SEISMOLOGICAL EVIDENCE FOR A DISCONTINUITY IN SUBDUCTION ZONES

CINNA LOMNITZ*

RESUMEN

La magnitud local media $\bar{M}$ es independiente del número anual de temblores, y puede servir para detectar cambios en los esfuerzos tectónicos regionales. Se examinan tres parámetros de estimación del tamaño medio de los temblores: la magnitud media $\bar{M}$, el factor b (proporcional al recíproco de $\bar{M}$), y el momento local medio $\bar{M}_0$. Este último es el único parámetro no sesgado. Al analizar la variación de b en ciertas zonas de subducción se detectan cambios sistemáticos con la profundidad. La evidencia sugiere que las placas descendentes no tienen continuidad hasta profundidades de 600 km, con la excepción de la zona de subducción de Tonga-Kermadec que no demuestra mayores variaciones de b con la profundidad.

* *Instituto de Geofísica, UNAM.*

ABSTRACT

The mean local magnitude $\overline{M}$ is independent of the rate of earthquake ocurrence, and can be used to monitor the changes of stress in a region. Three estimators of earthquake size are discussed: the mean local magnitude $\overline{M}$, the b-factor (related to the reciprocal of $\overline{M}$), and the mean local moment $\overline{M}_0$. Only $\overline{M}_0$ is an unbiased estimator of earthquake size. An analysis of the variation of b in some subduction zones discloses systematic changes with depth. The evidence suggests that downgoing slabs in subduction zones are not continuous down to depths of 600 km, excepting the Tonga-Kermadec subduction zone which exhibits no significant variation of b-values with depth.

INTRODUCTION

The realization of a point process $X(x_i)$ in three dimensional space x_i ($i = 1, 2, 3$) may be called a *space series*. Some statistical properties of geological space series were discussed by Mathéron (1970). In many geological applications the variable X is a random function of some underlying continuous process*. The process $X(x_i)$ may be described by its local mean:

$$\overline{X}(x_i) = \mathcal{E}[X(x_i)] \tag{1}$$

and by its local covariance:

$$C(\Delta x_i) = \mathcal{E}[X(x_i)\, X(x_i + \Delta x_i)] - \overline{X}(x_i)\,\overline{X}(x_i + \Delta x_i) \tag{2}$$

where (Δx_i) is a variable radius vector about (x_i).

The earthquake process is a point process which takes place both in the space and time domains. The energy E of an earthquake is a random function of the x_i as well as of t. This situation is common in physics, where transitions between the space domain and the time domain are governed by basic transformation laws. In the case of the

* For example, X might be the ore content of a sample extracted from a mineral deposit.

earthquake process, however, no basic relation between the space structure and the time structure has been proposed. Hence the global description of the process has encountered seemingly insurmountable difficulties.

In this paper we propose to unfold the earthquake process into two mutually imbedded processes:

(A) the *Process of Event Recurrence* N (E, x_i, t); where N is the (cumulative) number of earthquakes which occur in the state space defined by energy E, spatial coordinates (x_i), and time t;

(B) the *Process of Energy Partition* E (x_i, σ). where E is the energy of earthquakes in the state space defined by the spatial coordinates (x_i) and the tectonic stress σ.

This choice of processes will be justified later on. Process A is seen primarily as a time series which results from a random sampling of Process B. An analogy may be drawn from mining, e.g. from a sequence of gold discoveries. In this case the process can be unfolded into an underlying space series (the gold fields), and the time series of random strikes. It is clear that the probability of occurrence of an event of size E depends on the size, location, and time of occurrence of all earlier events; but it also depends, in a very real physical sense, on the actual distribution of $\bar{E}$ (x_i) in the earth. The latter is independent of the random sampling process, except insofar as "reserves" are depleted after each event.

THE PROCESS OF ENERGY PARTITION

Consider the random process E (x_i, σ), of earthquake energies as a function of their location and of the tectonic stress. Clearly, the energy also depends on other variables, such as the fault area and the efficiency of seismic wave generation; but these variables are part of the local tectonic setting and may be englobed in the general dependence of energy on location x_i.

