

Multiresolution analysis applied to interpretation of seismic reflection data

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RESUMEN

Hemos desarrollado el análisis multiresolutivo basado en la transformada ondicular discreta. En este trabajo ilustramos los usos potenciales en la exploración petrolera de la aplicación conjunta del análisis multiresolutivo y de los atributos sísmicos mediante un caso de estudio. Tres reflexiones mayores en la base de la ventana sísmica analizada están asociadas a formaciones geológicas con aceite (de edad Cretácico Inferior). Estas reflexiones mayores parecen continuarse hacia la izquierda por reflexiones menos claras y de menor amplitud. Basados únicamente en el análisis multiresolutivo de los datos sísmicos o en sus respectivos atributos convencionales (amplitud, fase y frecuencia instantáneas) no fue posible excluir la posible continuación. En efecto una continuación estratigráfica es sugerida por estos dos conjuntos de datos. Sin embargo, las resoluciones a diferentes escalas proporcionadas por el análisis multiresolutivo de los atributos sísmicos permitieron un análisis más fino. En efecto la continuidad sugerida en el análisis multiresolutivo de la sección sísmica y de los mismos atributos sísmicos, no fue apoyada completamente por el análisis de la fase. Es posible con la multiresolución de este atributo distinguir con confianza entre las reflexiones continuas de mayor amplitud y sus supuestas continuaciones. Las continuaciones están caracterizadas por reflexiones de baja amplitud, de media a alta frecuencia y continuas a tramos. Al estar aislada de las contribuciones de la información sísmica a otras escalas (o rangos de frecuencia) una característica sísmica dada es definida con más nitidez. Es este realzado adicional (es decir mayor calidad de la relación señal-ruido) lo que le permite a uno realizar un análisis discriminatorio más fino de mucha utilidad en la detección de hidrocarburos. El uso conjunto de la multiresolución y de los atributos sísmicos tiene aplicaciones potenciales en el campo de la sísmica de reflexión, tanto como indicador de hidrocarburos como en la caracterización de yacimientos carbonatados. El análisis multiresolutivo aquí desarrollado está basado en la transformada ondicular discreta, y representa una alternativa al análisis multiresolutivo basado en la transformada ondicular continua hoy disponible en la industria petrolera.

PALABRAS CLAVE: Transformada ondicular, análisis multiresolutivo, análisis de datos sísmicos, geofísica aplicada.

ABSTRACT

We have developed a method of seismic multiresolution analysis based on the discrete wavelet transform. We illustrate the potential use in petroleum exploration of the joint application of multiresolution analysis and seismic attributes for a real case study. Three major reflections at the base of the seismic window are associated with oil-bearing Lower Cretaceous geologic formations. These major reflections seem to be continued towards the left by less clear and lower amplitude reflections. On the strength of multiresolution analysis from the seismic data or from envelope amplitude, instantaneous phase and frequency could not be discarded, the possible continuations as a stratigraphic continuation was suggested in both data sets. Improved resolution at different scales provided by multiresolution analysis of the seismic attributes enabled a finer analysis, and continuity was no longer fully supported. It was possible to distinguish the high amplitude continuous reflections from their continuations. Higher resolution improved discrimination. The joint use of multiresolution and seismic attributes has potential applications in the field of seismic reflection. We use successfully the discrete wavelet transform as an alternative to the continuous wavelet transform.

KEY WORDS: Wavelet transform, multiresolution analysis, seismic data analysis, applied geophysics.

INTRODUCTION

Seismic attributes represent a mature technology which is being improved continuously. Seismic attributes are obtained from seismic data for geological interpretation and analysis. For example, the envelope amplitude represents a

measure of the reflection strength. Since seismic data are incredibly rich in information on amplitude, frequency and geometry, many attributes have been proposed in the last decade: e.g., AVO is an attribute which can evaluate the rock elastic parameters (Shuey, 1985; Jin *et al.*, 2000). Even when physical links between reservoir properties and attributes are

not established (Hart, 2002), attributes are being integrated in sophisticated workflows and techniques using meta-attributes more directly related to geology or to reservoir properties, and more amenable to well calibration. Some of these workflows can help in mapping faults (Rooij and Tingdahl, 2002), or in the classification of seismic facies (West *et al.*, 2002). Attributes are used to help predict physical properties (e.g., porosity, lithology, bed thickness) of strata. A classification of attributes may be based on time, amplitude, frequency, and attenuation (Brown, 1996; Chen and Sidney, 1997; Brown, 2001).

Instantaneous amplitude $A(t)$, instantaneous phase $\theta(t)$, and instantaneous frequency $f(t)$ are given by

$$A(t) = \sqrt{x(t)^2 + (Hx)(t)^2}, \quad (1)$$

$$\theta(t) = \text{tg}^{-1}\left(\frac{(Hx)(t)}{x(t)}\right), \quad (2)$$

$$f(t) = \frac{1}{2\pi} \frac{d}{dt} \theta(t). \quad (3)$$

The continuous wavelet transform has been recently proposed to perform a spectral decomposition from conventional attributes (Gao *et al.*, 1999; Castagna *et al.*, 2002). Active research includes the use of the complex wavelet transform, originally developed for image processing (Lawton 1993; Lina and Mayrand, 1995; Kingsbury, 1999, 2000; Spaendonck *et al.*, 2002). Projection-based (Spaendonck *et al.*, 2001a, 2001b), and local wavelet-based Hilbert transform operators (Selesnick, 2001; Spaendonck *et al.*, 2002) have also been proposed.

