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A NOTE ON THE PROPAGATION OF S–H WAVES IN A HETEROGENEOUS ISOTROPIC HALF–SPACE

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RESUMEN

En este artículo se determinan las velocidades de fase y grupo de ondas S-H en un semiespacio heterogéneo e isotrópico en el cual la velocidad de ondas de cizallamiento es constante.

ABSTRACT

The phase and group velocity of S-H waves in a heterogeneous isotropic half-space within which the shear wave velocity is constant throuhout are found.

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INTRODUCTION

Many investigators have dealt with problems related to the propagation of S-H waves in heterogeneous layered half-spaces (see for instance, Ewing *et al.*, 1957).

Meissner (1957), studied the problem of the propagation of Love waves in a semi-infinite medium by assuming that the modulus of rigidity (μ) and the density (ρ) of the medium behave linearly with the depth (z). By trying a different function for each, he obtained a solution in terms of the Wittaker function. In this paper we take $\frac{\mu}{\mu_0} = \rho/_{\rho_0} = \cos h^2 z/\lambda$, With μ_0, ρ_0 and λ as constants. This renders a constant shear wave velocity within the medium. We also derive a dispersion relation and establish an important relation between the phase and group velocities. Also the phase velocity $(^C/_\beta)$ and group velocity $(^U/_\beta)$ in units of the shear wave velocity $\left(\beta = \left(\frac{\mu_0}{\rho_0}\right)^{\frac{1}{2}}\right)$ are presented graphically as functions of λK, where K is the wavenumber. Finally a discussion of our results is given in the last section.

FORMULATION OF THE PROBLEM

We take the origin (z = 0) at the traction-free surface and the positive direction downwards. The positive X axis is in the direction of propagation of the waves. The equation of motion for the medium, is then

$$\nabla^2 v + \frac{1}{\mu}\frac{d\mu}{dz}\frac{\partial\vartheta}{\partial z} = \frac{1}{\beta^2}\frac{\partial^2\vartheta}{\partial t^2} \qquad (1)$$

where ϑ is the displacement.

We now choose the plane wave solution of equation 1 in the form:

$$\vartheta = v(z) \exp\left\{ iK(ct - X) \right\} \tag{2}$$

taking $\frac{\mu}{\mu_0} = \frac{\rho}{\rho_0} = \cos h^2\ z/\lambda$ and subtituting $v^1 = v\sqrt{\mu}$ equation 1 becomes

$$\frac{d^2 v}{dz^2} - \upsilon^2 v^1 = 0$$

$$\text{where } \upsilon^2 = \frac{1}{\lambda^2} + K^2 \left(1 - \frac{C^2}{\beta^2}\right) \tag{3}$$

Thus the displacement in the semi-infinite medium is given by

$$\vartheta = \frac{A}{\sqrt{\mu_0}} \operatorname{sech}(z/\lambda) \exp\left\{ -\upsilon z + iK(ct - X) \right\} \tag{4}$$

For S-H wave propagation the boundary condition is evidently, $\frac{\partial \vartheta}{\partial z} = 0$. at $z = 0$.

Consequently, the phase velocity C/β and the group velocity υ/β are given by

$$\frac{C}{\beta} = \left(1 + \frac{1}{\lambda^2 K^2}\right)^{1/2}$$

and (5)

$$\upsilon/\beta = \beta/C$$

DISCUSSION

The feasibility of assuming that $\frac{\mu}{\mu_0} = \frac{\rho}{\rho_0} = \cos h^2 \, z/\lambda$ depends solely on the choice of λ which is evidently a large quantity for a real medium. The problem as such requires no specific choice of λ. Figure 1 shows that the larger the phase velocity or the smaller the group velocity, the smaller the values of λK; this means that the wavenumber decreases and, consequently, the wavelenght grows, i.e., $C/\beta \rightarrow \infty$ and $U/\beta \rightarrow 0$ as $K \rightarrow 0$. Moreover, the rate of decrease of the phase velocity and the rate of increase of the group velocity are larger for small velues of λK and it decreases asymptotically to unity as λK becomes larger.

From Fig. 1 it can be seen that for λK = 1 the phase velocity is larger than the group velocity by about a factor 2, for λK = 2 it is larger by a factor of 1.25 and for λK = 3 by a factor of 1.11. The group velocity grows at the expense of the phase velocity in such a way that their product always equals the square of the shear wave velocity of the medium. The last value of C/β and largest value of U/β tend to unity for large values of λK.

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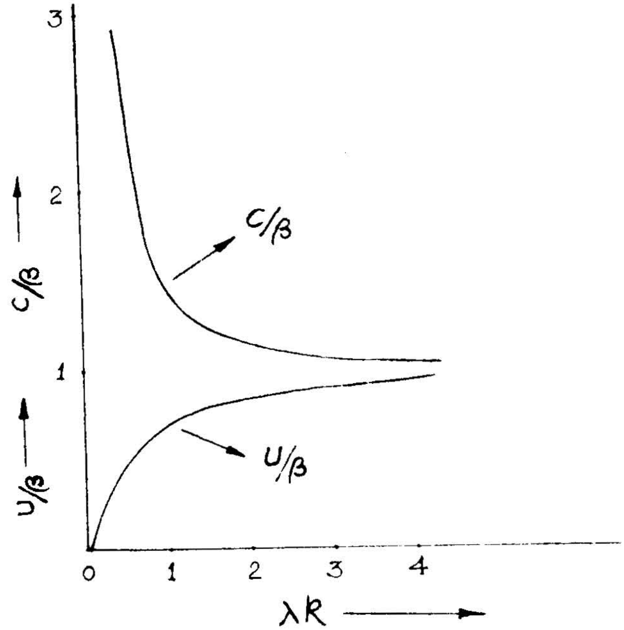

FIG.1. Graphs of Phase Velocity C/β and Group Velocity U/β Versus λR

BIBLIOGRAPHY

EWING, W. MAURICE, WENCESLAS S. JARDETZKY and FRANK PRESS, 1957. Elastic waves in layered Media, Mc Graw-Hill Book Co., New York.