

# Estimation of geographical coordinates using a differential GPS for short occupation times

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## RESUMEN

Una base corta de dos estaciones fue medida tres veces de manera repetitiva durante un período de tres días del 21 al 23 de marzo para determinar la precisión del componente vertical utilizando un Sistema de Posicionamiento Global Diferencial. Los tiempos de ocupación duraron de 270-600 minutos (de cuatro a diez horas). La ecuación que mejor define esta línea para el componente vertical es  $y=0.6397t^{2.009}$ , donde  $y$  es el error en metros y  $t$  es el tiempo en minutos.

**PALABRAS CLAVE:** Sistema de Posicionamiento Global, estimación de precisión, Yucatán, México.

## ABSTRACT

A short baseline consisting of two stations was measured three times during a three-day period March 21-23 to determine the precision of the vertical component using a differential Global Positioning System. Occupation times ranged from 270 to 600 minutes (4 to 10 hours). The power equation that best fits the regression for the vertical component is given by  $y=0.6397t^{2.009}$ , where  $y$  is the error in meters and  $t$  is time in minutes.

**KEY WORDS:** Differential Global Positioning System; precision estimates; short occupation times.

## INTRODUCTION

Instituto Nacional de Estadística, Geografía e Informática (INEGI) has a first-order network of benchmarks in Mexico (INEGI, 2000). In addition, they have produced a digital terrain model for Mexico. The latter has errors that can be as high as tens of meters (S. Peña, oral com.). Vertical control is essential for the geosciences, including hydrogeology. Marín and others (1998) found that the vertical errors using a hand-held GPS are greater than 100 meters. The Global Positioning System has been used extensively in geodesic issues, such as crustal deformation and plate tectonics (Cabral-Cano, in review). Geodetic surveys typically call for permanent stations, or use long occupation times. Engineering studies, on the other hand, typically use very short occupation times. In this paper, we discuss at the precision of GPS for geographical coordinates, in particular, the vertical component for short occupation times.

In this study we determine (1) the precision of latitude, longitude and ellipsoidal height determinations from GPS measurements using short occupation times; (2) the ratio of errors between latitude/longitude and ellipsoidal height determinations, (3) the optimal occupation time for a given error margin for ellipsoidal height determinations.

A Differential Global Positioning System Total Station was used which consists of a base receiver (Trimble model 4700) and a rover (Trimble model 4800). Both receivers are dual frequency receivers. For the base receiver, a tripod with three preset heights was used (1.20, 1.50, and 1.80 m).

The baseline between both receivers was 3.84 meters. The control point consists of a benchmark cemented on-site, and tied into INEGI's Toluca control station. This benchmark was surveyed using a level, and tied into the first order survey conducted by the Departamento del Distrito Federal. Five measurements were conducted with the base and rover receivers occupying the same station. A total of five different measurements (Table 1) were conducted during the following dates: March 21-23. Occupation times ranged from 240-600 minutes (four to ten hours, respectively). Data logged in the receivers were downloaded to a computer for post-processing after a measurement period. The Trimble Geomatics Office (TGO, 1999) software was used to process the data. Predicted orbits obtained from the satellites were used.

## RESULTS AND DISCUSSION

The mean values of latitude, longitude and ellipsoidal height were post-processed. Table 1 gives the number of

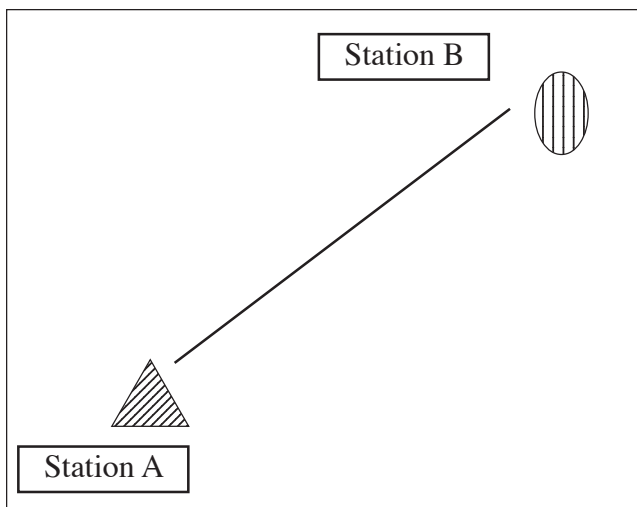


Fig. 1. DGPS baseline used for study.

measurements for each segment, and the mean, minimum and maximum values for the latitude, longitude and ellipsoidal height.

Figures 2 and 3 show the maximum difference in meters for the latitude and longitude as a result of different occupation time intervals. Figure 4 shows the maximum difference in ellipsoidal height as a function of occupation time. The measurements in all three figures correspond to 8, 15, 30, 60, 120, and 180 minutes, respectively. Differences between the minimum and maximum values are graphed. For each geographical coordinate (latitude, longitude, and ellipsoidal height) the values were sorted individually, and are presented as independent measurements.

For latitude, a maximum error of less than 25 mm is obtained for both the eight and 30 minute occupation time interval. The fifteen minute interval has a maximum error of ten millimeters, and the measurements for 60 minutes and larger time intervals have a lower error.

