

## Hydrostratigraphy of Haftad Gholle Karst, Markazi province, Iran, optimized by Fuzzy Logic

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### Resumen

Este trabajo presenta la litología kárstica de la región de Haftad Gholle en el SE de Arak, Irán empleando el enfoque de lógica borrosa. El área fue dividida en 7 mitades litológicas. El resultado permitió identificar los recursos del agua óptimos en las calizas macizas del Cretácico Inferior.

**Palabras clave:** Karst, hidroestratigrafía, lógica borrosa, Arak.

### Abstract

The objective of this study is to optimize the karstic lithology in the Haftad Gholle area located of South-eastern Arak, NW Iran, using fuzzy logic. The karst of Haftad Gholle area was divided into seven lithologic units; information on effective properties including thickness, RQD (Rock Quality Designation), opening joints, joint distance, porosity, permeability, and bedding were obtained. Using Similarity to Ideal Solution approach, the data were transformed to fuzzy numbers, and the analysis was performed. The results showed that the best karstic water resources correspond to a unit which includes lower massive limestone karstic units (KL1) belonging to the Lower Cretaceous.

**Keywords:** Karst, Hydrostratigraphy, Fuzzy logic, Arak.

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## Introduction

A Hydrostratigraphic unit includes all or part of a petrologic set which is separated from other units by hydrogeologic properties (Maxey, 1964).

Seaber (1988) proposed that porosity and permeability are the most important feature in determining a hydrostratigraphic unit, but Van Wagoner (1988) involves other properties such as usual hydrodynamic coefficients in the aquifers to define the properties of a hydrostratigraphic unit. The study area is located 40 km to the southeast of Arak, city in the Markazi province (Figure 1).

Main water resources in the area include Anjadan springs, with a water discharge of about 20 liters per second, and a deep well drilled into the lower Cretaceous with a water discharge of about 10 liters per second.

This study has been done to survey the hydrostratigraphic units belonging to the karstic water resources of Haftad Gholle to the Southeast of Arak in Markazi Province, Iran. The best karstic water resources were determined by the Similarity to Ideal Solution method.

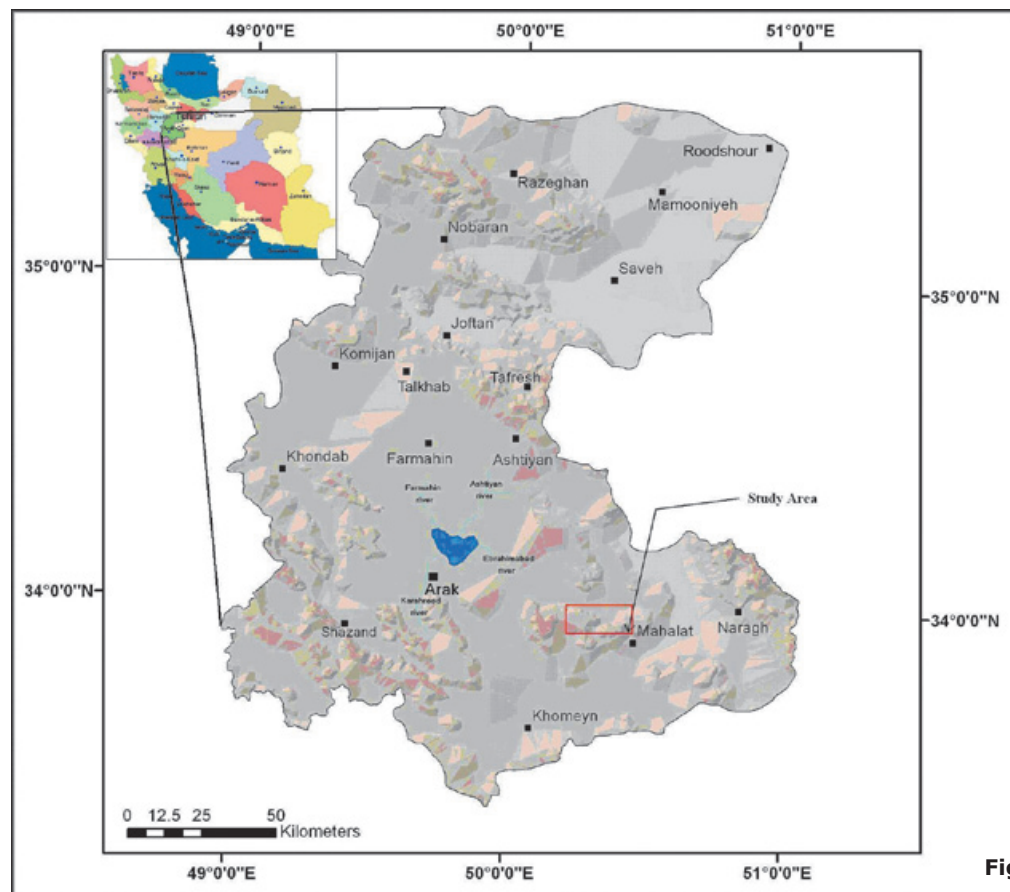
## Methodology

Usual methods were used to determine the lithostratigraphy and biostratigraphic properties. A Lugeon test was first performed in seven boreholes drilled in each lithologic unit to determine the permeability and other relevant hydrostratigraphical properties of the lithologic.