Among all possible measures of earthquake size the energy is physically the most natural and plausible, but technically the least convenient as it is not readily accessible to direct estimation from seismic records. Instead, the magnitude M is universally used as a measure of earthquake size. Since the relationship between magnitude and energy is a complex subject which cannot be discussed here, it will merely be assumed that there is a one-to-one correspondence between a given magnitude M and a given energy E. Thus, the continuity of tectonic stresses σ over the earth implies that a mean local magnitude $\overline{M}$ (x_i, σ) must exist everywhere on earth at some suitable scale.

It is also true, of course, that earthquakes are only generated on faults, and that faults (though common to all geologic environments) are discrete features. This indicates that the mean magnitude cannot be treated as a strictly continuous variable in the same sense as the tectonic stress can. However, the discontinuous microstructure of earthquakes in space may be statistically evaluated and taken into account, as has been done for the so-called "nugget effect" discussed by Mathéron (1970).

Ishimoto and Iida (1939), Gutenberg and Richter (1954) and many later authors have found that the observed number of earthquakes in a region obeys an empirical formula, called "magnitude frequency relation":

$$\log_{10} N(M) = a - bM, \tag{3}$$

where N is the *cumulative* number of earthquakes which exceed magnitude M.

An interpretation of the parameters a and b has been provided (Epstein and Lomnitz, 1966; Lomnitz, 1966a). In eq. (3), if we put $M = 0$ we find

$$\log_{10} N(0) = a, \tag{4}$$

which indicates that the parameter a measures the logarithm of the number of earthquakes of magnitude greater than zero which are expected to occur in the region during the sampling period.

We may normalize equation (3). Noting that 1-N/N(0) is the cumulative distribution function F(M) as commonly defined in statistics, we find

$$\log_{10} [N/N(0)] = -bM \qquad (5)$$

$$F(M) = 1 - e^{-\beta M}, \quad M \geqslant 0 \qquad (6)$$

where $\beta = b \ln 10$ (i.e. about 2.3 b). The derivative of the cumulative distribution function (6) yields

$$f(M) = \beta\, e^{-\beta M}, \quad M \geqslant 0 \qquad (7)$$

where $f(M) = dF/dM$ is the *frequency distribution* (or probability density function) of the magnitude M.

The average magnitude may be estimated directly as the mean of its frequency distribution:

$$\overline{M} \doteq \int_0^\infty M f(M)\, dM = 1/\beta \qquad (8)$$

This result affords an immediate interpretation of β as the reciprocal of the mean magnitude for the region.

Example: In California, Epstein and Lomnitz (1966) found the following values of the parameters (on a yearly basis):

$$a = 11.43/2.3 = 5;$$
$$b = 2.0/2.3 = 0.87$$

This is interpreted as follows: the estimated yearly number of earthquakes ($M \geqslant 0$) in California is $10^a = 100{,}000$; and the expected mean magnitude is $\overline{M} = 2.3/b = 0.5$.

If, instead of considering a sample of earthquakes of magnitude $M \geqslant 0$, one wished to introduce an arbitrary lower threshold M_{min} the frequency distribution (7) would become

$$f(M) = \beta \exp[-\beta(M - M_{min})], \quad M \geqslant M_{min} \qquad (9)$$

and the mean magnitude:

$$\overline{M} = M_{min} + \beta^{-1}, \qquad (10)$$

while the estimated total number of earthquakes in the sample becomes

$$\log_{10} N(M_{min}) = a - bM_{min}$$

Thus, in the above-quoted example, if one sets $M_{min} = 4.0$ one obtains $\overline{M} = 4.5$ and $N(4.0) = 31$; i.e., the expected yearly number of shocks ($M \geqslant 4.0$) in California is 31, and their expected mean magnitude is 4.5. These estimates are in excellent agreement with observations for the period 1932-1964.

The normalization of the magnitude-frequency relation is an essential step towards interpreting its parameters a and b in terms of the sample size N and the mean magnitude $\overline{M}$. We have shown that $a = \log_{10} N(0)$ is a normalizing factor, which drops out of the magnitude-frequency relation.

What is the dependence between the mean magnitude and the number of earthquakes per unit time? This question is now seen to be equivalent to asking about the dependence between the parameters a and b. Several authors (Gutenberg and Richter, 1954; Miyamura, 1964; Isacks and Oliver, 1964; Duda, 1964; Karnik, 1965; Evernden, 1970) have suggested that the b-factor is a constant for each region.