The diagonalization of standard linear operators in the wavelet domain (Ekstedt and Lindberg, 1997) was successfully applied to the processing of aeromagnetic data (Ridsdill-Smith and Dentith, 1999), as well as to the diagonalization of the Hilbert transform operator to obtain seismic attributes (Rivera-Recillas *et al.*, 2001; 2002; 2003a; 2003b; Rivera-Recillas, 2005). The continuous wavelet transform is also being applied to analyze seismic attributes (i.e., Castagna *et al.*, 2002). We have been studying several applications of the wavelet transform (Lozada Zumaeta and Ronquillo-Jarillo, 1996; 1997a; 1997b; 1997c; 2002; Ronquillo Jarillo *et al.*, 2003; Lozada-Zumaeta *et al.*, 2003a, 2003b). One initial research topic was the application of multiresolution to the analysis of seismic attributes (Ronquillo-Jarillo and Lozada-Zumaeta, 1997, 1998; Lozada-Zumaeta and Ronquillo-Jarillo, 1998; Lozada-Zumaeta *et al.*, 2001; Lozada-Zumaeta, 2002).

Early progress in research on the wavelet transform in seismic reflection may be found in symposiums and con-

gresses (i.e., Lozada-Zumaeta and Ronquillo Jarillo, 1997a, 1997b, 1997c; Ronquillo-Jarillo *et al.*, 2003; Lozada-Zumaeta *et al.*, 2003a, 2003b). At present, applications to seismology and seismic reflection begin to be reported formally (i.e., Chiao and Kuo, 2001; Chiao and Liang, 2003). Some recent reports are related to filtering (Fedi *et al.*, 2000; Yu and McMechan, 2002; Pazos *et al.*, 2003), phase detection (Yogomida, 1994; Lou and Rial, 1995; Singh and Dowla, 1997; Fedorenko and Husebye, 1999, Gendron *et al.*, 2000; Zhang *et al.*, 2003), earthquake detection, and source description and system characterization (Basu and Guota, 1997; Lui and Najmi, 1997; Ji *et al.*, 2002; Rezaei and Ventura, 2002; Botella *et al.*, 2003), analysis of seismic signals (Bartosch and Seidl, 1999; Le Gonidec *et al.*, 2002; Liu and Zhang, 2002), tomography (Bergeron *et al.*, 1999, 2000), group velocity and slowness determinations (Bear and Pavlis, 1997; Yamada and Yomogida, 1997), and inversion (Li *et al.*, 1996). A few formal papers have been reported on multiresolution analysis (Goupillaud *et al.*, 1984; Cohen and Chen, 1993; Faqui *et al.*, 1995; Grubb and Walden, 1997; Torrence and Compo, 1998). However, we know of just one informal application relating multiresolution to seismic attributes (Castagna *et al.*, 2002), this was based on the continuous wavelet transform. Here we formally present some early results on the use of the discrete wavelet transform in multiresolution to analyze seismic attributes, and we provide a field application.

MULTIRESOLUTION OF SEISMIC DATA: AN EXAMPLE

Figure 1 shows a window of a seismic section from a Lower Cretaceous limestone hosting oil and gas. The production zone is expected between traces 240 and 300 and between 0.55 s and 0.65 s. The seismic data were processed to preserve amplitude and waveform. The processing included editing, geometry correction, surface-consistent deconvolution, velocity analysis, normal move out and depth move out corrections, stacking, filtering, and time migration.

The section featured two groups of reflections: Z1 dips towards the left from about 0.2 s around trace 300 to 0.6 at trace 10. The reflections in this group in general are clear, continuous, and of high frequency.

The group Z2 (Figure 1b) contains three high amplitude, clear reflections between the traces 200 and 400, and between 0.6 s and 1.0 s. At trace 210 approximately these reflections disappear. Instead, further to the left, we find reflections of about the same wavelength but slightly displaced. The amplitudes of these reflections are lower and they dip in the opposite direction. The geologic interpretation is not straightforward (see Figure 1b): a fold involving a facies change, or a fault? No coherent, small-amplitude reflections

are observed between Z1 and Z2. The interval of economic interest for oil corresponds to the top reflection of the second group (I on Figure 1b). In the following we illustrate the joint use of multiresolution and seismic attributes.

Multiresolution may be used: (1) to perform the analysis of a seismic attribute from a given seismic section (Rivera-Recillas, 2005); or (2) to obtain first the multiresolution analysis and then calculate the attributes for each resolution scale (Lozada-Zumaeta and Ronquillo-Jarillo, 1997a, 1997b; Lozada-Zumaeta, 2002).

We first obtained the multiresolution analysis of the seismic data from Figure 1, and we compared the resolution (1) from the conventional attributes themselves (amplitude, instantaneous phase, and instantaneous frequency), and (2) for the multiresolution analysis of the seismic attributes, obtained as a second step.

See Appendix A for an introduction to multiresolution analysis. Figure 2 shows the multiresolution from the original seismic data. To see how the original data are decomposed into its components at different frequencies or scales consider that the information contained in panels h) and i) are used to recover the section of panel f), and so on. The original section in a) is recovered from the sum of the information in panels b) and c). Panels at the left correspond to average or coarse information, while panels at right feature fine-scale information. From top to bottom we have coarser scales. The finest scale information is contained in panels b) and c), while panels h) and i) feature coarse scale information. The frequency content of resolution W_{10} (Figure 2c) lies between 30 and 70 Hz with peaks at 45 and 60 Hz. For the resolution W_9 (Figure 2e) we have a frequency content between 20 and 60 Hz with peaks at 30 and 45 Hz. For the resolution W_8 the seismic information has a content between 10 and 35 Hz with a peak at 25 Hz and a minor one at 15 Hz (Figure 2g). For the resolution corresponding to W_7 (Figure 2i), the frequency content corresponds to a narrow band around 10 Hz. In resolution V_{10} (Figure 2b) the frequency content ranges between 10 and 50 Hz with a peak at 10 Hz and minor peaks at 20 and 30 Hz. For resolution V_9 (Figure 2d) the frequency content ranges between 10 and 50 Hz, with a peak at 8 Hz, and a minor one at 25 Hz. In the resolution V_8 (Figure 2f) the information is concentrated between 10 and 30 Hz, with a peak at 10 Hz, with minor information between 20 and 30 Hz. Finally, for resolution V_7 (Figure 2h) the information is concentrated around 10 Hz.

