A COMPUTATIONAL PROCEDURE FOR THE DETERMINATION OF THE OPTICAL AIR-MASS

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RESUMEN

En este trabajo se presenta un cálculo por máquina en lenguaje Algol de la masa de aire óptica. El método incorpora la reducción del tiempo de observación, una corrección diaria para la declinación del sol, la ecuación trigonométrica y, mediante el uso de la tabla de Bemporad modificada con 900 entradas se obtiene la masa de aire relativa y absoluta. El tiempo de cálculo es sólo de 1/60" por observación.

Los resultados permiten valorar cuantitativamente los efectos del camino óptico y la latitud del lugar así como el error de cálculo cometido al leer directamente el ángulo zenital con el actinómetro en el momento de la observación.

ABSTRACT

In this report a computation in Algol language for the optical air mass is presented. The method contains the reduction of the observation time, a daily correction for the solar declination and the trigonometric equation. By means of a modified Bemporad table with 900 entries the relative air mass is obtained. The running time for each observation is only 1/60".

The results show quantitatively the effects of the optical path length and the latitude. The error derived from the direct reading of the zenithal angle with the actinometer at the time of observation is also estimated here.

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INTRODUCTION

The determination of the optical air mass is customarily performed in Actinometry by the direct reading of the zenital angle at the moment of observation. However, this instrumental reading causes several errors due either to the instrument itself or to the observer. During the I.G.Y., the International Commision for Solar Radiation studies (C.S.A.G.I. 1957) recomended the use of the trigonometric formula $\sin h = \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos t$, where φ is the latitude of the observation point, δ the solar declination and t the hourly angle referred to the true solar time; from the local observation time the equation of the time must be used in order to obtain the hourly angle. Then in both determinations, one has to use the table of Bemporad (in Linke, 1942) where the optical air mass is tabulated against the zenital angle. The absolute air mass $m = m_r \, p/p_o$ is then obtained.

The use of the above equations requires tedious calculations, even if one establishes a general linear relationship of the form $\sin h = A + B. \cos t$, where $A = \sin \varphi$ and $B = \cos \varphi \cos \delta$, the terms are not absolutely constant since the solar declination also changes slowly day by day. Among the methods that have been proposed, perhaps the best is the diagram of Schütte (in Perrin de Brichambaut, 1963) which in essence is no more than a linear transformation of the form $\sin h = A' f(\delta) + B' g(\delta) \cos t$; the functions $f(\delta)$ and $g(\delta)$ are then parametrically fixed and the plot deals as coordinate only sin h and cos t of the diagram for each latitude φ, i.e., for each place. Since variations of the solar declination are neglected, the error in these methods is relatively large.

In the present paper we present a machine computation in Algol language for the optical air mass. The procedure incorporates the reduction of the observation time, a daily correction for the solar declination, the trigonometric equation, and, finally, the table of Bemporad modified with 900 entries. The running time for the program is very short, it takes only 1/60" per observation data. The procedure has already been proved for several thousand data from our network of solar radiation stations, namely Chihuahua (1960-1967), Mexico City (1968-1969), and Orizabita (1967-1969). The program is a subroutine of the general algorithm for the complete determination of the actinometric radiation field (Galindo, I. and A. Muhlia, 1970).

2. METHODS

The input data are:
the date,
H = the observation time,
δ = the solar declination taken from the corresponding table of the *Nautical Almanac* (1968) with daily values,
t = the hourly angle for the zero time of each day of the year,
δ and θ = the latitude and longitude of the station, both constant.

For a given ith observation one has the pair of values t_i, δ_i; one makes also an interpolation with the next day pair of values t_j, δ_j ··

$$t_i, \delta_i \rightarrow t_j, \delta_j$$

The daily variation of the time and the solar declination are then given as

$$\frac{t_j, \delta_j - t_j, \delta_i}{24} = \Delta t_{ij}, \Delta\delta_{i_j} \tag{2.1}$$

then, one obtains the correct hourly angle and the solar declination with the following equations:

$$t = t_{0i} + (15 + \Delta t_{i_j}) \cdot H + \frac{\Delta t_{ij} \cdot \theta}{360} \tag{2.2}$$

$$\delta = \delta_{0_i} + \Delta\delta_{i_j} \cdot H + \frac{\Delta\delta_{ij} \cdot \theta}{360} \tag{2.3}$$

here t_{0_i} and δ_{0_i} are the hourly angle and declination for the zero time of the ith day, as for the date of entry.