All boreholes were sampled to obtain the degree of effective porosity by estimating the amount of drainable water in the laboratory. As the karstic aquifer is open, the storage coefficient was actually determined by effective porosity and total porosity. Also, its water transfer capability of lithologic units were estimated by assuming full saturation from permeability, and thickness of each lithologic unit. Finally, the type of aquifer per lithologic unit was identified using the available information.

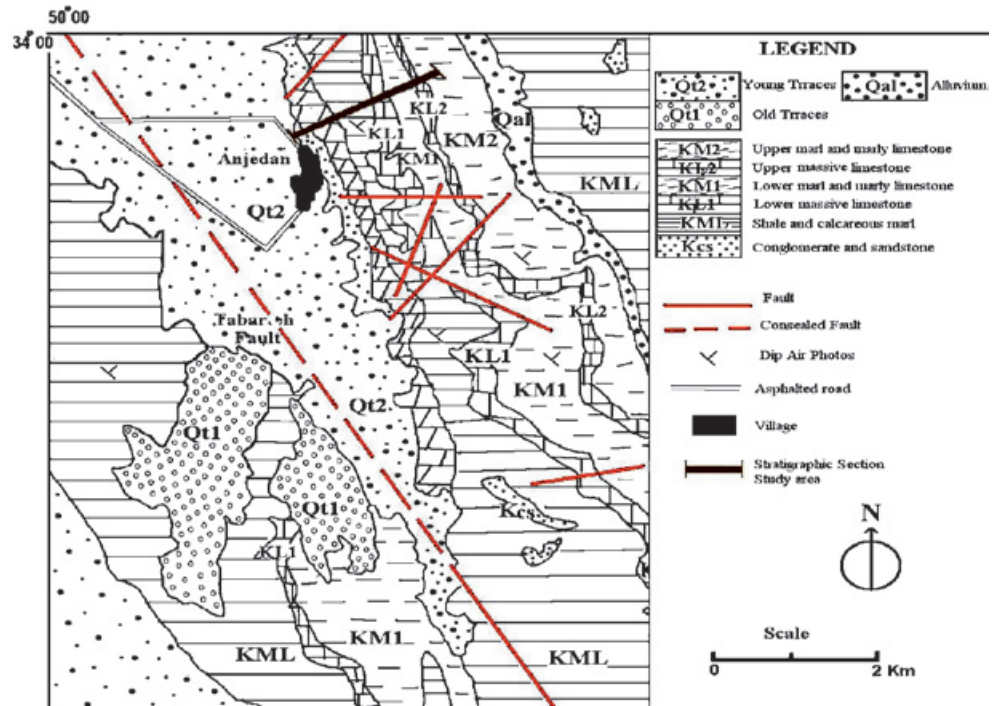
## Geology

From the geology viewpoint, the study area is located between two central geologic provinces; Iran and Sanandaj-Sirjan (Emami, 1992). The Tabarteh fault forms the border separating these two geologic provinces zones (Figure 2).



**Figure 1.** Location of the study area.

**Figure 2.** Geological map of the study area.



*Petrology*

A stratigraphic section location in (Figure 2) shows that in the study area, there age of the lower Cretaceous age outcropped and these units are subdivided into conglomerates and sandstones (Kcs); dolomites and limey dolomites (Kd); shales and calcareous marls (KML); lower massive limestones (KL1); lower marls and marly limestones (KM1); upper massive limestones (KL2); and upper marls and marly limestones (KM2) from top to bottom respectively.

The carbonate lithologic units are among the most important lithologic units because of more dissolubility, and they contain the principal underground water reserves in the area. The major stratigraphic units are shown in Figure 3.

*Structural Properties*

Structural features in southeastern Arak are considered determinant for the formation and development of the karst. Tabarteh fault activity has given rise to several sub-faults in the lithologic units (Figure 2).

This fault has also caused joints and fracture systems in lithologic units. Faulting and fracturing

are also effective in increasing the effective porosity and developing and accelerating the karst process.