Since the rate of earthquake activity fluctuates in time the constancy of the b-factor would imply *independence* between the parameters a and b. This assumption has been tested for aftershock sequences, where the rate of activity decays rapidly with time. It was found that the mean magnitude $\overline{M}$ remained indeed constant during each of the California sequences tested (Lomnitz, 1966b). In statistical terms, the probability of obtaining an aftershock of magnitude M is the same at any time, even though the rate dN/dt of aftershock activity changes rapidly:

$$\varphi(M,N) \approx f(M)\, g(N) \tag{12}$$

which indicates that the joint distribution of aftershock magnitudes and earthquake incidence is the product of the marginal distributions of M and N.

If M and N are independent, so are a and b. This result was tested and confirmed for Chile (Lomnitz, 1960), Japan (Hamada and Hagiwara, 1967), and New Zealand (Hamilton, 1966). Presumably, the independence of $\overline{M}$ and N means that the depletion of regional stress by aftershocks is negligible; otherwise the mean tectonic stress $\bar{\sigma}$ should decay significantly, producing a noticeable decrease in the mean magnitude during the sequence. This result may seem surprising at first sight, because the strain release of aftershock sequences is not negligible in terms of the strain release of the main shock. However, the total *energy* of the aftershock sequence rarely exceeds 10% of the energy of the main shock. It has also been suggested on thermodynamical considerations that the stress drop due to the main earthquake should be expected to induce a compensating flow of strain energy towards the aftershock region; this influx of new energy might be sufficient to sustain the regional tectonic stress at a quasi-stationary level (Lomnitz, 1961).

On the other hand, the *frequency* of aftershocks (not their mean magnitude) does depend on the time of occurrence of previous shocks. A detailed analysis suggests that the probability of occurrence is strongly increased by aftershocks whose magnitude is large in comparison with the mean magnitude of the events being tested (Lomnitz and Hax, 1966).

In conclusion, the choice of the earthquake energy space series as the underlying process appears justified on the following grounds:

(a) Earlier work on the magnitude-frequency relation indicates that the local mean magnitude $\overline{M}$ tends to be constant.

(b) Statistical tests on aftershock sequences show that the local mean magnitude $\overline{M}$ is quasi-stationary during a sequence.

(c) On the other hand, studies on the structure of aftershock sequences indicate that the rate of occurrence dN/dt depends on the magnitude and time of occurrence of previous shocks.

(d) Laboratory studies in rocks have suggested that fluctuations in b may be attributed to changes in stress (Scholz, 1968).

These observations do not, however, justify the following assumptions, which are sometimes implicitly made:

(1) that $\overline{M}$ is stationary in space. The only assumption about $\overline{M}$ that can safely be made is that $\overline{M}$ is an identically distributed random variable in space. This, in itself, is a consequential assumption: thus, if f(E) is lognormal the worldwide mean energy may be estimated by the mean of all local mean energies.

(2) that the maximum likelihood estimate of $\beta = 1/\overline{M}$ is unbiased. This is true only when $M(x_i)$ is normally distributed in space. If $M(x_i)$ is arbitrarily distributed we may approximate the mean in the region of (x_o) by a polynomial

$$\overline{M}\ (x_i) = \sum_{n=0}^{k} p_n \quad f^n\ (x_i) \tag{13}$$

where f(x) is some simple algebraic function. Then the optimum estimator of $\overline{M}(x_o)$ is given by the weighted mean over neighboring data points:

$$\mathcal{E}\left[\overline{M}\ (x_0)\right] = \sum_j \omega_j\ M(x_j) \tag{14}$$

where the weights ω_j are estimated from an array of equations of condition:

$$\begin{aligned} &\sum \omega_j\ c_{ij} = \mu_n \quad f^n\ (x_i) \\ &\sum \omega_i \quad f^n\ (x_i) = \quad f^n (x_0) \end{aligned} \tag{15}$$

where c_{ij} is the covariance and the μ_n are parameters of Lagrange (Mathéron, 1970).

(3) Furthermore, it is not justified to assume that the mean local magnitude corresponds to the earthquake with the mean local energy $\overline{E}$ in the region.