For longitude, a maximum error of 23 mm is obtained for the eight minute occupation time interval. For both the 15 and 30 minute intervals, the maximum difference is on the order of 18 mm. A 60 minute occupation time yields a maximum error of 11 mm, and the maximum error drops to five mm for an occupation time of 180 minutes.

For the ellipsoidal height determination, a continuous drop in the maximum error is observed, with a maximum difference of almost 600 mm for an eight minute occupation time. The maximum difference in ellipsoidal height is

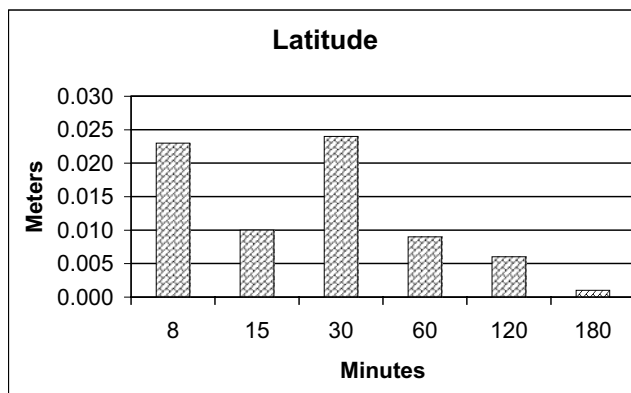


Fig. 2. Latitude measurements as a function of occupation time.

Table 1

Time, minutes	total vectors	solved vectors				
			mean	std dev		
8	30	24	mean	2,131,005.876	485,975.172	2,272.706
			std dev	0.026	0.011	0.169
15	30	26	mean	2,131,005.878	485,975.172	2,272.717
			std dev	0.035	0.000	0.082
30	25	19	mean	2,131,005.877	485,975.175	2,272.714
			std dev	0.029	0.013	0.104
60	12	8	mean	2,131,005.875	485,975.173	2,272.733
			std dev	0.000	0.006	0.058
120	7	5	mean	2,131,005.877	485,975.173	2,272.726
			std dev	0.031	0.000	0.018
180	4	3	mean	2,131,005.877	485,975.175	2,272.727
			std dev	0.000	0.008	0.015

18 mm, 10 mm, four mm, two, and one mm, for the 15, 30, 60, 120, and 180 occupation time intervals. The equation that describes these results is given by  $y = 0.6397x^{-2.009}$  (with an  $R^2 = 0.9674$ ), where  $y$  is the error in meters, and  $x$  is the time occupation in minutes.

When the DGPS is triangulating to obtain a fix on its position, the initial positions are not very accurate. Figure 5 shows how the initial vectors improve with time. As the occupation time is increased, the solution becomes more and more stable. This may explain the wander in the latitude and longitude solutions. The issue is what should be the acceptable error. Figures 2-4 quantify the error bars as a function of occupation times. Both the latitude and longitude determinations show a slight increase in maximum error for the 30 minute determination. However, the errors associated with the vertical dimension are one order of magnitude higher. The maximum error for the latitude and longitude are 22 and 23 millimeters, respectively, whereas the maximum error for the ellipsoidal height determination is 58 centimeters. A rule-of-thumb is that the vertical error is twice as large as the latitude or longitude error, but our results show that the vertical error is 25 times greater than the latitude or longitude error. Thus, for the rest of this discussion, we focus on the vertical dimension.

Ellipsoidal height determinations are the first step towards using a DGPS for orthometric surveying (determining the elevation of the ground surface above mean sea level, for example). Figure 3 shows a best fit line to determine the ellipsoidal height error for a given occupation interval.

Figure 5 (a-c) shows solutions for eight, 15, 30, 60, 120, 180, 240, 300, 360, and maximum occupation times in minutes. All solutions shown in this figure passed the quality control tests in the Trimble Geomatic Software. The figure shows how the measurement with the smallest occupation interval is the least precise. With increasing occupation times, the solution tends towards a stable value. For the first measurement, there are almost ten centimeters difference between the eight minute and the thirty minute occupation in-

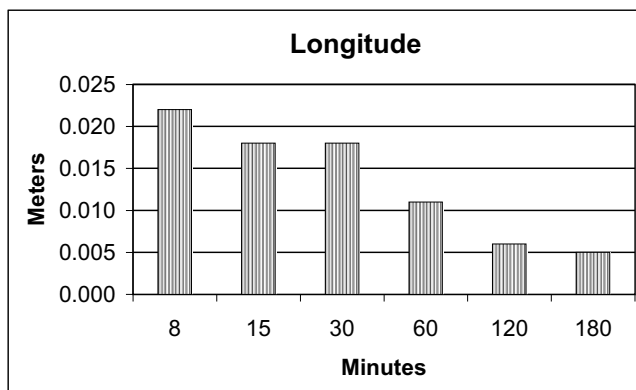


Fig. 3. Longitude measurements as a function of occupation time.