This is evident from the tests of permeability and will not be mentioned further.

But there are other structural properties including thickness of layers, surface RQD,

**Figure 3.** Lithostratigraphic properties of the Cretaceous sequence.

Age	Thickness	Stratigraphic	Sample	Lithology
Cenomanian	400	[Stratigraphic column]	A10	Marl and marly limestone
			A9	Massive limestone
Albian	300	[Stratigraphic column]	A8	Marl and marly limestone
			A7	
			A6	Massive limestone
A5				
Aptian	200	[Stratigraphic column]	A4	Shale and marly limestone
			A3	
Barremian	100	[Stratigraphic column]	A2	Dolomite and dolomitic limestone
			A1	Conglomerate and sandstone
	0	[Stratigraphic column]		

opening joints, joint distance, porosity, permeability, bedding, which have been effective in the formation and development of a karstic aquifer in the area.

The greater the thickness of lithologic units with karstic capability the larger the aquifer (Seaber, 1988). Table 1 summarizes these properties for the lithologic units in the study area.

*Hydraulic Properties*

The most important purpose of the study is to identify the hydraulic properties of the lithologic units.

This section of the study attempts to identify the hydrostratigraphic unites and determine the role of main lithologic units in forming the karstic aquifer. This provides the best data to classify the hydrostratigraphic units for their contribution in formation of karstic aquifer (Van Wagoner, 1990).

The hydraulic properties in each lithologic units were determined by Lugeon tests and other required tests (Agassi and Afrasiabi, 2004).

Results of these experiments for the lithology of the cretaceous sequence have been presented in Table 1.

According to Table 1, lithologic units KL1 and KL2 are distinguished other units, so that is hydrostratigraphic unit has very good structural and hydraulic properties to form the karstic aquifer in the area.

Finally, we determine the best karstic water resources storage units using the Similarity to Ideal Solution method (TOPSIS).

This method is a popular approach to multiple criteria decision making (MCDM). It has been widely used in the literature. TOPSIS is a technique for order preference by similarity to ideal solution

proposed by Hwang and Yoon(1981), Agrawal *et al.*(1991), Lai, *et al.*(1994), Kim *et al.*(1997), Parkan and Wu,(1997), Zanakis *et al.*(1998), Deng, *et al.*(2000), Jee and Kang(2000), Feng and Wang(2001), Cheng *et al.*(2003), Liao(2003), Olson(2004), Opricovic and Tzeng(2004), Abo-Sinna and Amer(2005),Tzeng, *et al.* (2005).

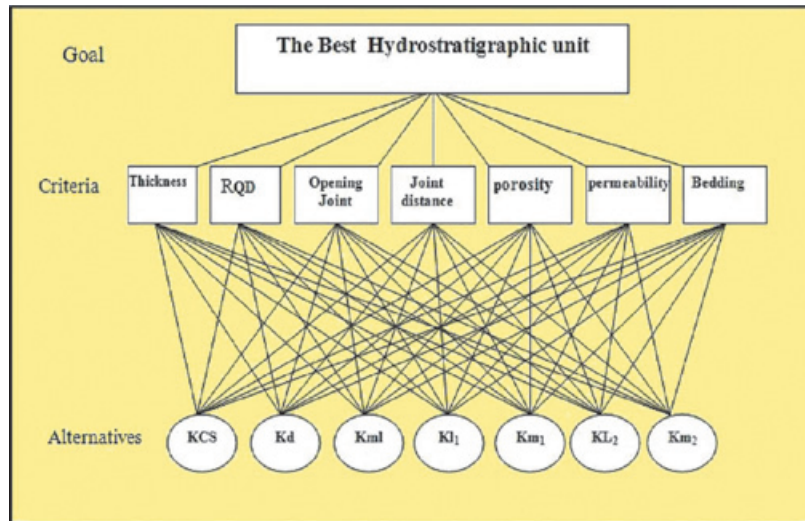
The Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS) (Triantaphyllou and Lin, 1996) is a fuzzy multi-objective decision technique based on simple geometric concepts: the best alternative exhibits the shortest distance from the Best Ideal Solution (BIS) and the farthest distance from the Worst Ideal Solution (WIS) in a Euclidean sense. Just as the LW method, this approach requires as input data the mxn decision matrix **DM** and the weights  $w_j$  with  $j=1, \dots, n$  measuring the criteria importance. Moreover, a fuzzification process associates to each value  $dm_{ij}$  of **DM** a fuzzy value  $d'_{ij}$ , with  $0 \leq d'_{ij} \leq 1$ , defining the mxn fuzzified decision matrix **D'**, depending on the user satisfaction degree with respect to the criteria.