Suppose that we attempt to estimate $\overline{E}\ (x_i)$ by adding the magnitudes in the region, dividing by the number of shocks, and using the average thus obtained to compute $\overline{E}$ through some formula such as

$$\log_{10} \overline{E} = 1.5\,\overline{M} + 11.4 \tag{16}$$

or its equivalent. Note that all these formulas make M proportional to the logarithm of E. But the mean of the logarithm of a positive variable x is always smaller than the logarithm of the mean:

$$\frac{1}{n}\,(\log x_1 + \log x_2 + \ldots + \log x_n) < \log[(x_1 + x_2 + \ldots + x_n)/n]. \tag{17}$$

This may be proved by taking antilogarithms of both sides:

$$(x_1 x_2 \ldots x_n)^{1/n} < (x_1 + x_2 + \ldots + x_n)/n\ , \tag{18}$$

leading to the well-known result that the geometric mean of a positive random variable is smaller than the arithmetic mean. Therefore the mean magnitude $\overline{M}$ is a biased estimator of $\overline{E}$, which it systematically underestimates.

MEAN LOCAL MOMENT AND STRESS DROP

From the experiments by Scholz (1968) we know that raising the stress will lower the b-value of microfractures in a rock, and vice versa. Thus, in a laboratory situation it is possible to simulate a dependence between mean magnitude and stress. In the following we attempt to show that a similar dependence is theoretically to be expected in the case of earthquakes.

Hanks and Wyss (1972) have shown that the model of the seismic source proposed by Brune (1970) gives excellent agreement with field observations. This model consists in a plane rupture of area A in a prestressed elastic medium The initial shear stress over A is σ_1 and the final stress after rupture is σ_2. The stress drop is

$$\Delta\sigma = \sigma_1 - \sigma_2 = k\mu\ D/r\ , \tag{19}$$

where D is the average displacement, μ is the rigidity, r is the average radius of the area of rupture, and k is a correction for fault geometry. The stress drop can also be expressed in terms of the seismic moment M_0 :

$$\Delta\sigma = k\, M_0/r^3. \tag{20}$$

Finally, the seismic moment is related to the seismic energy E, and to the average stress $\bar{\sigma}$ in the region, by

$$M_0 = \mu E/\eta\bar{\sigma}\ , \tag{21}$$

where η is the seismic efficiency and

$$\bar{\sigma} = (\sigma_1 + \sigma_2)/2\,. \tag{22}$$

Using average values of μ and $\eta\bar{\sigma}$ for the earth's lithosphere one finds, from Eq. (21), that the seismic moment is roughly proportional to the seismic energy. The empirical relation between seismic moment and surface-wave magnitude M is

$$\log_{10} M_0 = 19.2 + M \tag{23}$$

where M_0 is in dynes - cm (Brune, 1968).

Hence the seismic moment is essentially proportional to the fault area and to the stress drop. The distribution of fault areas is presumably a constant in any given region. For a highly complex, thoroughly fractured region the fault sizes are well graded, the slope of the magnitude frequency distribution is well-defined and the b-value is low. On the other hand, if the region contains but a few major structures the slope of the magnitude-frequency distribution may be broken into several segments or otherwise poorly defined. For example, deep-focus earthquakes in the Andean region in the magnitude range 6.5 – 7.0 are nearly twice as frequent as in the range 5.5 – 6.0 (Acharya, 1971). Neither the b-value nor the mean magnitude give an idea of the true distribution of energies. Figure 1 compares the magnitude-frequency graph for Andean deep earthquakes with the distribution of energy release. Notice that by far the largest amount of energy is released in the magnitude range 6.5 – 7.0; yet the b-value is 0.28±0.09, which yields an estimated mean magnitude of 5.55. Hence the mean magnitude is not representative of the mean energy content of the shocks in the region.

It is clear that no single parameter is sufficient to fully define the seismic source. However, a quantitative experimental measure of the "size" of an earthquake is desirable. Because of the inherent uncertainty of energy determinations the magnitude M has traditionally been used for this purpose.

Unfortunately,. the mean local magnitude $\overline{M}$ yields a biased measure of the average size of earthquakes in a region. Since the seismic moment is a linear combination of the relevant source parameters, including the energy, the rigidity, the mean tectonic stress, and the efficiency of seismic energy conversion, it seems convenient to use the mean local moment $\overline{M}_0$ as a consistent and unbiased measure of earthquake size. The mean local moment affords a quantitative discussion of mean source parameters in a region, while the mean magnitude $\overline{M}$ should be used as a relative yardstick for comparative purposes only.

