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Viscoelastic modeling and factor Q for reflection data

Vladimir Sabinin*

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Resumen

Los modelos numéricos viscoelásticos, basados en el concepto de mecanismos de dispersión, toman en cuenta las variables de memoria y algunos parámetros de dispersión, a saber los tiempos de relajación de tensión. En la práctica de la geofísica, el factor de calidad Q se usa normalmente para describir una propiedad de atenuación de medios viscoelásticos. Para el modelado numérico, es útil saber qué dependencia existe entre el factor Q y los tiempos de relajación.
En lugar de derivar la dependencia teóricamente, en el reciente trabajo, esta se evalúa de los resultados de un experimento numérico para la estimación del factor Q de datos sintéticos de reflexión. Para obtener los sismogramas sintéticos, un nuevo modelo 3D numérico de propagación de las ondas en medios viscoelásticos se desarrolló, difiriendo de los anteriores en que utiliza los valores medios de parámetros de relajación en los casos de mecanismos de dispersión múltiples y aplicando una nueva modificación del límite absorbente PML. Para la estimación del factor Q, se usaron métodos numéricos con la opción manual de ventanas espectrales. Estos métodos se adaptaron para los datos de reflexión. La fórmula desarrollada de la dependencia de Q en los tiempos de relajación es cualitativamente buena en la gama amplia de los tiempos de relajación.

Palabras clave: medios viscoelásticos, mecanismo de dispersión, tiempos de relajación, modelo numérico, PML, factor de calidad Q.

Abstract

Viscoelastic numerical models, based on a concept of dissipation mechanisms, take into account memory variables, and some dissipation parameters, namely stress and strain relaxation times. In geophysical practice, the quality factor Q is widely used for describing an attenuation property of viscoelastic media. For numerical modeling, it is useful to know what dependence exists between the factor Q, and the relaxation times.
Instead of deriving this dependence theoretically, in recent work, it is evaluated from results of numerical experiment for estimating the factor Q in synthetic reflection data. For obtaining the synthetic seismograms, a new 3D numerical model of wave propagation in viscoelastic media is developed, differing from previous ones by utilizing average values of relaxation parameters in cases of multiple dissipation mechanisms, and by applying a new modification of PML absorbing boundary. For the estimation of factor Q, numerical methods are used with manual choice of spectral windows. These methods are adapted for surface reflection data. The developed formula of the dependence Q on relaxation times is qualitatively good in a wide range of relaxation times.

Key words: viscoelastic media, dissipation mechanism, relaxation times, numerical model, PML, quality factor Q.

V. Sabinin*
Instituto Mexicano del Petróleo
Eje Central Lázaro Cárdenas 152
Col. San Bartolo Atepehuacan
C.P. 07730
México D.F., México
Corresponding author: vsabinin@yahoo.com

Introduction

Viscoelastic properties of oil-gas reservoirs cause high attenuation of seismic waves. Mathematical models of wave propagation in viscoelastic media can be useful for investigation of seismic attenuation in oil-gas reservoirs. The attenuation can be introduced into an elastic model in different ways. One of the most theoretically interesting is the method of dissipation mechanisms (Carcione *et al*., 1988; Robertsson *et al*., 1994; Xu and McMechan, 1998; Mikhailenko *et al*., 2003; Sabinin *et al*., 2003). It supposes that the viscoelastic property can be described by action of some dissipation mechanisms which are characterized by stress and strain relaxation times, and by type of interaction.

In the practice, it is more convenient to have lesser number of parameters for describing attenuation, for instance, one - the quality factor Q. Definition of the dependence between the relaxation times and the factor Q will facilitate an application of the models. It is known that the Q factor is nearly independent on frequency (Knopoff, 1964) in seismic spectrum, and it can be composed by some (>1) dissipation mechanisms with suitably fitted values of relaxation times (Emmerich, 1992; Blanch *et al*., 1995; Xu and McMechan, 1995, 1998). But nobody did comparison the composed input values of factor Q with the output values obtained from viscoelastic modeling. This issue may be also connected namely with a problem of correct definition of the factor Q.

Below, an attempt is made to derive the formula for dependence of the factor Q on relaxation times by estimating directly the factor Q output from synthetic seismograms computed by the viscoelastic numerical model. For the correct modeling, I revised the numerical model Virieux (1986), and Robertsson *et al*. (1994), and added to it my modification of the absorbing boundary layer by Collino and Tsogka (2001). Additionally, I developed a methodology of correct estimation of factor Q in surface reflection data for using this typical problem of seismology in numerical experiment.

Viscoelastic Model

In the linear theory of viscoelasticity, for the standard linear solid model of relaxation, stress σ_{ij} depends on strain ε_{ij} by the following modified Hooke's law:

$$\sigma_{ij} + \tau_\sigma \dot{\sigma}_{ij} = G_{ijkl}(\varepsilon_{kl} + \tau_\varepsilon \dot{\varepsilon}_{kl}) \qquad (1)$$

where G_{ijkl} - the stiffness tensor, τ_ε and τ_σ the strain and stress relaxation times.