This multiresolution treatment was based on Daubechies wavelets of order 6 (see Appendix A). We note that the first group of reflections Z1 is characterized by high and medium frequencies (see resolutions V_{10} , V_9 , and V_8). The reflections of the second group Z2 comprise medium and

low frequencies. The projections onto V_7 and W_7 suggest continuity of these reflections. However, they do not present the same amplitudes and frequencies.

Figure 3 shows the conventional seismic attributes (envelope amplitude, instantaneous phase, and instantaneous frequency) from the seismic window. The envelope amplitude enhances the presence of the two reflection groups Z1 and Z2 as defined in the original seismic section. The reflections from the second group and their possible continuations have contrasting reflection amplitudes. The three reflections from the second group stand out for their high amplitude, but contrastingly, the eventual continuations towards the left feature lower amplitudes (Figure 3c). The instantaneous frequency signatures do not imply a major difference between reflections and supposed continuations. The main reflection group Z1 is featured by high instantaneous frequency values. The high frequency signature of the second group of reflections is featured by a shadow (i.e., low values). Their supposed continuations present low but uncorrelated values. According to these seismic attributes, we cannot exclude a stratigraphic continuity towards the left from the Z2 main reflections as suggested by the instantaneous phase.

Now let us illustrate how multiresolution analysis of the seismic attributes may provide a higher resolution. The improvement is due to the fact that at a given scale the resolution of the seismic attributes is enhanced, as they are isolated from information at other scales. This enhanced resolution enables one to trace seismic reflections with more confidence.

The multiresolution analysis from the instantaneous phase seems to confirm continuity of the second and third reflections in group Z2 (Figure 4). However, now it appears that the supposed continuations towards the left are only piece-wise continuous. Continuity toward the left is not clear anymore at projections V_7 and W_7 (i.e., at low frequencies). Thus V_7 and W_7 helps to rule out continuity towards the left of the topmost reflection.

Multiresolution analysis of the envelope amplitude is shown in Figures 5. The importance of the reflections of the second group from top to bottom is clear in the resolutions V_{10} , V_9 , V_8 and V_7 from the envelope amplitude.

On the other hand, in the instantaneous frequency (Figure 6), this second group of reflections is transparent. At all resolutions they present low values (a shadow). The supposed continuations are featured by low values.

A stratigraphic continuation at high and medium frequencies cannot be excluded totally by the instantaneous phase, but the different signatures from the envelope ampli-