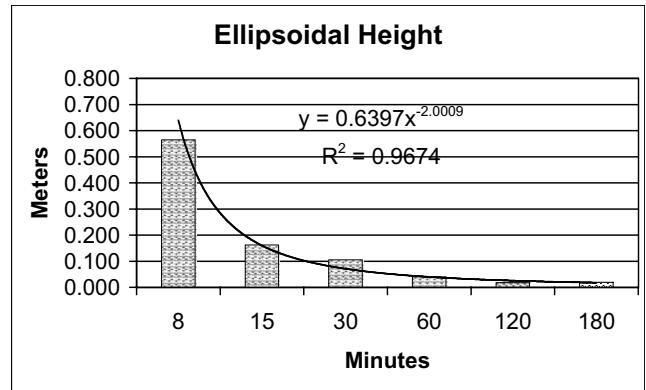


Fig. 4. Ellipsoidal height measurements as a function of occupation time.

terval. For the second case, there is a difference of more than 15 cm between the eight and fifteen minute occupation intervals. For the third case, the difference between the eight and the fifteen minute interval is 50 cm. We conclude that a thirty minute occupation interval is the minimum time required to obtain a solution with a precision better than ten centimeters in ellipsoidal height determination.

### CONCLUSIONS

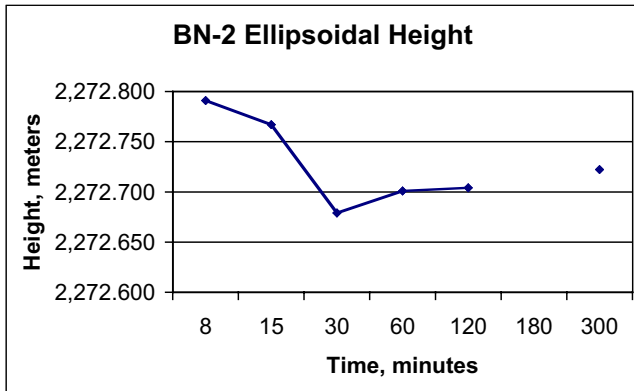
Several conclusions can be drawn from this experiment. First, fast-static surveys may not always record enough data to obtain a solution if the occupation time is eight minutes. If this technique is to be employed at all, the observation time should be at least 12 minutes to obtain reasonable latitude and longitude coordinates. However, if ellipsoidal height determination with a precision of ten centimeters is required, the occupation interval should be at least 30 minutes. This ought to help ensure that enough data is collected to secure that a DGPS solution without having to go back in the field.

Our short baseline was measured five different times with occupation times that ranged from 270 to 600 minutes. Three baselines were used for post-processing. The data from each measurement was post-processed using time intervals of eight to 240 minutes and a final measurement from 270 to 600 minutes. The precision for the ellipsoidal height was determined by taking the minimum/maximum values from the final time interval determinations, used as reference value.

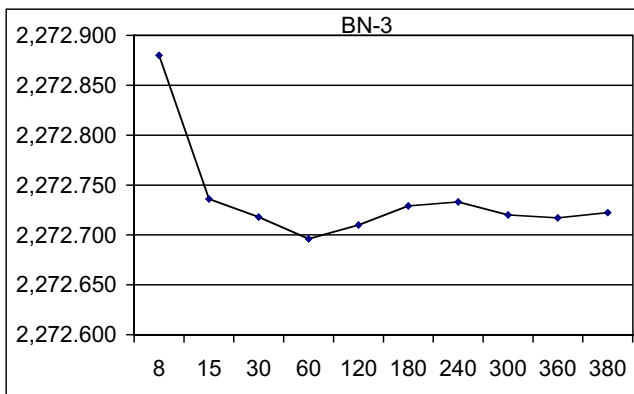
The standard deviation for the ellipsoidal height determinations for the eight, 15, 30, 60, 120, 180 and final times was: 0.17, 0.082, 0.10, 0.06, 0.018, and 0.015 meters, respectively. The line that describes this equation is:  $y = 0.6397x^{-2.009}$ .

The ratio of the error between the vertical ( $z$ ) and the horizontal coordinates ( $x,y$ ) is 1:25.

a)



b)



c)

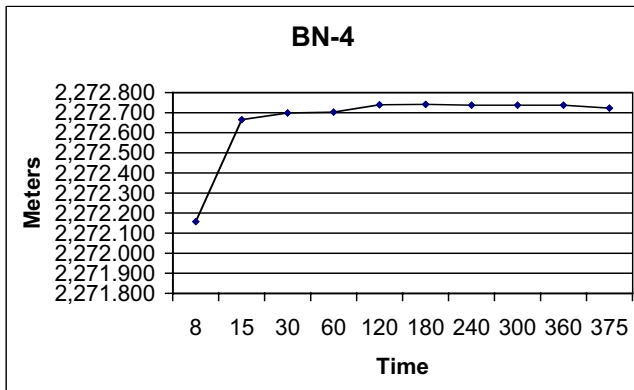


Fig. 5. a) BN-2 ellipsoidal height measurements as a function of occupation interval. b) BN-3 ellipsoidal height measurements as a function of occupation interval. c) BN-4 ellipsoidal height measurements as a function of occupation interval.

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