Following Liu *et al*. (1976), one can derive the Boltzmann's after-effect equation directly from (1):

$$\dot{\sigma}_{ij} * \phi(t) = G_{ijkl}\varepsilon_{kl} \qquad (2)$$

where * denotes the convolution in time, and $\phi(t)$ is the specific creep function. For one dissipation mechanism,

$$\phi(t) = [1 - \tau \exp(-t/\tau_\varepsilon)]\theta(t) \qquad (3)$$

where θ - the Heaviside function, and $\tau = 1 - \tau_\sigma/\tau_\varepsilon$, provided $\tau_\varepsilon > \tau_\sigma$. For small $t << \tau_\varepsilon$, it corresponds to the hyperbolic creep function used by Lomnitz (1957).

By taking the time derivative of (2), and substituting the dependence of strain on the particle velocity

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\partial_i v_j + \partial_j v_i\right) \qquad (4)$$

one can get:

$$\dot{\sigma}_{ij} * \dot{\phi} = L_{\mathrm{ij}} \qquad (5)$$

where following Carcione (1993), and Xu and McMechan (1995) :

$$L_{ii} \equiv \left(\lambda + 2\mu\right)\partial_i v_i + \lambda \partial_j v_j \ , \ j \neq i \ ,$$
$$L_{ij} \equiv \mu\left(\partial_i v_j + \partial_j v_i\right) \ , \ i \neq j \ ,$$

where λ and μ - Lame elastic constants, {i=x,z; j=x,z} for two-dimensional viscoelastic media, and {i=x,y,z; j=x,y,z} for three-dimensional viscoelastic media.

By expanding in the convolution the time derivative of ϕ (see Robertsson *et al*., 1994), one can get from (5):

$$(1 - \tau)\dot{\sigma}_{ij} = L_{ij} - \tau r_{ij} \qquad (6)$$

where r_{ij} - the so-called memory variable:

$$\tau_\varepsilon r_{ij} = \exp(-t/\tau_\varepsilon)\theta(t) * \dot{\sigma} \qquad (7)$$

By taking the time derivative of (7), one can get the equation for the memory variable:

$$\tau_\sigma \dot{r}_{ij} + r_{ij} = L_{ij} \qquad (8)$$

Combining (6) and (8), one can obtain a more convenient form of (6):

$$\dot{\sigma}_{ij} = L_{ij} + (\tau_\varepsilon - \tau_o)\dot{r}_{ij} \tag{9}$$

Adding Newton's second law

$$\rho \dot{v}_i = \partial_j \sigma_{ij} \tag{10}$$

yields the system of equations (8)–(10) for the seismic wave propagation in the viscoelastic media, which governs the stress σ_{ij}, the particle velocity v_i, and the memory variable r_{ij} in the area of modeling for the case of one dissipation mechanism.

In the case of $\tau_\sigma = \tau_\varepsilon$, the system (8)–(10) becomes the system for elastic media.

By substituting $E_{ij} = \sigma_{ij} - (\tau_\varepsilon - \tau_\sigma) r_{ij}$, one can transform (9) into the equation for elastic media $\dot{E}_{ij} = L_{ij}$, and obtain from (8):

$$\tau_\sigma \dot{\sigma}_{ij} + \sigma_{ij} = \tau_\varepsilon \dot{E}_{ij} + E_{ij} \tag{11}$$

The equation (11) is similar to equation (1), and can be derived directly from it by defining $E_{ij} = G_{ijkl}\varepsilon_{kl}$. Therefore, the value *E* has a sense of the elastic part of the stress σ. Thus, the equation (11) governs an effect of viscosity, and it is an equation on only time.

Possibly, we ought to consider two sets of relaxation times: for shear and compressional waves, and modify the equations (8) and (9) by a way following Carcione (1995), for example. But Xu and McMechan (1995) simplified the problem by supposing equal relaxation times for shear and compressional waves. Another way of simplification is to consider cases of multiple dissipation mechanisms which can be transformed to the case of one dissipation mechanism, as below.

Extention of the model to *N* dissipation mechanisms

For the general case of *N* dissipation mechanisms which differ only by values of τ_ε, and τ_σ, we should write equations (1) and (2) for each n-th dissipation mechanism as follows:

$$\sigma_{nij} + \tau_{\sigma n}\dot{\sigma}_{nij} = G_{ijkl}(\varepsilon_{nkl} + \tau_{\varepsilon n}\dot{\varepsilon}_{nkl}) \tag{12}$$

$$\dot{\sigma}_{nij} * \phi_n(t) = G_{ijkl}\varepsilon_{nkl} \tag{13}$$

Values of total σ_{ij} and ε_{ij} over *N* dissipation mechanisms depend on scheme of interactions (interconnections) of the mechanisms: which mechanisms interact as parallel or as sequential, and which groups of mechanisms interact with other groups as parallel or sequential.

Really, the scheme of interactions of mechanisms is not known beforehand, and the problem of modeling does not need such fine developing.

For three simple cases below, it is possible to reduce the system of equations for *N* mechanisms to the case of one mechanism. They are: 1) *N* mechanisms interacting sequentialy, 2) *N* mechanisms interacting in parallel, and 3) two independent groups: one including N_1 mechanisms interacting sequentialy, and the second including N_2 mechanisms interacting in parallel, $N_1 + N_2 = N$. These are general enough cases.