There are seven called alternative hydrostratigraphic units to select the best aquiferous layer, where the hydrostratigraphic units are conglomerate and sandstone (Kcs); dolomite and limey dolomite (Kd); shale and calcareous marl (KML); lower massive limestone (KL1); lower marl and marly limestone (KM1); upper massive limestone (KL2); upper marl and marly limestone (KM1) from lower to upper respectively.

We want to select one of these seven units based on seven criteria (thickness, surface RQD, opening joints, joints distance, porosity, permeability, and bedding). The thickness, porosity, permeability, and bedding criteria have positive aspects and the surface RQD, opening joints and joints distance criteria have negative aspects, where the hierarchical structure for the decision is shown in Figure 4.

**Table 1.** Structural and Hydraulic properties of lithologic units in the study area.

KM2	KL2	KM1	KL1	KML	Kd	KCS	Lithologic unit Properties
55	25	115	105	36	45	123	Thickness
69.3	42.6	59.3	41.5	68.2	48.2	61.2	RQD
2	6	3	7	3	4	4	Opening Joint
15	40	20	31	20	40	15	Joint distance
18.7	32.1	19.1	37.6	15.7	14.8	13.1	porosity
3.06	11.49	2.59	10.31	6.72	3.7	4900	permeability
7	105	20	105	20	65	65	Bedding



**Figure 4.** The Hierarchical structure for selecting the best aquiferous layer.

**Step 1: Decision matrix and weights vector of criteria**

The alternatives have been evaluated by different criteria and the results as decision matrix is presented as Table 2.

**Table 2.** The results of evaluated criteria (Decision Matrix).

							Criteria
Bedding	Permeability	Porosity	Joint distance	Opening Joint	RQD	Thickness	Alternative
Thick layer	With high permeability	Very good	high	Very high	Average	high	Kcs
Thick layer	With high permeability	Average	Average	Very high	Weak	Average	Kd
Middle layer	With very high permeability	good	high	Very high	Average	Average	KML
Mass	With very high permeability	Very good	Average	high	Weak	Very high	KL1
Middle layer	With high permeability	good	high	Very high	Average	high	KM1
Mass	With very high permeability	Very good	Average	high	Weak	Average	KL2
Thin layer	With high permeability	good	high	Very high	Average	high	KM2

The weight vector of criteria is shown in Table 3.

**Table 3.** The weight vector of criteria.

Bedding	Permeability	Porosity	Joint distance	Opening Joint	RQD	Thickness	Criteria
Somewhat less important	Very important	Important	Indifferent	Somewhat important	Important	Somewhat less important	Weight

Tables 4 to 10 are used to form the fuzzy decision matrix and weight vector.

**Table 4.** The classification of lithologic units based on bedding.

Linguistic Variable	Bedding		Corresponding Fuzzy Number
	Qualitative	Qualitative	
Very important	>100 cm	Mass	(9,10,10)
Important	30-100	Thick layer	(7,9,10)
Somewhat important	10-30	Middle layer	(5,7,9)
Indifferent	3-10	Thin layer	(3,5,7)
Somewhat less important	1-3	Very thin layer	(1,3,5)
Low importance	0.3-1	Laminate	(0,1,3)
Very low importance	<0.3	Very Laminate	(0,0,1)

**Table 5.** The classification of lithologic units based on permeability.

Linguistic Variable	Permeability		Corresponding Fuzzy Number
	Qualitative	Qualitative	
Very low importance	< 0.001	Impermeable	(0,0,1)
Low importance	0.001-0.01	With low permeability	(0,1,3)
Somewhat less important	0.01-0.1	Permeable	(1,3,5)
Indifferent	0.1-1	With average permeability	(3,5,7)
Somewhat important	1-10	With high permeability	(5,7,9)
Important	10-100	With very high permeability	(7,9,10)
Very important	100-1000	With extremely high permeability	(9,10,10)

**Table 6.** The classification of lithologic units based on porosity.

Linguistic Variable	porosity %t	Corresponding Fuzzy Number
Very good	20-25	(9,10,10)
Good	15-20	(5,7,9)
Average	10-15	(3,5,7)
Weak	5-10	(1,3,5)
Very weak	0-5	(0,1,3)

**Table 7.** The classification of lithologic units based on thickness.

Linguistic Variable	Thickness	Corresponding Fuzzy Number
Very high	100 m More than	(9,10,10)
High	30-100 cm	(5,7,9)
Average	10 -30 cm	(3,5,7)
Low	3- 10 cm	(1,3,5)
Very low	Less than 3 cm	(0,0,1)