With t form (2.2) and δ form (2.3) one obtains the set of values t_i, δ_i, which enter in the trigonometric equation

$$\sin h = \sin \varphi \sin \delta + \cos \varphi . \cos \delta . \cos t \tag{2.4}$$

Once equation (2.4) is solved for h the program instruction is to go to Bemporad's table and to find there the corresponding entries for the relative air mass m_r; finally one calculates the absolute air $m = m_r \cdot p/p_O$.

3. RESULTS

In order to study the efficiency of the procedure, linear correlations between the manually computed air masses m_m and those computed by the program m_i were performed. Table I, summarizes the statistics for different stations.

Table I. Linear correlation of optical air masses

Place	$\overline{m}_m$	$\overline{m}_i$	C	m	σ	a	b	N
Basle*	1.843	1.838	0.9811	0.5742	0.5610	1.838	0.9586	70
México, D.F.	1.245	1.273	0.9715	0.2177	0.2423	1.273	1.0810	79
Chihuahua	2.022	2.055	0.9929	0.8243	0.8521	2.022	0.9605	80
Orizabita	1.293	1.298	0.9893	0.2939	0.2907	1.293	1.0000	75

* Through the courtesy of Dr. Schüepp.

The bars denote the mean, C the correlation coefficient, σ the standard deviation, a and b are the linear regression coefficients and N de number of data.

Despite the fact that the program has been run for thousands of data, we have found that the changes in the statistics are not significant, therefore the sampling presented here is only for small runnings ($70 \leqslant N \leqslant 80$). Table I shows that the correlation between both variables is quite good.

The maximum scattering of the data from the mean is explicitly shown through the ratio σ m:

Place	$\sigma m / \overline{m}_m$	$\sigma c / \overline{m}_c$
Basle	0.311	0.305
México, D.F.	0.175	0.190
Chihuahua	0.408	0.415
Orizabita	0.181	0.224

Obviously, the scattering is lower for data in which the mean value lies near the unity; in other words, when the sun is in the zenith the linearity is optimal; this is due to the definition of the optical air mass (see Robinson, 1965, p. 48). The minimum values then correspond to Mexico, D.F. and Orizabita where sampling lies between the boundaries $1.00 \leqslant m_{D.F.} \leqslant 2.43$; $0.88 \leqslant m_0 \leqslant 2.16$. Note that for Orizabita one uses reduced air masses. Basle and Chihuahua are given with the sampling boundaries $1.00 \leqslant m_B \leqslant 3.00$; $1.00 \leqslant m_{CH} \leqslant 3.71$, and they show the larger values for scattering of the data from the mean. The difference in the upper boundaries for both places is of the order of one unit of air mass, but their standard deviations are quite different; Basle $\sigma_m = 0.5742$, Chihuahua $\sigma_m = 0.8243$. The above considerations seem to show that the scattering and the standard deviation depend strongly on the path length of the atmosphere.

From Table I the following linear equations are established:

Basle	m_B	=	0.071	+ 0.960	m_m	(3.1)
México, D.F.	m_{DF}	=	-0.073	+ 1.081	m_m	(3.2)
Chihuahua	m_{CH}	=	-0.050	+ 1.041	m_m	(3.3)
Orizabita	m_O	=	0.005	+ 1.000	m_m	(3.4)

Figures 1 to 4 show the regression lines together with the distribution of data.

From the above equations one sees that the maximal error is committed for $m_m = 1.0$, that is to say, when the sun is on the zenith, according to equation (2.4) h = 90°. Table II shows the maximal error for zenithal air masses.