For the case 1), the creep function can be represented as follows (for details see Appendix A):

$$\phi(t) = [1 - \tfrac{1}{N}\sum_{n=1}^{N} \tau_n \exp(-t/\tau_{\varepsilon n})]\theta(t) \tag{14}$$

If introduce the average values of relaxation times:

$$\bar{\tau}_\varepsilon = \tfrac{1}{N}\sum_{n=1}^{N}\tau_{\varepsilon n}, \quad \bar{\tau}_\sigma = \tfrac{1}{N}\sum_{n=1}^{N}\tau_{\sigma n}, \quad \bar{\tau} = 1 - \frac{\bar{\tau}_\sigma}{\bar{\tau}_\varepsilon},$$

then, from (5) with (14), one can derive (for details see Appendix B) similar equation to (6):

$$(1-\bar{\tau})\dot{\sigma}_{ij} = L_{ij} - \bar{\tau} R_{ij} \tag{15}$$

where R_{ij} is the average memory variable, which can be found from the following similar equation to (8):

$$\bar{\tau}_\sigma \dot{R}_{ij} + R_{ij} = L_{ij} \tag{16}$$

For the case 2), the equation (5) becomes as follows:

$$\dot{\sigma}_{ij} = \dot{\phi}_1 * L_{\mathrm{ij}} \tag{17}$$

in which the creep function is (see Appendix A, and Carcione, 1995)

$$\phi_1 = [1 + \tfrac{1}{N}\sum_{n=1}^{N}\tau_{1n}\exp(-t/\tau_{\sigma n})]\theta(t) \tag{18}$$

where $\tau_{1n} = \dfrac{\tau_{\varepsilon n}}{\tau_{\sigma n}} - 1$.

For the case 3), the equation (5) is valid for the first group, and the equation (17) is valid for the second group.

In the case 2), from (17) with (18), and in the case 3), from (5) with (14), and from (17) with (18), one can derive the same equations (15) and (16) for the stress, and for the average memory variable (see Appendix B).

Consequently, (if use is done the average memory variable and the average values of relaxation times), one may clearly see that the equations for the considered cases of *N* mechanisms (15)–(16) are equivalent to equations (6) and (8) for the case of one dissipation mechanism.

From this equivalence, it follows that, for a fixed geometry of the problem, the value of Q depends only on values of $\overline{\tau}$, and $\overline{\tau}_\sigma$, and on the source of seismic wave.

Anyway, the solution of equation (11), which is similar to (1), is basic for the case of *N* mechanisms. Therefore, we will consider the equations for one dissipation mechanism (8) and (9) as the basic equations, and for this case, will calculate the dependence of factor Q on the average relaxation times of the media.

Solution method for the model

The system of equations (8)–(10) is solved by the finite-difference method with using the PML for boundary conditions. The finite-difference scheme has the following form:

$$\tau_\sigma \dot{R}_{ij}^n + R_{ij}^n = L_{ij}(v_i^n), \tag{19a}$$

$$\frac{\sigma_{ij}^{n+1} - \sigma_{ij}^n}{\Delta t} = L_{ij}(v_i^n) + (\tau_\varepsilon - \tau_\sigma)\dot{R}_{ij}^n, \tag{19b}$$

$$\rho \frac{v_i^{n+1} - v_i^n}{\Delta t} = \partial_i \sigma_{ii}^{n+1} + \breve{\partial}_j \sigma_{ij}^{n+1}, \quad j \neq i, \tag{19c}$$

where

$$L_{ii}(v_i^n) = (\breve{\lambda}_i + 2\breve{\mu}_i)\breve{\partial}_i v_i^n + \breve{\lambda}_j \breve{\partial}_j v_j^n,$$
$$L_{ij}(v_i^n) = \hat{\mu}_i \partial_i v_j^n + \hat{\mu}_j \partial_j v_i^n, \quad j \neq i$$

Here $i,j=1,2,3$ – are the indexes of direction, $n=0,1,2\ldots$ – the mesh index in time, Δt – the time increment. Space derivatives are defined as follows:

$$\partial_i u = \left(u_{k-1} - 27u_k + 27u_{k+1} - u_{k+2}\right)/\left(24h_i\right),$$
$$\breve{\partial}_i u = \left(u_{k-2} - 27u_{k-1} + 27u_k - u_{k+1}\right)/\left(24h_i\right),$$

where h_i - the space step in the i-direction, and k – the space mesh index in the i-direction. If an inner boundary of the area coincides with a middle line between the nodes, the average coefficients $\hat{\mu}_i$, $\breve{\lambda}_i$, and $\breve{\mu}_i$ for the inner boundary are calculated as follows:

$$\breve{\mu}_i = 2\mu_k\mu_{k-1}/(\mu_k + \mu_{k-1}),$$
$$\hat{\mu}_i = 2\mu_k\mu_{k+1}/(\mu_k + \mu_{k+1}).$$

The scheme (19) is written for a fixed grid $(k_i = 0, \ldots, I; k_j = 0, \ldots, J; k_k = 0, \ldots, K; n = 0, 1, 2, \ldots)$, and has the order of approximation $O(\Delta t^2, h^4)$. It is practically equivalent to the "staggered" scheme used by Virieux (1986), and by Robertsson *et al*. (1994), but it operates only with integer values of indexes, as classical schemes, what is achieved by appropriate shift of "staggered" indexes.