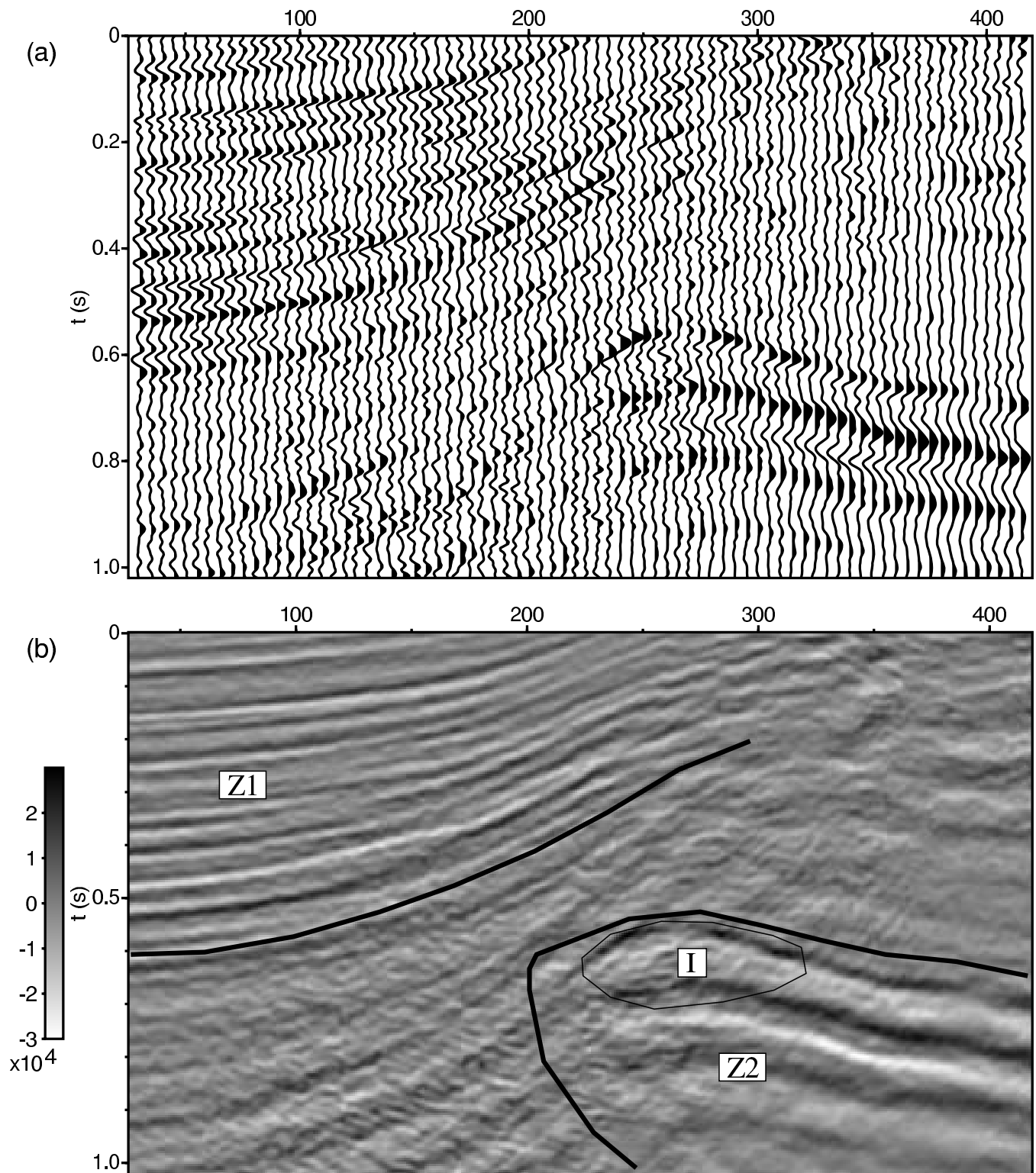


Fig. 1. A seismic section: (a) wiggle-trace plot, (b) image plot.

tude (reflection strength), and the instantaneous frequency suggest that the layers contain oil only in the right hand portion. The existence of a fault seems confirmed by 1) the slight displacement observed between the main reflections and their

supposed continuations, and 2) the lack of continuity to the left. This fault might be sealing the layer to the right. In conclusion, the structure associated to these reflections can be interpreted as an asymmetrical anticline affected by an in-

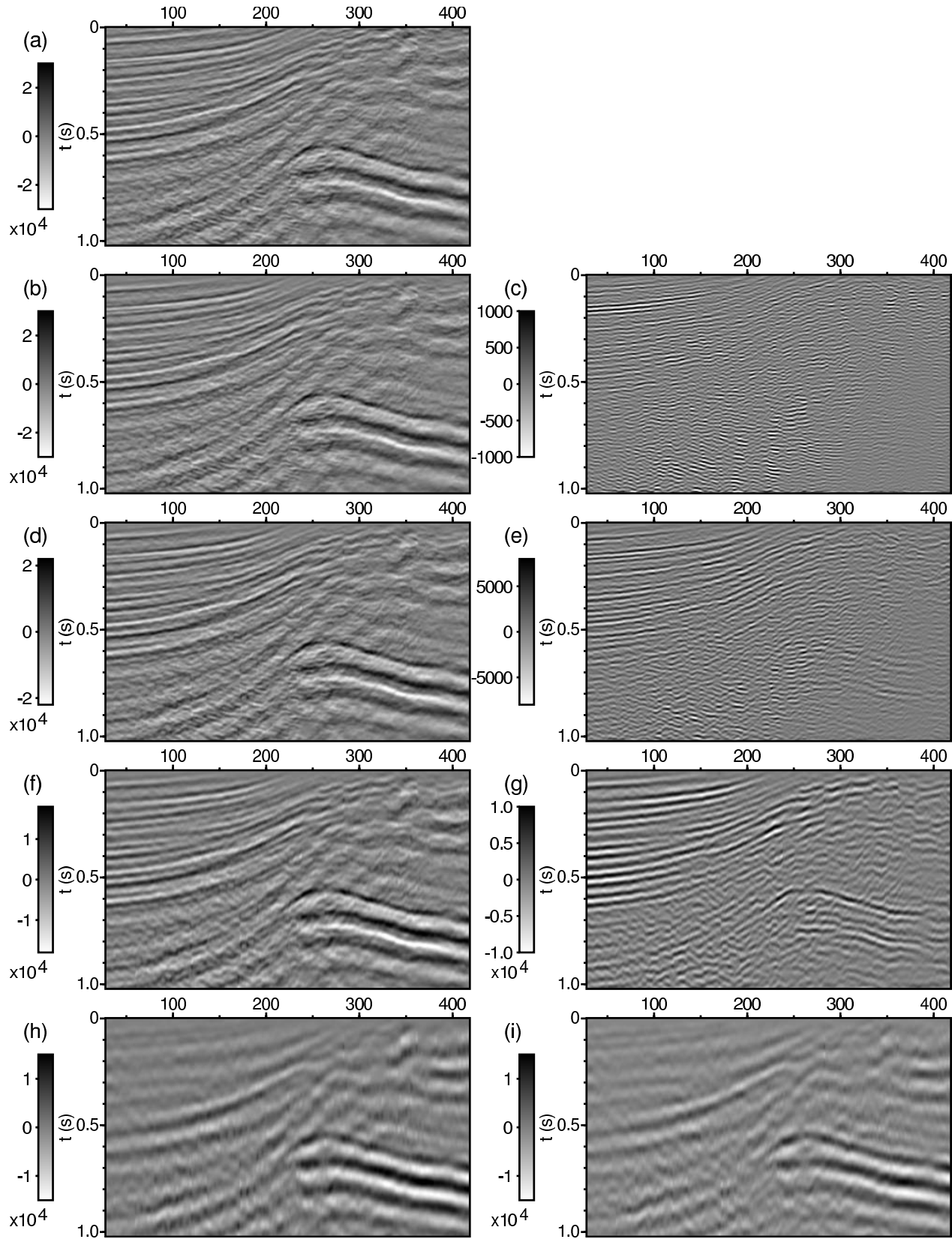


Fig. 2. Multiresolution analysis of the seismic section, based on the wavelet Daubechies of order 6 wavelet: (a) the seismic section (considered as resolution V_{11}), (b), (d), (f), and (h) are the resolutions V_{10} , V_9 , V_8 , and V_7 , respectively; (c), (e), (g), and (i) are the resolutions W_{10} , W_9 , W_8 , and W_7 , respectively.