Table II. Maximal error for zenithal air masses

Place				m_i	$\Delta h(°)$
Basle	(47 33' N.,	7 35' E.,	318 m.a.s.l.)	1.0306	12.5
México, D.F.	(19 20' N.,	99 11' W.,	2268 m.a.s.l.)	1.0082	7.0
Chihuahua	(28 30' N.,	106 04' W.,	1450 m.a.s.l.)	0.9910	6.5
Orizabita	(20 35' N.,	99 12' W.,	1745 m.a.s.l.)	1.0050	2.5

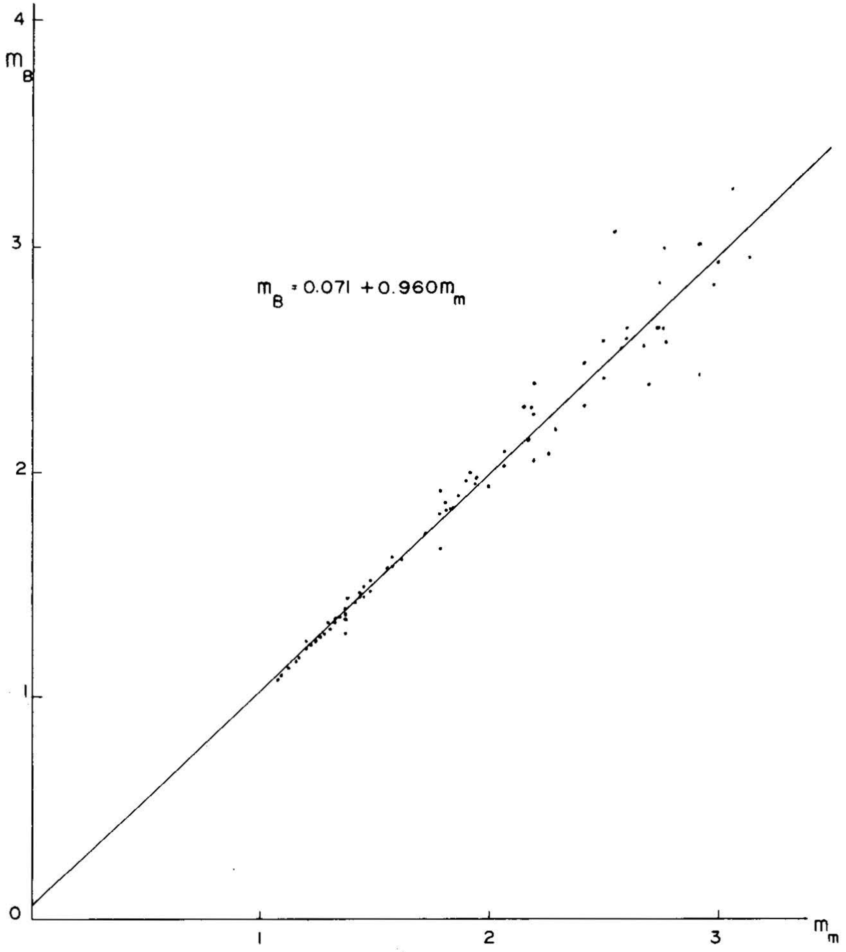

FIGURE 1. BASLE (1958). COMPUTED OPTICAL AIR MASS m_B . MANUALLY CALCULATED OPTICAL AIR MASS m_m .