The equation (19a) can be solved by implicit scheme. It gives:

$$R_{ij}^n = [\Delta t L_{ij}(v_i^n) + \tau_\sigma R_{ij}^{n-1}]/(\Delta t + \tau_\sigma) \tag{20}$$

The necessary stability condition for the scheme (19)–(20) in the 3D case is:

$$\sqrt{3.5}V_P \sqrt{\frac{\Delta t + 2\tau_\varepsilon}{\Delta t + 2\tau_\sigma}}\,\frac{\Delta t}{h} \le 1,$$

where $h = \sqrt{3/(h_x^{-2} + h_y^{-2} + h_z^{-2})}$.

If applying the Ricker wavelet for a seismic source, the following condition is used additionally:

$$\max(\Delta x, \Delta y, \Delta z,) < \min(V_s)/(16f), \tag{21}$$

where f – the Ricker wavelet frequency. This condition provides the undamaged form of PP, PS, and SS waves in seismograms. The coefficient 16 is empirical and can be explained by 4-point approximation of space derivatives and 4-slope form of the Ricker wavelet. It should not be less than 4 points at each slope. Other authors suggest different values of the coefficient for schemes of different accuracy: 10 – Virieux (1986), 12 – Moczo *et al*. (1997), 15-20 – Xu and McMechan (1998).

Above, V_p and V_s are velocities of the compressional and shear waves used in definition of the Lame constants.

In the nodes of the PML absorbing layer, the equations (19b)–(19c) are modified by Collino

and Tsogka (2001). Indeed, I apply another formula for absorbing sink (S_a) in the left hand side of the modified equations (19b,c). It forms the implicit (not centered) scheme with the finite-difference time derivative (Sabinin *et al*., 2003):

$$S_a = du^{n+1}, \qquad (22)$$

where u – the same variable that is in the finite-difference time derivative of the corresponding equation of the system (19).

The parameter d in (22) is calculated by a new formula ($k = x, y, z$ – index of direction):

$$d_k = d_{0k}[\exp(a\frac{n_k - 0.5}{m_k}) + \exp(-a\frac{n_k - 0.5}{m_k}) - 2]$$

$$d_{0k} = \frac{baV_P}{m_k(e^a - e^{-a} - 2a)} \qquad (23),$$

where m_k– the thickness of PML in mesh steps in k-th direction, n_k– number of the node across the PML, $1 \le n_k \le m_k$, a, b – matching coefficients (approximately, $a=1$, $b=10$). The formula (23) is not sensitive to values of the parameters a, b. The PML thickness m_k is recommended to be 20, or more.

Advantage of the finite-difference scheme (19)–(23) is its convenience for parallel computations what are easily done with instructions of Open MP.

Estimation of factor Q

For obtaining dependence of the quality factor Q on relaxation times, let us consider a formula by Liu *et al*. (1976), which is valid under the assumption that τ_ε and τ_σ do not depend on frequency $\omega=2\pi f$ at the specified bandwidth:

$$\tau Q(\omega) = \omega\tau_\sigma + \frac{1}{\omega\tau_\varepsilon}$$

Taking an integral from this expression over some interval of frequency, one can get the formula

$$\tau Q = a\tau_\sigma + \frac{(1-\tau)b}{\tau_\sigma}, \qquad (24)$$

where the constant coefficients a, and b are to be evaluated from a numerical experiment. The formula (24) is derived without the assumption $\tau<<1$ used by Blanch *et al*. (1995).

From different methods of estimation for factor Q (see, for example, Tonn, 1991), two methods seem as more reliable.

The Spectral Ratio method (SR) will be applied in the following form. From the theory, the ratio of spectral amplitudes of waves reflected from the bottom and from the top of target layer can be expressed as follows:

$$s \equiv \frac{A_b}{A_t} = s_0 \exp(-\beta r),$$

where A_t, A_b - amplitudes of spectra of reflection waves from top and bottom of the target layer in the same ray path, r is a travel distance, and β is an absorption coefficient.

Futterman (1962) defined the quality factor Q as

$$\frac{2\pi}{Q} = 1 - e^{-2d}, \qquad (25)$$

where

$$d = \frac{\beta r}{\tau_0 f},$$

f is frequency, and τ_0 is a travel time. It means that $Q>2\pi$.

From the other hand, one can approximate a logarithm of the same spectral ratio by a line function over a proper interval of f by the least squares method:

$$\ln(s) = -\eta f + \eta_0$$

It means, that estimated d in the interval is:

$$d = \eta / \tau_0 \qquad (26)$$

The art of application of this method consists in proper choice of a window for impulse in the time domain, and a window (interval) for the least squares method in the spectral domain.

For the Ricker wavelet which will be used further, the time window must include all three phases of the impulse up to visible noise at the edges. It is a visible width of impulse at the seismogram.

The spectrum of the Ricker impulse has a shape of a bell, and a logarithm of the spectral ratio has a near line part (see Fig. 1). The spectral

window should be chosen inside this line part. It can be made manually. For automatic choice, by observing synthetic seismograms, it was found that a good choice is the window between 0.8 of the peak frequency for the bottom spectrum and the peak frequency for the top spectrum.