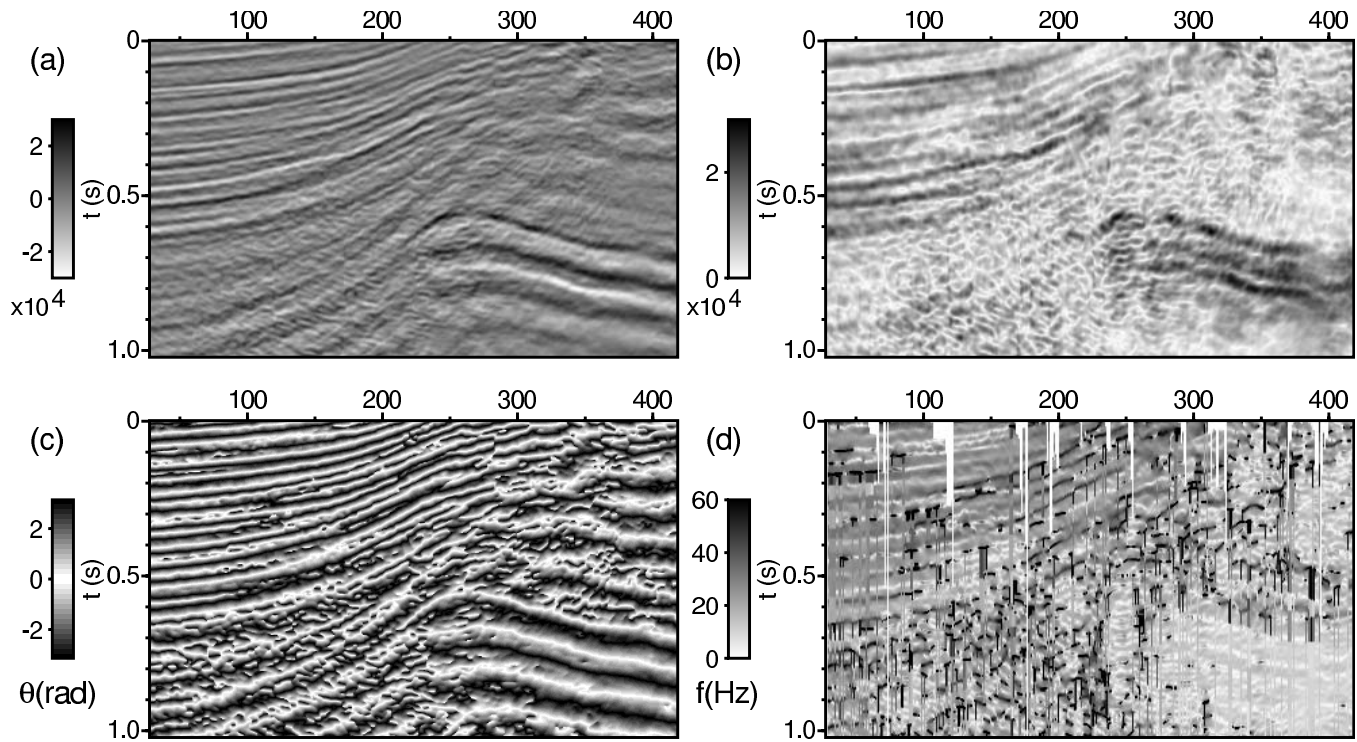


Fig. 3. Conventional seismic attributes for the seismic section: (a) the seismic section, (b) envelope amplitude, (c) instantaneous phase, and (d) instantaneous frequency.

verse fault. The behavior of the attributes (amplitude, phase, instantaneous frequency) suggests that the upper left portion may be acting as a trap for gas.

CONCLUSIONS

The performance of discrete wavelet-based multiresolution analysis was illustrated with a window of a real seismic section. The high amplitude and wavelength reflections from the second group featuring the seismic window seem to continue towards the left with less clear and lower amplitude reflections. Production of oil and gas is expected in the domain of this second group (between 0.6 and 0.8 s, and between traces 240 and 300). Based on multiresolution analysis from the seismic window, or from the seismic attributes themselves, it was not possible to rule out the continuity of this group of reflections.

However, the resolutions at different scales provided by a multiresolution analysis of the seismic attributes enabled a finer analysis. The continuity, suggested in the multiresolution analysis of the seismic section and of the seismic attributes themselves, was not fully supported by the multiresolution analysis of the phase attribute. It was possible to distinguish with some confidence between the high-amplitude continuous reflections and their continua-

tions, characterized by high-to-medium frequency, piecewise, low-amplitude reflections. This difference, and the fact that the high-amplitude reflections featured low frequency shadows, suggested the existence of oil in the respective layers. This finding was confirmed by well data.

The frequency range at which a given reflection is dominant is very useful because it enables us to study the frequency-dependent attenuation, by analysing a given reflection at different resolutions. For example, the high amplitude reflections were clear around 10 Hz. Also, low-frequency shadows may be associated with hydrocarbon-related bright spots (Castagna *et al.*, 2002). Multiresolution may help detect the presence of such low frequency shadows at a given frequency range.