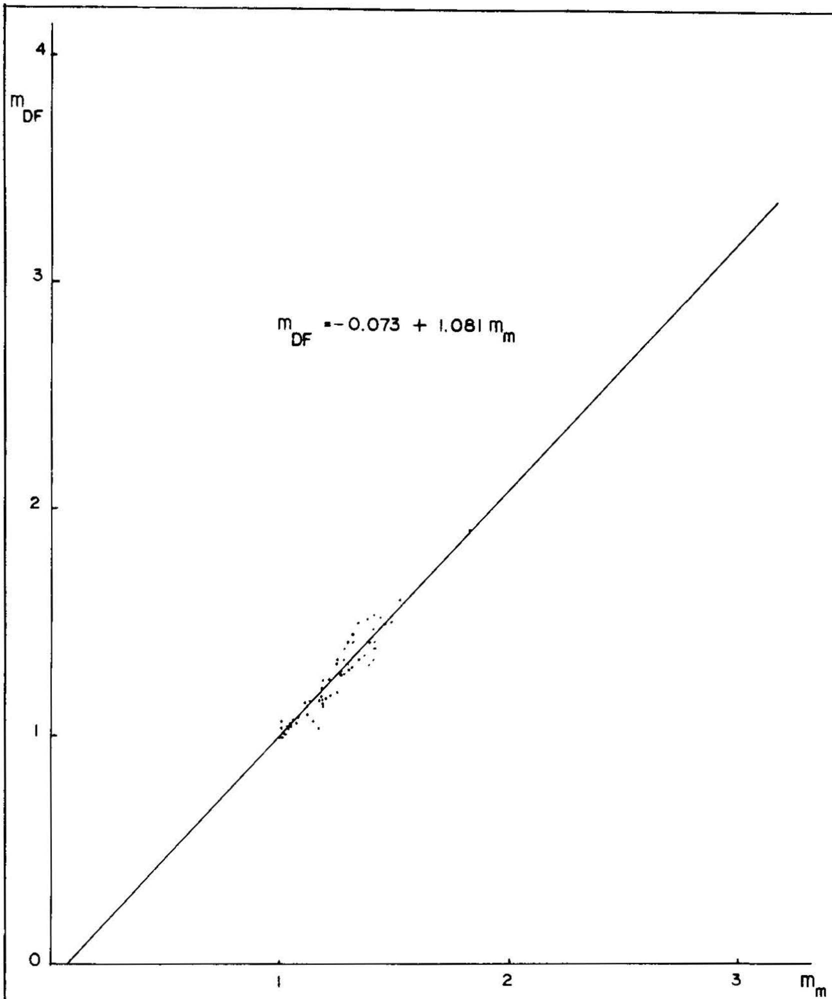

FIGURE 2. MEXICO D.F (1962) COMPUTED OPTICAL AIR MASS m_{DF}. MANUALLY CALCULATED OPTICAL AIR MASS m_m.