MEAN MAGNITUDE VARIATIONS IN SUBDUCTION ZONES

The large amount of data on b-factors in the literature can still be used to resolve major qualitative problems. The well-known result by Suyehiro (1966), who found a significant increase in the b-value after a large earthquake, may be interpreted as a decrease in the mean local magnitude due to the stress drop associated with the main shock. Important variations of the b-factor with depth were found in certain subduction zones (Gutenberg and Richter, 1954; Duda, 1964). These variations indicate that the mean magnitude $\overline{M}$ may change as the lithosphere sinks into the earth's mantle.

As the slab sinks into the mantle the increases in temperature and hydrostatic pressure might conceivably weld the fault surfaces together; this effect should be proportionally the same for all faults, irrespective of size. It should tend to reduce the mean fault area, without affecting the general shape of the distribution. An *increase* in the mean fault area seems very unlikely as the slab of lithosphere penetrates to increasing depths.

The observed decrease in the b-factor should therefore reflect an increase of stress with depth. Figure 2 shows the mean magnitudes for several subduction zones around the Pacific Ocean, as computed from b-values (Acharya, 1971). If the slabs are continuous down to depths of 700 km and if the distribution of fault sizes is the same at all depths, these increases in mean magnitude indicate a tendency toward an increase in deviatoric stress $\bar{\sigma}$ with depth. Wyss (1970), Wyss and Molnar (1973) and Molnar and Wyss (1973) have computed apparent average stresses $\eta\bar{\sigma}$ for South America and Fiji-Tonga, and have reached the conclusion that the stresses of deep-focus earthquakes are the same or lower than at normal depths in the same regions. They have found apparent stresses of less than 100 bars at depths of 500-700 km.

The observed increase in mean magnitudes for deep-focus earthquakes thus requires a different explanation. The following possibilities are suggested:

1. The downgoing slab is not continuous.
2. The seismic efficiency η decreases with depth.
3. The mechanism of earthquakes is different at great depths.

These possibilities are not mutually exclusive. However, the data of fig. 2 strongly favor the first conclusion, because the Tonga-Fiji subduction zone exhibits no change of mean magnitude with depth. Seismic evidence suggests that this subduction zone is continuous down to depths of 700 km, whereas most of the other subduction zones are interrupted by wide gaps at intermediate depths.

The question about the relationship between lithospheric slabs and seismicity has been discussed in many recent articles. It has become increasingly clear that a significant proportion of shallow earthquakes occurs outside the downgoing slab; the source parameters of these shocks scatter widely from those associated with the slabs themselves

(Wyss and Hanks, 1972). However, until now it was assumed that intermediate and deep focus earthquakes at least do occur within the plates of sinking lithosphere. Isacks and Molnar (1971) have suggested that the observed gaps in seismicity below 200 km may be due to changes in physical parameters at depth, and not necessarily to a lack of physical continuity of the lithospheric plate itself, as indicated by the present results. Acharya (1971) assumed that the differences in b-values observed in deep-focus earthquakes were caused by the existence of stress variations between different subduction zones at a given depth. According to this assumption, one would predict high stresses in the deep-focus zone under South America, contrarily to the findings of Wyss (1970).

CHANGES IN MAXIMUM MAGNITUDE

Note also the change in maximum magnitudes along a given subduction zone (fig. 3). The largest observed magnitudes consistently decrease from the surface toward the deep-focus zones, while the *mean* magnitudes increase in the same direction. In the case of the Tonga-Fiji subduction zone there is a smooth decrease of maximun magnitudes with depth, even though the mean magnitude remains constant.

The maximum magnitude in a region is a function of two parameters: (a) the maximum fault size; (b) the maximum stress in the region. In the case of the Tonga-Fiji subduction zone, there is evidence that the slab is continuous and that the distribution of fault areas changes little with depth. The apparent average stress may be slightly, but not significantly, lower than near the surface. Hence it would appear that the maximum fault size decreases with depth in the Fiji-Tonga subduction zone.

In South America this decrease is combined with a sharp drop in the available stress. South American deep-focus earthquakes are concentrated in discrete "nests", with diameter as small as 10 km. It seems likely that these nests have a very simple tectonic structure, a fact which would account for their high mean magnitude in the presence of low stresses.