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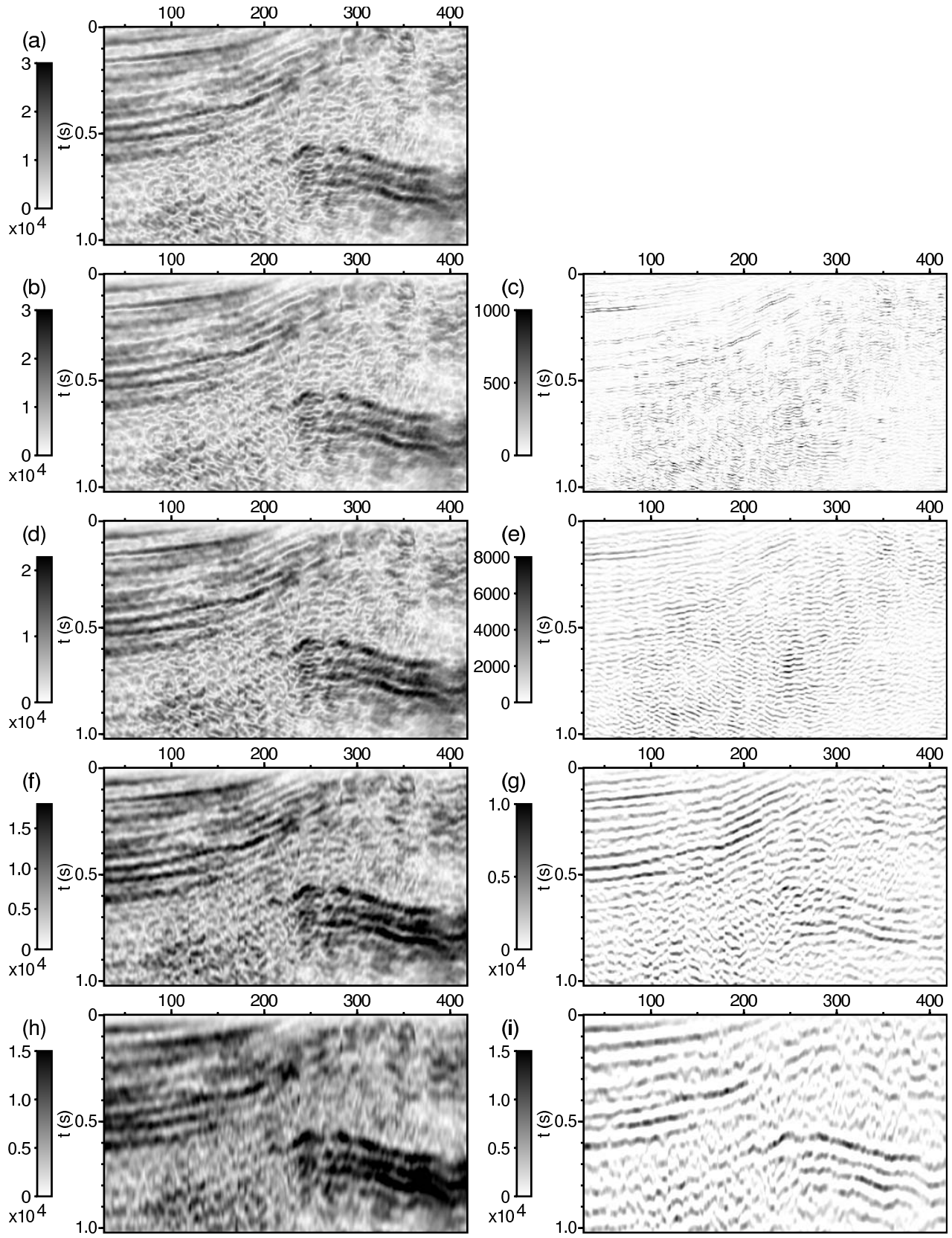


Fig. 4. Multiresolution analysis for the envelope amplitude of the section, based on the Daubechies of order 6 wavelet: (a) the envelope amplitude, (b), (d), (f), and (h) are the resolutions V_{10} , V_9 , V_8 and V_7 , respectively; (c), (e), (g), and (i) are the resolutions W_{10} , W_9 , W_8 and W_7 , respectively.