**Table 8.** The classification of lithologic units based on opening joints.

Linguistic Variable	Opening Joint	Corresponding Fuzzy Number
Very low	25	(9,10,10)
Low	20	(5,7,9)
Average	12	(3,5,7)
High	6	(1,3,5)
Very high	0	(0,1,3)

**Table 9.** The classification of lithologic units based on surface RQD.

Linguistic Variable	RQD	Corresponding Fuzzy Number
Very important	91-100	(9,10,10)
Important	76-90	(5,7,9)
Average	51-75	(3,5,7)
Weak	26-50	(1,3,5)
Very weak	0-25	(0,1,3)



**Table 10.** The classification of lithologic units based on joint distance.

Linguistic Variable	Joint distance	Corresponding Fuzzy Number
Very low	>3	(8,10,10)
Low	1-3	(6,7,8)
Average	0.3-1	(4,5,6)
High	0.5-0.3	(2,3,4)
Very high	0.05	(0,1,2)

Using the above tables, the fuzzy weight vector is obtained as Table 11:

**Table 11.** Fuzzy weight vector.

Bedding	permeability	porosity	Joint distance	Opening Joint	RQD	Thickness Alternative	Criteria
(7 <sup>th</sup> Criteria)	(6 <sup>th</sup> Criteria)	(5 <sup>th</sup> Criteria)	(4 <sup>th</sup> Criteria)	(3 <sup>th</sup> Criteria)	(2 <sup>nd</sup> Criteria)	(1 <sup>st</sup> Criteria)	weight
(0.1,0.3,0.5)	(0.9,1,1)	(0.7,0.9,0.1)	(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.7,0.9,0.1)	(0.1,0.3,0.5)	

Using the above tables, the fuzzy decision matrix is obtained (table 12).

**Table 12.** The results of fuzzy decision matrix.

Bedding	permeability	porosity	Joint distance	Opening Joint	RQD	Thickness	Criteria
(7 <sup>th</sup> Criteria)	(6 <sup>th</sup> Criteria)	(5 <sup>th</sup> Criteria)	(4 <sup>th</sup> Criteria)	(3 <sup>th</sup> Criteria)	(2 <sup>nd</sup> Criteria)	(1 <sup>st</sup> Criteria)	Alternative
(7,9,10)	(3,5,7)	(3,5,7)	(2,3,4)	(0,1,3)	(3,5,7)	(5,7,9)	Kcs
(7,9,10)	(5,7,9)	(3,5,7)	(4,5,6)	(0,1,3)	(1,3,5)	(3,5,7)	Kd
(5,7,9)	(5,7,9)	(5,7,9)	(2,3,4)	(0,1,3)	(3,5,7)	(3,5,7)	KML
(9,10,10)	(7,9,10)	(9,10,10)	(4,5,6)	(1,3,5)	(1,3,5)	(9,10,10)	KL1
(5,7,9)	(5,7,9)	(5,7,9)	(2,3,4)	(0,1,3)	(3,5,7)	(5,7,9)	KM1
(9,10,10)	(7,9,10)	(9,10,10)	(4,5,6)	(1,3,5)	(1,3,5)	(3,5,7)	KL2
(3,5,7)	(5,7,9)	(5,7,9)	(2,3,4)	(0,1,3)	(3,5,7)	(5,7,9)	KM2

**Step 2: Unscaling the decision matrix**

The first, Second, Third, fifth, sixth and seventh criteria have a positive aspect, and the following equation is used to unscaling the decision matrix.

$$\tilde{r}_{ij} = \left( \frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*} \right) \tag{1}$$

Where

$$c_j^* = \max_i c_{ij}$$

$a_{i,j}$  stands for the first component of vector in position column  $i$ , and row  $j$ ,  $b_{i,j}$  for the second component of the triade, and so forth.

For example, in the first row and column of the decision matrix, we have:

$$\tilde{r}_{22} = \left( \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \right) = (0.5, 0.7, 0.9)$$

The second, third and fourth criteria have a negative aspect, and following equation is used to unscaling the decision matrix.

$$\tilde{r}_{ij} = \left( \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}} \right) \tag{2}$$

Where

$$a_j^- = \min_i a_{ij}$$

For example, in the array for second row and fourth column of the decision matrix, we have:

$$\tilde{r}_{24} = \left( \frac{2}{6}, \frac{2}{5}, \frac{2}{4} \right) = (0.33, 0.4, 0.5)$$

Other arrays for the unscaled decision matrix are calculated by the same calculations that the results would be as table 13.