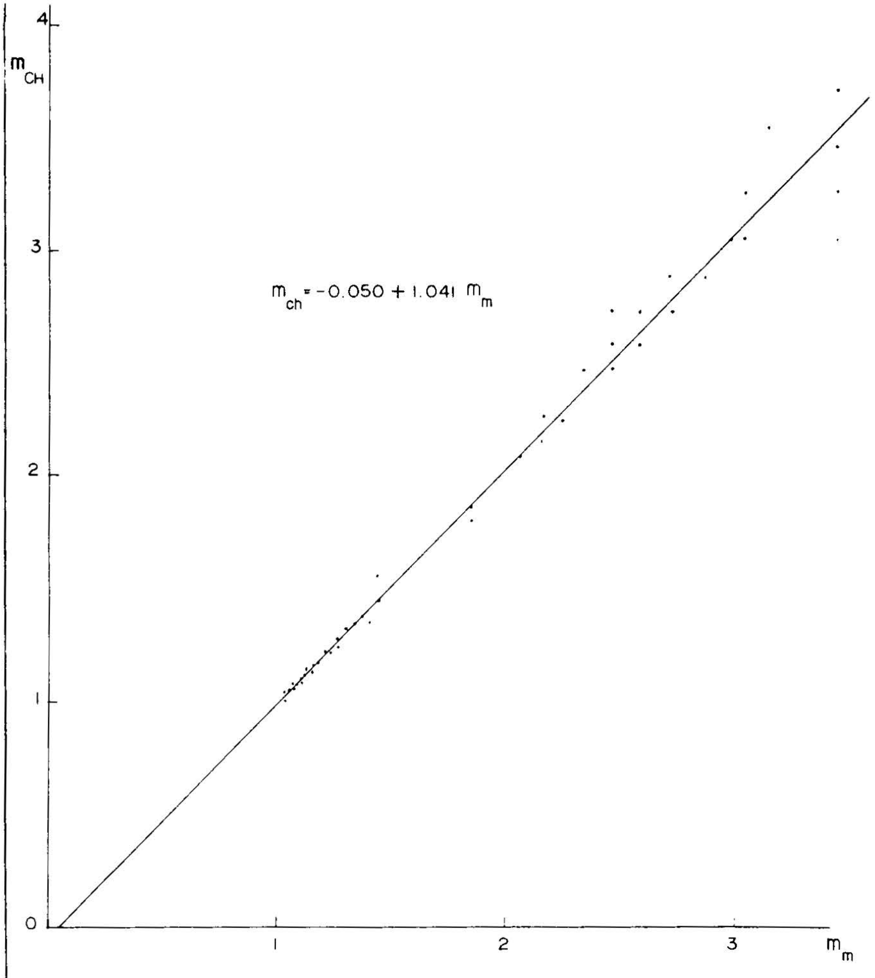

FIGURE 3. CHIHUAHUA (1960). COMPUTED OPTICAL AIR MASS m_{ch} MANUALLY CALCULATED OPTICAL AIR MASS m_m

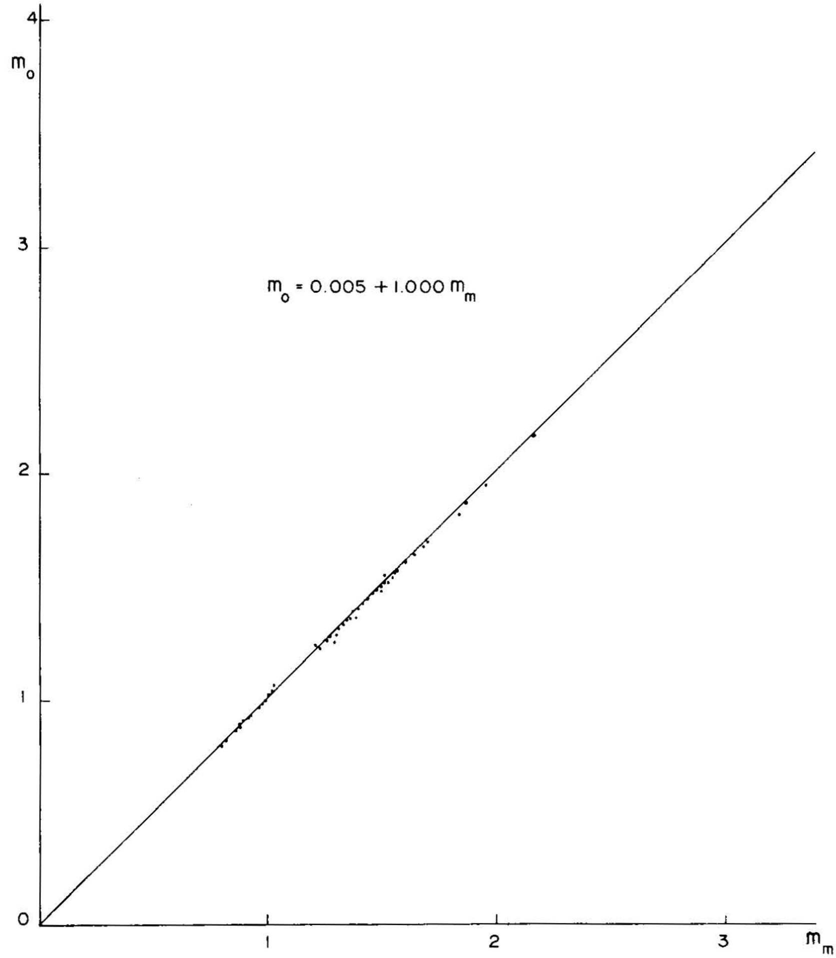

FIGURE 4. ORIZABITA (1968). COMPUTED OPTICAL AIR MASS m_o. MANUALLY CALCULATED OPTICAL AIR MASS m_m.

Δh was calculated by using Table 4 from the IGY Instruction Manual (1957) and Table 137 from the Smithsonian Meteorological Tables (1951). The factor that contributes to this error is the direct reading of the zenithal angle from the actinometer during the observation[1]

Figures 5 to 8 show that for air masses larger than unity the error tends to zero quite rapidly; according to equation 2.4 the form of this distribution depends strongly on the latitude, thus México, D.F. and Orizabita because of their vicinity have the same pattern. However Orizabita reaches the asymptote very fast (see Figures 5 and 6). Figures 7 and 8 show the distribution for Chihuahua and Basle respectively, their patterns are alike.