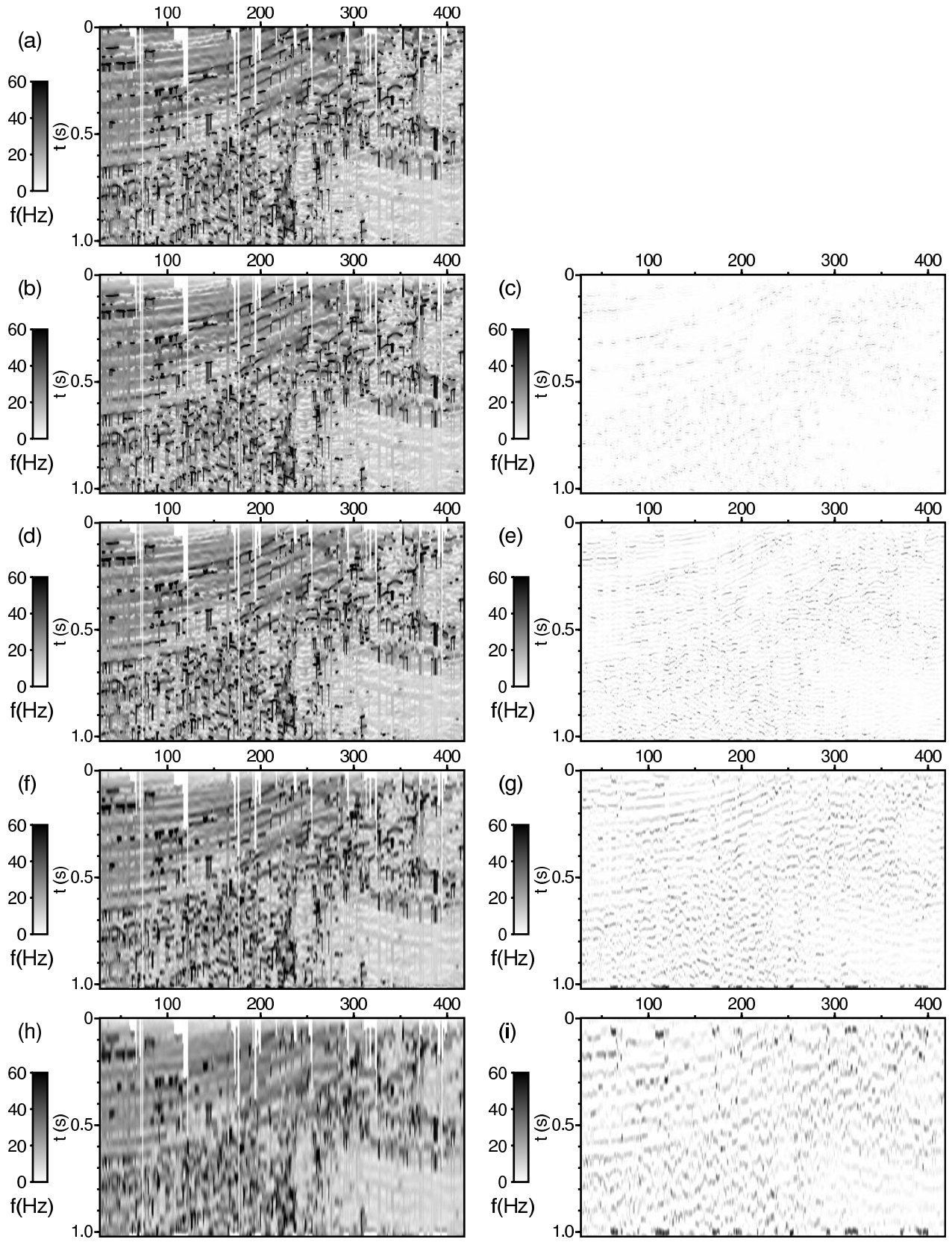


Fig. 5. Multiresolution analysis for the instantaneous frequency of the section, based on the Daubechies of order 6 wavelet: (a) the instantaneous frequency, (b), (d), (f), and (h) are the resolutions V_{10} , V_9 , V_8 and V_7 , respectively; (c), (e), (g), and (i) are the resolutions W_{10} , W_9 , W_8 and W_7 , respectively.