**Table 13.** The results of the unscaled decision matrix.

Bedding	Permeability	porosity	Joint distance	Opening Joint	RQD	Thickness	Criteria
							Alternative
(0.7,0.9,1)	(0.5,0.7,0.9)	(0.9,1,1)	(0.25,0.333,0.5)	(0.333,1,0)	(0.143,0.2,0.33)	(0.5,0.7,0.9)	Kcs
(0.7,0.9,1)	(0.5,0.7,0.9)	(0.3,0.5,0.7)	(0.166,0.2,0.25)	(0.333,1,0)	(0.2,0.333, 1)	(0.3,0.5,0.7)	Kd
(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.25,0.333,0.5)	(0.333,1,0)	(0.143,0.2,0.33)	(0.3,0.5,0.7)	KML
(0.9,1,1)	(0.7,0.9,1)	(0.9,1,1)	(0.166,0.2,0.25)	(0.2,0.33,1)	(0.2,0.333, 1)	(0.9, 1,1)	KL1
(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.25,0.333,0.5)	(0.333,1,0)	(0.143,0.2,0.33)	(0.5,0.7,0.9)	KM1
(0.9,1,1)	(0.7,0.9,1)	(0.9,1,1)	(0.166,0.2,0.25)	(0.2,0.333,1)	(0.2,0.333, 1)	(0.3,0.5,0.7)	KL2
(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.25,0.333,0.5)	(0.333,1,0)	(0.143,0.2,0.33)	(0.5,0.7,0.9)	KM2

**Table 14.** The results of the weighted unscaled decision matrix.

Bedding	Permeability	porosity	Joint distance	Opening Joint	RQD	Thickness	Criteria
							Alternative
(0.07,0.27,0.5)	(0.27,0.5,0.7)	(0.21,0.45,0.7)	(0.075,0.166,0.35)	(0.166,0.7,0)	(0.1,0.18,0.333)	(0.05,0.21,0.45)	Kcs
(0.07,0.27,0.5)	(0.45,0.7,0.9)	(0.21,0.45,0.7)	(0.05,0.1,0.175)	(0.166,0.7,0)	(0.14,0.30, 1)	(0.03,0.15,0.35)	Kd
(0.5,0.21,0.45)	(0.45,0.7,0.9)	(0.35,0.63,0.9)	(0.075,0.166,0.35)	(0.166,0.7,0)	(0.1,0.18,0.333)	(0.03,0.15,0.35)	KML
(0.09,0.3,0.5)	(0.63,0.9,1)	(0.63,0.9,1)	(0.05,0.1,0.175)	(0.1,0.233,0)	(0.14,0.30, 1)	(0.09,0. 3,0.5)	KL1
(0.5,0.21,0.45)	(0.45,0.7,0.9)	(0.35,0.63,0.9)	(0.075,0.166,0.35)	(0.166,0.7,0)	(0.1,0.18,0.333)	(0.05,0.21,0.45)	KM1
(0.09,0.3,0.5)	(0.63,0.9,1)	(0.63,0.9,1)	(0.05,0.1,0.175)	(0.1,0.233,0)	(0.14,0.30, 1)	(0.03,0.15,0.35)	KL2
(0.03,0.15,0.35)	(0.45,0.7,0.9)	(0.35,0.63,0.9)	(0.075,0.166,0.35)	(0.166,0.7,0)	(0.1,0.18,0.333)	(0.05,0.21,0.45)	KM2



**Step 3: Obtain the weighted unscaled decision matrix**

The method of calculating a few arrays of the matrix is as follows:

$$\tilde{v}_i = \tilde{r}_{ij} \cdot \tilde{w}_j \quad (3)$$

$$\tilde{V}_{ij} = [\tilde{v}_{ij}]_{m \times n} \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (4)$$

Where  $w_j, j=1, 2, \dots, n$  is the importance degree of each criterion and  $v_{ij}$  is Weighted normalized matrix element.

For example for  $v_{17}$  (Kcs Thickness) and  $v_{23}$ (KML Permeability):

$$\tilde{v}_{17} = (0.5, 0.7, 0.9)(0.1, 0.3, 0.5) = (0.05, 0.21, 0.45)$$

$$\tilde{v}_{23} = (0.5, 0.7, 0.9)(0.9, 1, 1) = (0.45, 0.7, 0.9)$$

Other arrays for the unscaled decision matrix are calculated by the same calculations that the results would be as Table 14.