CONCLUSIONS

Statistical properties of earthquake time series may be used to introduce important restraints on the physical realizability of models of subduction zones.

An interpretation of the b-factor in terms of the mean magnitude leads to the following qualitative conclusions:

1. At any given depth level, a low b-factor may be correlated with a high mean magnitude for a region.

2. With the exception of the Tonga-Fiji arc (and possibly Kamchatka-Kuriles-Japan), most circum-Pacific subduction zones lack continuity (in the sense of conserving their identity as one continuous slab of lithosphere) down to depths of the order of 700 km.

3. Deep-focus nests under South America probably have a simple tectonic structure, which allows them to generate large-magnitude earthquakes at low stresses.

4. The observed decrease in maximum magnitude with depth is consistent with the variations in apparent stress found by Wyss and Molnar.

In this respect, it should be noted that the maximum magnitude does not begin to decrease immediately, but remains constant to a depth of 100–150 km (fig. 3). This is significant in view of the fact that Wyss (1970) and Wyss and Molnar (1973) found relatively high stresses associated with intermediate depth earthquakes.

The use of b-values in making inferences on stresses has its limitations, because the b-factor leads to a systematic underestimation of the mean energy in a region. For consistent stress estimates the mean local moment $\overline{M}_0$ should preferably be used instead of the b-factor or the mean magnitude $\overline{M}$.

It should be pointed out that most large earthquakes may be expected to occur at a stage of high stress level in a region, while aftershocks (which are the most numerous events in any earthquake sample) are representative of a state of stress depletion. Hence the b-factor will (in general) be correlated with low stress levels during periods of high activity, rather than with average stresses during the long periods of seismic quiet which precede a large shock.

The relatively low b-values observed in deep earthquakes may partly be attributed to the fact that aftershocks are infrequent at great depth. The b-value is more representative of average stresses in this case, than in the case of shallow earthquakes which contain a large proportion of aftershocks.

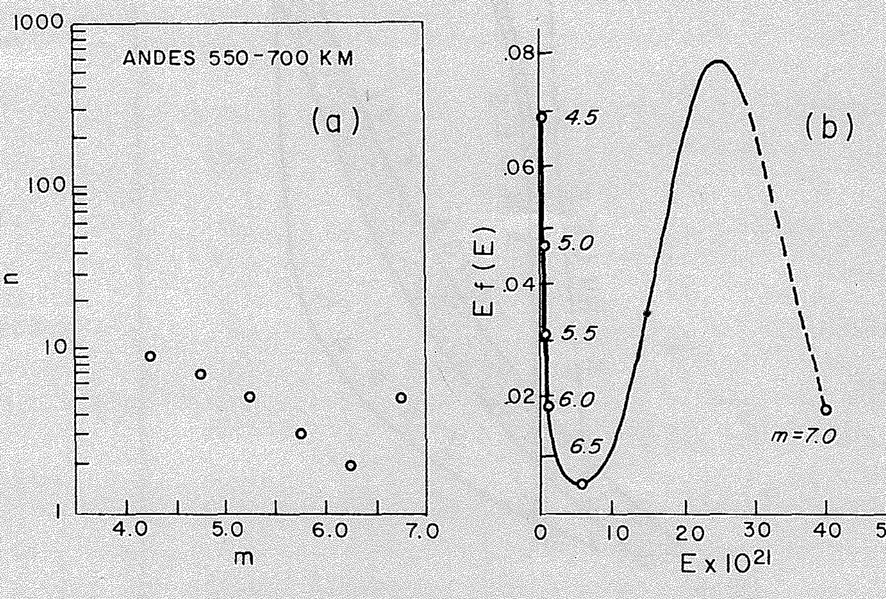

Fig. 1. Deep-focus earthquakes under the Andes, 1961-66. (a) Magnitude-frequency graph, after Acharya (1971). The b-value predicts a mean magnitude of 5.55. (b) Spectrum of energy release (in units of $\bar{E}$) for the same data, showing that the highest density of energy release occurs in the range above m = 6.5.