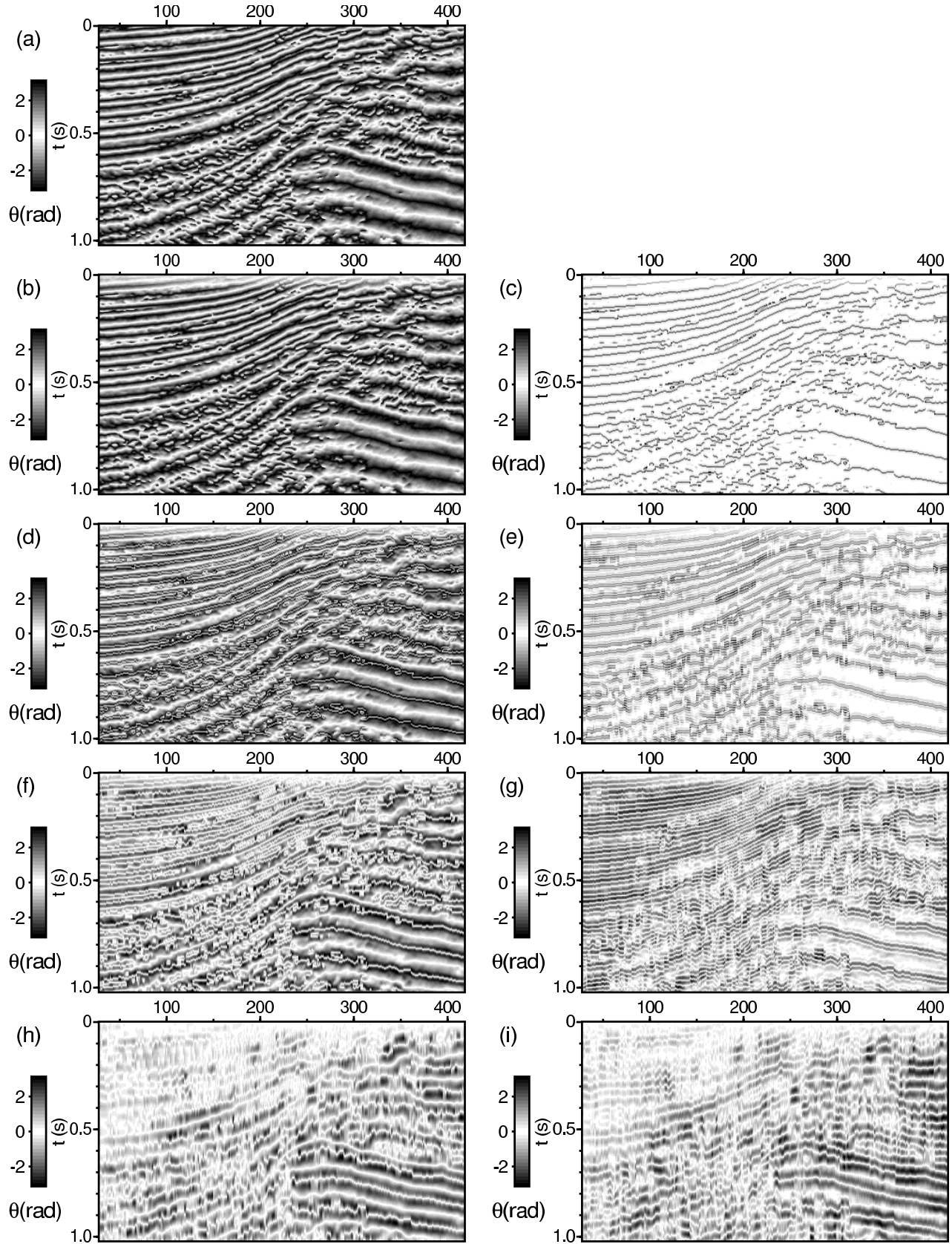


Fig. 6. Multiresolution analysis for the instantaneous phase of the section, based on the Daubechies of order 6 wavelet : (a) the instantaneous phase, (b), (d), (f), and (h) are the resolutions V_{10} , V_9 , V_8 and V_7 respectively; (c), (e), (g), and (i) are the resolutions W_{10} , W_9 , W_8 and W_7 , respectively.

APPENDIX A: INFORMAL INTRODUCTION TO WAVELET TRANSFORM AND MULTIREOLUTION ANALYSIS

The wavelet transform enables, in contrast with the Fourier transform, a time frequency analysis of a given function (Daubechies, 1992; Strang and Nguyen, 1996). The continuous wavelet transform of $f(t)$ is the integral transform

$$Wf(\lambda, t) = \int_{-\infty}^{\infty} f(u) \frac{1}{\sqrt{\lambda}} \psi\left(\frac{u-t}{\lambda}\right) du, \quad \text{for } \lambda > 0, \quad \text{A1}$$

The function $\psi(t)$ is a wavelet, t is a time localization parameter, and λ is a scale parameter.

Wavelets are introduced by the definition of the continuous wavelet transform, eq. A1. In order to obtain localization in time, wavelets must have unit energy, compact support, or sufficiently fast decay, and zero mean. Some wavelets are defined analytically (Table 1), other are given numerically (i.e., Daubechies order six wavelet, Table 2). The meaning of the lambda and t parameters is as follows. By varying λ we can compress or dilate the original wavelet, while we can translate it by varying t . In this way low and high frequencies can be selectively analyzed by a wavelet (i.e., features at different scales). The wavelets $\psi(t)$, $\psi(t-1)$, $\psi(t-2)$, etc., obtained by varying the time localization parameter (and holding $\lambda = 1$) constitute a base for a function space W_j . In a general way, by varying λ we obtain additional function spaces W_{j-1} , W_{j-2} , W_{j-3} , etc., where

$$w_j(t) = \sum_l b_l^j \psi(2^j t - l)$$

and $w_j \in W_j$. At this point, it is useful to consider the normalized wavelet localized in time b and scale λ as follows

$$\psi_{\lambda, b} = |\lambda|^{-1/2} \psi\left(\frac{t-b}{\lambda}\right).$$

The scaling functions $\varphi(t)$, in a similar way give rise to function spaces V_j , V_{j-1} , V_{j-2} , etc., orthogonal complements respectively of W_j , W_{j-1} , W_{j-2} , etc. In these spaces we have

$$v_j(t) = \sum_l a_l^j \varphi(2^j t - l),$$

where $v_j \in V_j$ and $V_j \subset V_{j+1}$.

Multiresolution analysis enables to represent a function in different grades of resolution based on a scaling function $\varphi(t)$ and a wavelet $\psi(t)$. Suppose we have a function $v_{j+1}(t)$ generated by the scaling function

$$v_{j+1}(t) = \sum_l h_l \varphi(2^{j+1} t - l)$$

It can be considered as a fine resolution representation of $v_j(t)$. In general a function can be represented as a combination of an average or coarse scale function $v_j(t)$ plus a detail function $v_j(t)$

$$v_j(t) = v_{j-1}(t) + w_{j-1}(t).$$

In a recursive way we have

$$v_j(t) = v_{j-1}(t) + w_{j-1}(t) + w_{j-2}(t)$$

and so on. The mechanism to practically calculate the coefficients is the fast wavelet transform. These relations are based on the dilation equation

Table A1

Example of wavelets

Name	Formula
Haar (defined in time)	$\psi(t) = \begin{cases} 1, & 0 \leq t < 1/2 \\ -1, & 1/2 \leq t < 1, \\ 0, & \text{otherwise.} \end{cases}$
Mexican hat (defined in time)	$\psi(t) = \frac{2}{3} \pi^{-1/4} (1-t^2) e^{-t^2/2}$
Morlet (defined in frequency)	$\psi(t) = \pi^{-1/4} (e^{-i\omega_0 t} - e^{-i\omega_0^2/2}) e^{-t^2/2}$

$$\varphi(t) = \sqrt{2} \sum_l h_l \varphi(2t - l), \quad \text{A2}$$

and the wavelet equation

$$\psi(t) = \sqrt{2} \sum_l g_l \varphi(2t - l). \quad \text{A3}$$

Equations A2 and A3 relating the wavelet (scaling function) at two different scales. Multiresolution analysis let us to study a given function at several scales.

Table A2

Coefficients of Daubechies six order lowpass filter

$$\sum h_n e^{-in\omega}$$

h_0	0.33267055295000825
h_1	0.8068915093110924
h_2	0.4598775021184914
h_3	-0.1350110200102546
h_4	-0.0854412738820267
h_5	0.0352262918857095

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