**Step 4: Calculate the ideal (A\*) and anti-ideal (A-) alternatives**

The method of calculating a few arrays for the matrix is as follows:

$$\tilde{V}_j^* = \max_i \{ \tilde{V}_{ij} \} \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (5)$$

$$\tilde{V}_j^- = \min_i \{ \tilde{V}_{ij} \} \quad i = 1, 2, \dots, m, j = 1, 2, \dots, n \quad (6)$$

For example for  $v_7^*$  (The maximum amount of Thickness) and  $\tilde{v}_7^-$  (The minimum amount of Thickness):

$$v_7^* = \left( \begin{matrix} \max(0.45, 0.35, 0.35, 0.5, 0.45, 0.35, 0.45), (0.45, 0.35, 0.35, 0.5, 0.45, 0.35, 0.45) \\ (0.45, 0.35, 0.35, 0.5, 0.45, 0.35, 0.45) = (0.5, 0.5, 0.5) \end{matrix} \right)$$

$$\tilde{v}_7^- = \left( \begin{matrix} \min(0.05, 0.03, 0.03, 0.09, 0.05, 0.03, 0.05), (0.05, 0.03, 0.03, 0.09, 0.05, 0.03, 0.05) \\ (0.05, 0.03, 0.03, 0.09, 0.05, 0.03, 0.05) = (0.03, 0.03, 0.03) \end{matrix} \right)$$

Similar calculations have been done for other alternatives with results as follows:

$$\begin{aligned} v_2^* &= (1, 1, 1) \\ v_3^* &= (0.9, 0.9, 0.9) \\ v_4^* &= (0.7, 0.7, 0.7) \\ v_5^* &= (1, 1, 1) \\ v_6^* &= (1, 1, 1) \end{aligned}$$

$$\begin{aligned} v_7^* &= (0.5, 0.5, 0.5) \\ \tilde{v}_2^- &= (0.1, 0.1, 0.1) \\ \tilde{v}_3^- &= (0, 0, 0) \\ \tilde{v}_4^- &= (0.01, 0.01, 0.01) \\ \tilde{v}_5^- &= (0.21, 0.21, 0.21) \\ \tilde{v}_6^- &= (0.27, 0.27, 0.27) \\ \tilde{v}_7^- &= (0.03, 0.03, 0.03) \end{aligned}$$

The fuzzy ideal (A\*) and anti-ideal (A-) alternatives are defined as follows:

$$A^* = \{ \tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^* \} \quad (7)$$

$$A^- = \{ \tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^- \} \quad (8)$$

For example for A\* and A- :

$$A^* = [(0.05, 0.05, 0.05), (1, 1, 1), (0.7, 0.7, 0.7), (0.35, 0.35, 0.35), (1, 1, 1), (1, 1, 1), (0.5, 0.5, 0.5)]$$

$$A^- = [(0.03, 0.03, 0.03), (0.1, 0.1, 0.1), (0, 0, 0), (0.05, 0.05, 0.05), (0.21, 0.21, 0.21), (0.27, 0.27, 0.27), (0.03, 0.03, 0.03)]$$

**Step 5: Determine the distance of any alternative from the ideal and anti-ideal (S\* and S-) alternative and similarity attribute.**

The distance of first alternative from the fuzzy ideal alternative of each of the criteria is calculated as follows:

$$S_i^* = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*), \quad i = 1, 2, \dots, m \quad (9)$$

Where  $d_i$  represents the distance measurement between two fuzzy numbers. For example for S\*11 - S\*17 :

$$S_{17}^* = \sqrt{\frac{1}{3}[(0.05 - 0.5)^2 + (0.21 - 0.5)^2 + (0.45 - 0.5)^2]} = 0.32$$

$$S_{12}^* = 0.45, S_{13}^* = 0.70, S_{14}^* = 0.38, S_{15}^* = 0.582, S_{16}^* = 0.583, \text{ and } S_{17}^* = 0.28$$

As a result, the distance of first alternative from the fuzzy ideal alternative is equal to:

$$S_j^* = 0.28 + 0.58 + 0.58 + 0.38 + 0.70 + 0.45 + 0.32 = 3.3$$

The distance of first alternative from the fuzzy anti-ideal alternative of each of the criteria is calculated as follows:

**Table 15.** the values of any alternative distance from the ideal alternative.

$d(A_i, A^*)$	Bedding	permeability	porosity	Joint distance	Opening Joint	RQD	Thickness	S *
$d(A_1, A^*)$	0.28	0.58	0.58	0.38	0.70	0.45	0.32	3.3
$d(A_2, A^*)$	0.28	0.37	0.58	0.49	0.70	0.65	0.36	3.43
$d(A_3, A^*)$	0.31	0.37	0.44	0.38	0.70	0.45	0.36	3.01
$d(A_4, A^*)$	0.26	0.221	0.22	0.49	0.54	0.65	0.27	2.65
$d(A_5, A^*)$	0.31	0.366	0.44	0.38	0.70	0.45	0.32	2.97
$d(A_6, A^*)$	0.26	0.221	0.22	0.49	0.54	0.65	0.36	2.74
$d(A_7, A^*)$	0.36	0.366	0.44	0.38	0.70	0.45	0.32	3.02