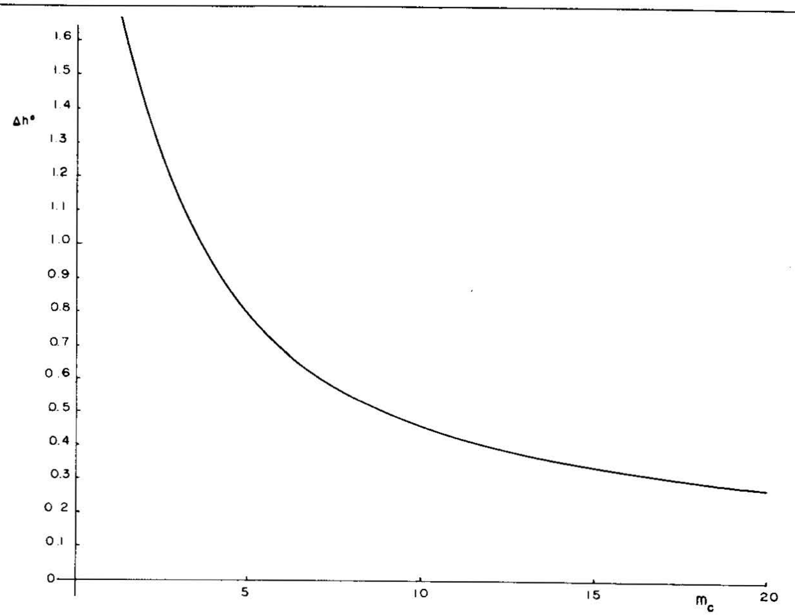

FIGURE 5. MEXICO D.F. (19° 20'N, 99°11'W.) ERROR DISTRIBUTION OF ZENITAL ANGLE VS OPTICAL AIR MASS

[1] Schüepp, private communication. Oct. 1970

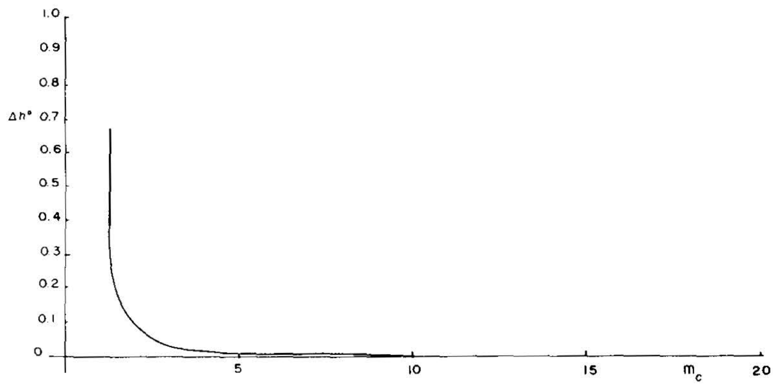

FIGURE 6. ORIZABITA(20°35'N.,99°12'W.) ERROR DISTRIBUTION OF ZENITAL ANGLE VS. OPTICAL AIR MASS

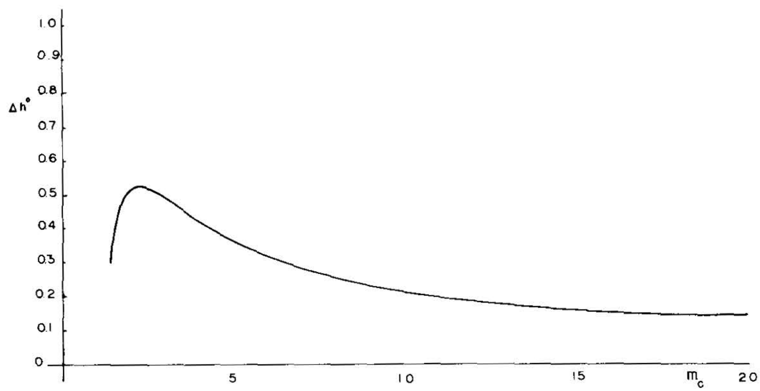

FIGURE 7. CHIHUAHUA (28°38'N.,106°05'W.) ERROR DISTRIBUTION OF ZENITAL ANGLE VS. OPTICAL AIR MASS

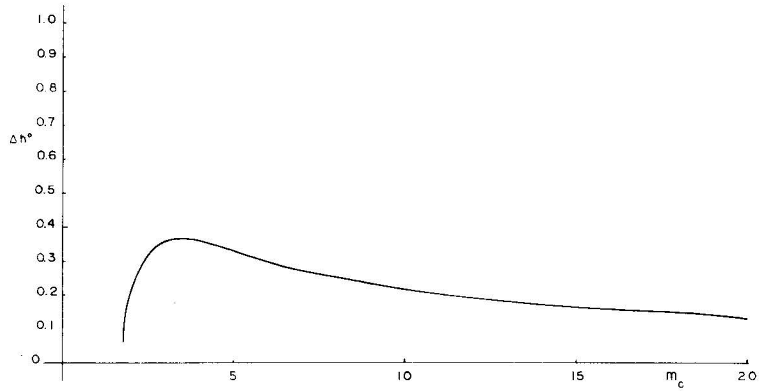

FIGURE 8. BASLE (47°33N., 7°35'E) ERROR DISTRIBUTION OF ZENITAL ANGLE VS OPTICAL AIR MASS

CONCLUSIONS

1. A machine computation in Algol language of the optical air maiss is presented. Its running time is about 1/60" for each observation.

2. The entries are the date, the observation time, and, for different stations the geographical coordinates; the solar declination is taken from the *Nautical Almanac* with extrapolated daily values.

3. By extrapolation methods one obtains the hourly time variation and the declination, then one obtains the hourly angle and the corrected declination, these parameters are then fixed with the date, after that the trigonometric equation is solved and with the modified Bemporad table one reads the relative optical air mass; finally a simple calculation gives the absolute air mass.

4. The maximal error arises when the sun is in the zenithal position; this is caused mainly by the direct reading of the zenithal

angle by means of the actinometer and the extrapolation in the tables. This error is quantitatively evaluated here for different stations.

