ± 2.2 km. The extent to which such precise estimates have not been attained in teleseismic locations is a measure of the degree of possible improvement in location procedures, especially for intra-plate events.

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