The second method used is the Centroid Frequency Shift (FS) method (Quan and Harris, 1993). Here the coefficient η for the formula (26) is calculated by the following formulas:

$$a = \sum_f A_t, \quad b = \sum_f A_b \quad ,$$

$$f_t = \frac{1}{a}\sum_f fA_t, \quad f_b = \frac{1}{b}\sum_f fA_b \, ,$$

$$v = \frac{1}{a}\sum_f (f - f_t)^2 A_t, \eta = s\frac{f_t - f_b}{v} \, , \qquad (27)$$

where s depends on the shape of spectra, for Gaussian spectra $s=1$ (Quan and Harris, 1993).

The FS method operates with integral values, therefore it is less sensitive to the noise than the SR method, but it is more sensitive to the shape of spectra.

Because of influence of errors on the shape of spectra, it is used the same spectral window for the FS method as for the SR method.

For estimating Q from equations (25)–(26), the value of travel time τ_0 must be calculated, too.

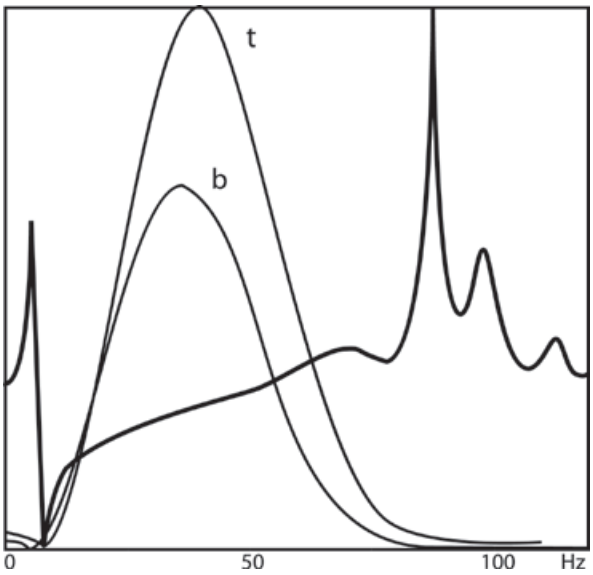

Figure 1. Spectra and a logarithm of its ratio. b: bottom, t: top.

To calculate τ_0 in a multilayered reservoir, a system of non-linear equations can be derived by the ray-tracing method (see Appendix C).

If the target layer is the second from the surface, then the calculations become simpler. Ray paths for this case are presented in Figure 2. One can see that

$$0.5\Delta t = \frac{r_1 - r_0}{V_1} + \frac{r_2}{V_2} \, , \qquad (28)$$

where indexes 1, and 2 denote the number of layer from above, V is V_P, r_0 is the path from a source to the top of target layer for the impulse reflected from the top, r_1 is the same for the impulse reflected from the bottom, r_2 is the path inside the second layer from the top to the bottom of target layer, and Δt is time between the reflected impulses at the trace.

For reflection data, the receivers are at the surface, therefore τ_0 is twice more than travel time between points x , and x_0 .

Using Snell's law, one excludes the velocity V_2 of the target layer from (28):

$$p = \frac{z_1}{\cos\theta_1} + \frac{z_2 \sin\theta_1}{\cos\theta_2 \sin\theta_2} \, , \qquad (29)$$

where $p = 0.5\Delta t V_1 + \sqrt{x_0^2 + z_1^2}$, x_0 is the half of the offset, z_1, z_2 are thicknesses of the layers, and θ_1, θ_2 are travel (incidence) angles.

Denoting the offset value of refraction point for the bottom ray as x (see Fig. 2), one obtains:

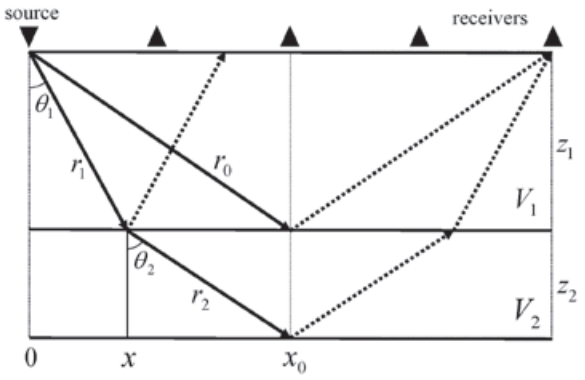

Figure 2. Scheme of P-wave reflections in a three-layer medium.

$$(x_0 - x)(p\sqrt{x^2 + z_1^2} - xx_0 - z_1^2) = xz_2^2 \qquad (30)$$

The non-linear equation (30) is solved numerically. The obtained value x is used to calculate the travel time $\tau_0 = 2r_2/V_2$ of the ray inside the target layer. From (28),

$$\tau_0 = \Delta t + 2(\sqrt{x_0^2 + z_1^2} - \sqrt{x^2 + z_1^2}) / V_1 , \qquad (31)$$

where V_1 is V_p of the upper layer. The value Δt can be calculated by the correlation function between the impulses at the trace.

Difference between Δt and τ_0 is illustrated in Figure 3. The difference increases with offset significantly.