5. In the trigonometrical equation the latitude is a very important parameter, it gives a characteristic distribution pattern of the zenital angle versus the optical air mass. Here we show two of these distributions.

6. The results show that the statistical scattering is due mainly to the path length, with its lower values for air masses near the zenith.

ACKNOWLEDGMENTS

The authors want to testimony the aid received during the preparation of this work from Dr. W. Schüepp, Mr. Heriberto Ornelas and, finally, the Centro de Cálculo of the National University of Mexico.

BIBLIOGRAPHY

C.S.A.G.I. 1957. *Manuel d'Instructions Météorologie, Instruments et Mesures de Rayonnement,* Asoc. Int. Météorologie, U.I.G.G. Uccle.

LINKE, F. 1942. *Handbuch der Geophysik,* Band VIII: pp. 239-245, Gebrüder Borntraeger.

PERRIN DE BRICHAMBAUT, Ch. 1963. *Rayonnement Solaire et Echanges radiatifs naturels.* pp. 54 Gauthier-Villars, Ed.

GALINDO, I. & A. MUHLIA. 1970. Contribution to the Turbidity Problem in Mexico City, *Arch. Met. Geoph. Biokl.,* Band 18, p. 169.

U.S. NAVAL OBSERVATORY, 1968. *The Nautical Almanac,* U.S. Government Printing Office.

ROBINSON, N. (Editor). 1965., *Solar Radiation,* pp. 48, Elsevier Publishing Co.

IGY INSTRUCTION MANUAL, *Part VI Radiation Instruments and Measurements,* Appendix, Table 4, pp. 461, Pergamon Press.

SMITHSONIAN INSTITUTION. 1951. *Smithsonian Meteorological Tables.* Sixth Revised Ed. (prepared by R.J. List) Table 137, p. 422.