**Table 16.** The values of any alternative distance from the anti-ideal alternative.

$d(A1, \bar{A})$	Bedding	permeability	porosity	Joint distance	Opening Joint	RQD	Thickness	S
$d(A_1, \bar{A})$	0.31	0.28	0.33	0.42	0.37	0.61	0.26	2.58
$d(A_2, \bar{A})$	0.31	0.45	0.33	0.16	0.37	0.39	0.19	2.2
$d(A_3, \bar{A})$	0.26	0.45	0.48	0.42	0.37	0.61	0.19	2.78
$d(A_4, \bar{A})$	0.31	0.76	0.65	0.42	0.57	0.39	0.33	3.17
$d(A_5, \bar{A})$	0.26	0.45	0.48	0.16	0.37	0.61	0.26	2.86
$d(A_6, \bar{A})$	0.31	0.76	0.65	0.42	0.57	0.39	0.19	3.03
$d(A_7, \bar{A})$	0.19	0.45	0.48	0.42	0.37	0.61	0.26	2.78

**Table 17.** The values of distance from the ideal and anti-ideal alternative and the similarity attribute for any alternative.

KM2	KI1	KM1	KI2	KM2	Kd	Kcs	
3.02	2.74	2.97	2.65	3.01	3.43	3.3	the distance of alternative from the ideal alternative
2.78	3.03	2.86	3.17	2.78	2.2	2.58	the distance of alternative from the anti-ideal alternative
0.479	0.525	0.491	0.545	0.481	0.391	0.439	similarity index

$$S_i^- = \sum_{j=1}^n (\tilde{v}_{ij}^-, \tilde{v}_j^-), \quad i = 1, 2, \dots, m \tag{10}$$

For example for  $S_{11}^- - S_{17}^-$  :

$$S_{17}^- = \sqrt{\frac{1}{3}[(0.05 - 0.03)^2 + (0.2 - 0.03)^2 + (0.45 - 0.03)^2]} = 0.263$$

$$\begin{matrix} S_{11}^- = 0.31, & S_{12}^- = 0.28, & S_{13}^- = 0.33, \\ S_{14}^- = 0.42, & S_{15}^- = 0.33, & S_{16}^- = 0.61 \end{matrix}$$

Consequently, the distance of first alternative from the fuzzy ideal alternative or the closeness coefficients(similarity index) of each supplier according to distance from the fuzzy positive-ideal solution (FPIS),  $S^*$  and the fuzzy negative-ideal solution (FNIS),  $S^-$ , can be calculated as follows:

$$CC_i = \frac{S_i^-}{S_i^* + S_i^-} \quad i = 1, 2, \dots, m \tag{11}$$

For example  $S_{11}^-$  and  $CC_1$  (For Conglomerate and Sandstone Unit or Kcs) is calculated as follows:

$$S_{11} = 0.31 + 0.28 + 0.33 + 0.42 + 0.37 + 0.61 + 0.26 = 2.58$$

$$CC_{1(Kcs)} = \frac{2.58}{3.3 + 2.58} = 0.439$$

Thus, the similarity attribute for the alternative is equal to:

$$\begin{matrix} CC_{2(Kd)} = 0.391, & CC_{3(KML)} = 0.481, & CC_{4(KL1)} = 0.545, \\ CC_{5(KM1)} = 0.491, & CC_{6(KL2)} = 0.525, & CC_{7(KML2)} = 0.479 \end{matrix}$$

Similar calculation have been done for the other alternatives, and the calculation results for the distance of any alternative from the ideal and anti-ideal alternative has been inserted in the Tables 15-17.

Each of the lithologic units whose its similarity attribute is better than the other units, has more potential unit to become the karstic aquifer. Thus, according to the results obtained from Table 17, the unit KL1 and KL2 are the best units to form the karstic aquifer, and the other units in the area have moderate and weak karstic aquifer conditions.

### Conclusions

The objective of present research is to identify the most developed karstic lithologic unit

of Haftad Gholleh area in South-eastern Arak in Iran, and the study has been done using the Similarity to Ideal Solution method.

This research indicates that the karst of Haftad Gholleh area can be classified into seven separate lithologic units which they are different by structural and hydraulic properties.

According to the research, the lower massive limestone or KL1 unit is the most important lithologic unit to form the karst of the area and after that, KL2 unit has been effective in forming the karst. Also, Base Lithologic unit of Cretaceous, include conglomerate and sandstone, the least karstic water reservoirs, does not play a important role to form the karst of Haftad Gholleh.

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