An advantage of the method (30)–(31) is the exclusion of the unknown velocity V_2 from consideration.

As can be also seen from Fig. 2, for more exact estimating the factor Q, one should use the wave reflected from the point x at the top of target layer to calculate spectral amplitude A_t, but not from the point x_0 which is commonly used for this purpose. Knowing value x from (30), one may obtain this wave (or its spectrum) by interpolation from waves (or spectra) of adjacent traces, with taking into account different values of geometrical spreading.

Another possible problem in estimating Q of the target layer is how to exclude from the consideration the coefficients of reflection and refraction at the boundaries of the viscoelastic target layer. It will be the best result in the estimation of Q if the spectral amplitudes will differ only because of viscoelastic attenuation inside the layer. Really, the reflection and refraction at the boundaries of the layer may also act:

$$\frac{A_b^R}{A_t^R} = \frac{A_b R_b P_{tb} P_{bt}}{A_t R_t} ,$$

where A^R- real amplitude, R - coefficient of reflection, P - coefficient of refraction.

If R and P would depend on f then they influenced on value of Q. However, practice shows that influence of reflection/refraction coefficients on the estimated values of Q is less than difference in estimated values Q caused by different smart choices of the time and spectral windows.

Both methods SR and FS give close values Q but FS insufficiently less.

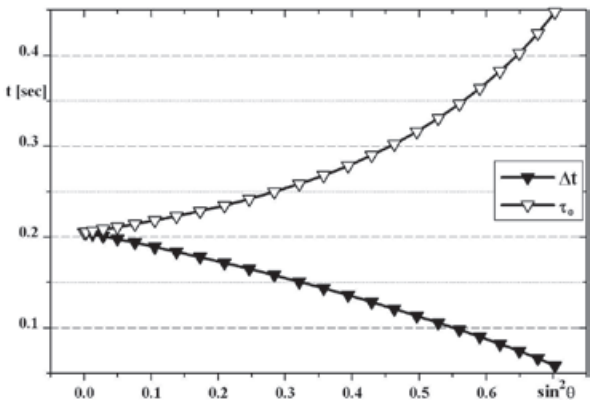

Figure 3. Travel time τ_0, and difference Δt of arrival times calculated in one of the variants for different incidence angles.

Numerical simulation

I use the developed numerical model for obtaining the synthetic seismograms of waves reflected from the top and bottom boundaries of a horizontal viscoelastic layer. Then I estimate the factor Q from the seismograms with different τ, and τ_σ, and put these values into the left-hand side of the formula (24) to estimate the values of the coefficients a, b of the formula (24) by least squares method.

The computation of synthetic seismograms by the viscoelastic model was performed at a workstation which gave possibility to parallelize calculations into 24 threats. But it was too slow for a 3D model. For economy, a 2D problem was chosen for the numerical experiment, as follows.

At the earth surface, there was one wave source, and a set of equally spaced (every 100 meters) receivers (the common source point observation system). The area of width 5000 m and depth 2250 m consisted of three homogeneous layers, the target viscoelastic layer was the second, beginning at the depth 1500 m, and had the thickness 400 m. The other layers were considered as elastic, with $\tau = 0$.

The density, and the elastic velocities of compressional and shear waves were as in Table 1.

Table 1. Media parameters.

Layer	V_p *m/sec*	V_s *m/sec*	ρ *kg/m³*
1	3500	2000	2000
2	3800	1600	2500
3	2000	1200	2000

Seismic wave source of explosion type was the Ricker wavelet in time, its frequency f_r was set to 15, 30, 45, and 60 Hz, duration of the impulse – 160, 80, 53.3, and 40 ms correspondingly.

The mesh sizes of the finite-difference grid were $h_x=5m$, $h_z=2.5m$, and the time step was 0.2 ms. The PML thickness was equal to 40 nodes. The PML was not mounted at the earth surface where the source was set.

The typical seismogram of v_z obtained for this 2D problem is shown in Figure 4. First two fronts of PP-waves were used for estimating the factor Q. PP waves reflected from the bottom of area (1400–1500 ms), and direct waves reflected from the vertical sides of area are not visible. It means a good effectiveness of applied modification of PML absorbing boundaries.

For comparison, the 3D problem was computed in the variant of $f_r = 30$ Hz, $\tau = 0.7$, and τ_σ = 0.625ms. The 3D area had and additionally y-direction with thickness 2000 m. The receivers were spaced in the middle line. For saving time of computation, a rough grid was used: h_x=h_y=20m, and the time step 0.4ms.

The seismogram of v_z obtained for the 3D problem is shown in the Figure 5. Visible distortions are caused by the roughness of grid The Figure 5 corresponds to the variant of Figure 4. Values of factor Q calculated for seismograms of Figures 4, and 5 are practically the same, and are equal to 14.8.

The comparison is good enough to decide to apply more economical 2D model in the numerical experiment.

Numerical results

Factor Q was calculated with the SR method for all offsets and did not show any stable or significant dependence on the offset. Existing errors were caused by small distortions of waves by interference with waves reflected from the sides of area, and by errors in setting the time windows.

Difference in Q on offsets was up to 6% of magnitude for small values τ_σ(<0.0002), and up to 12% for large values τ_σ(>0.01), with 0.5%–3% for a middle values, what can be adopted as an error for the estimation of Q.