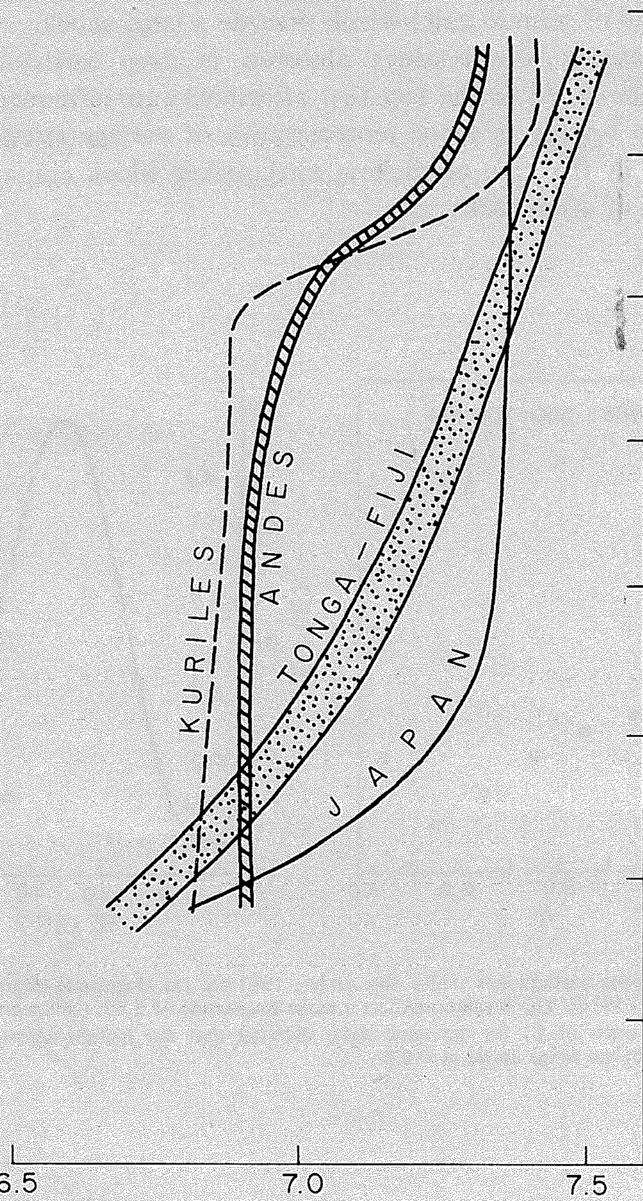

Fig. 2. Mean magnitudes vs depth (1961-66) for various subduction zones around the Pacific Ocean, as estimated from published b-values (Achayra, 1971). Dashed lines mark the depth ranges where no earthquakes occur.

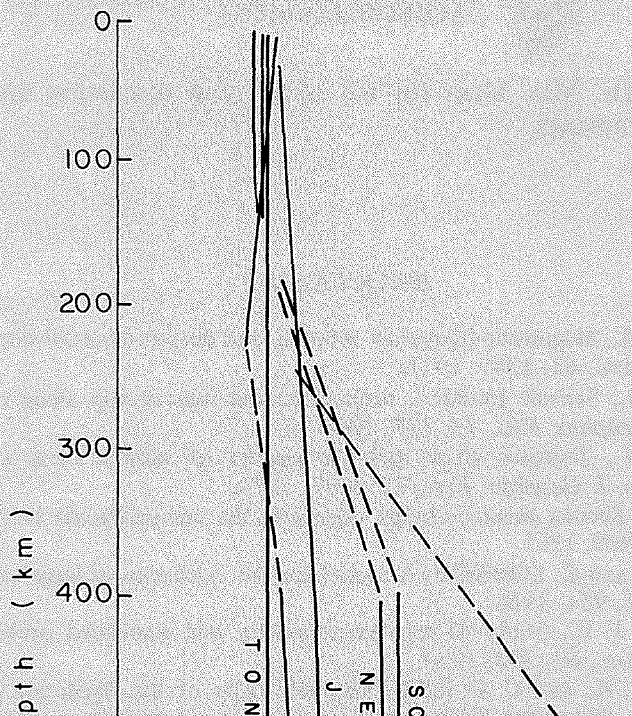

Fig. 3. Maximun observed magnitudes vs depth, for the same sample as in fig. 2.

ACKNOWLEDGMENT

I thank Dr. Max Wyss for his stimulating discussion and for his valuable comments.

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