Difficulty in calculation of the factor Q is that the linear part of logarithm of spectral ratio which one can see in Figure 1 is not clearly present for extreme values of relaxation times. For example, it is a curve for large values τ_σ. Therefore, this gave unreliable values of Q sometimes.

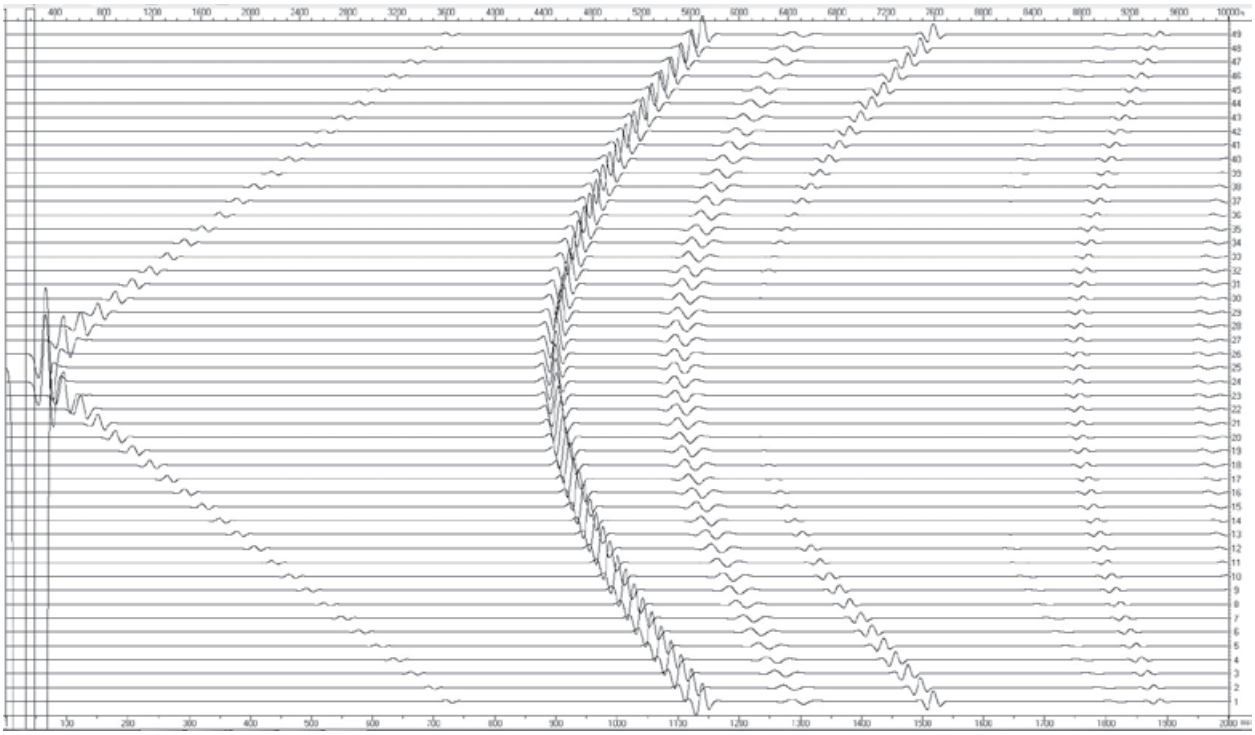

Figure 4. Seismogram of direct P, and reflected PP, and PS waves (v_z) for the 2D problem.

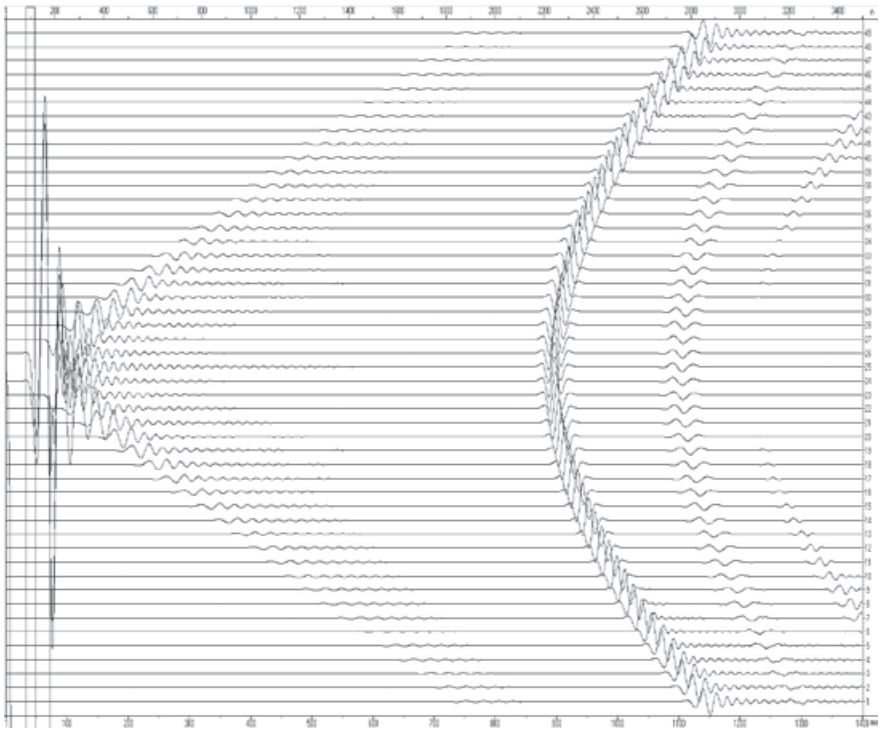

Figure 5. Seismogram of direct P, and reflected PP waves (v_z) for the 3D problem.

Table 2. Computed values of Q for f_r=15, and different values of τ, and τ_σ.

τ \ $3/(f_r\tau_\sigma)$	10	20	40	80	160	320
0.05	172	61	46.5	63.3	115	190
0.1	107	34.1	25.5	35.4	64.2	115
0.2	68.7	24.2	16.9	21.5	36.8	66
0.3	57	16.9	13.5	15.9	26.8	46.6
0.4	51.4	14.3	10.4	13.1	21.3	37.5
0.5	48.3	12.7	9.35	11.3	18.1	31.6
0.6	47.5	11.5	8.6	10.4	15.1	25.8
0.7	43.4	10.7	8.0	9.9	14.1	23.9
0.8	33.9	10.3	7.7	9.4	13.2	21.3
0.9	31.9	10.1	7.4	9.1	12.1	19.6
0.95	29.9	9.7	6.8	8.8	11.6	18.6

Table 3. Computed values of Q for f_r=30, and different values of τ, and τ_σ.

τ \ $3/(f_r\tau_\sigma)$	10	20	40	80	160	320
0.05	201	59.9	48.7	73	128	236
0.1	115	43.7	31	40.6	72	132
0.2	67	22.5	17.9	23.3	39.6	72.2
0.3	51.8	14.8	13.14	17.3	28.3	52.5
0.4	47.7	11.5	10.65	14.7	22.6	39.2
0.5	42.6	9.73	9.72	12.4	19.3	33.3
0.6	41	8.46	8.85	11.7	17.5	29.9
0.7	39.6	7.97	8.2	11.2	15.6	26.1
0.8	38.4	7.38	7.81	9.55	14.7	23.8
0.9	37.8	7.02	7.35	9.44	13.8	22.5
0.95	37.2	7.02	7.31	9.08	13.2	21.4

Table 4. Computed values of Q for f_r=45, and different values of τ, and τ_σ.

τ \ $3/(f_r\tau_\sigma)$	10	20	40	80	160	320
0.05	196	84	46	74	128	221
0.1	128	43	32	41.8	72.2	131
0.2	73	18.6	18.5	24.2	41.1	71.4
0.3	55.6	12.8	13.45	17.4	30.6	52.8
0.4	46.8	10	11.4	14.8	24.5	41.2
0.5	43	8.9	9.8	14.4	21	36.2
0.6	41.8	7.8	9.1	12	17.7	31.3
0.7	41	7.4	8.5	11.3	16.8	27.8
0.8	40	7.0	8.0	10.4	15.1	25.2
0.9	39.9	6.8	7.5	9.85	14.35	23.4
0.95	39.7	6.7	7.2	9.77	13.8	22.6

Table 5. Computed values of Q for f_r=60, and different values of τ, and τ_σ.

τ \ $3/(f_r\tau_\sigma)$	10	20	40	80	160	320
0.05	197	79	61.2	74.8	129	211
0.1	128	34.4	35.2	44.5	78.8	132
0.2	71	16.8	20.1	23.5	43.7	75.5
0.3	55.5	11.7	14.2	21.4	31.7	55.9
0.4	48.8	9.45	11.84	17.4	26.9	44.8
0.5	47	8.1	10.4	15.4	22	36.6
0.6	46	7.5	9.4	13.1	18.5	31.6
0.7	44	7.15	8.9	11.9	17.4	28.5
0.8	42.3	6.88	8.35	11.3	16.1	26.2
0.9	41.4	6.7	8.2	10.6	15.2	22.6
0.95	41.1	6.65	7.7	10.1	14.7	21.4

The results of calculation of factor Q in a wide range near normal incidence are presented in Tables 2–5 for the source frequencies of f_r=15, 30, 45, 60 Hz respectively.

As equations (8), (9) depend on τ and τ_σ, so the values Q in the Tables 2–5 are presented depending on these parameters. Also, the results depend on the frequency of the source significantly.

It was found that equation (24) does not match satisfactory to Tables 2–5. Instead, the following similar equation was derived for this:

$$1/d=A/(xy)+By/x-Cy, \qquad (32)$$

where d depends on Q by formula (25), $x=\tau^\alpha$, $y=[3/(f_r\tau_\sigma)]^\beta$. Values α=0.7, and β=1.2 represent a near optimal choice.

The obtained coefficients A, B, and C of (32) are presented in Table 6 with values of relative estimation error by (32).

One can compare values Q calculated by (32) for f_r =45 in Table 7 with values from Table 4.

For practical use, one can apply Tables 2–5, or equation (32), or derive an own approximation formula.

Table 6. Coefficients for equation (32).

f_r	A	B	C	Rel. error
15	100.4	0.0079	0.0040	0.138
30	108.1	0.0096	0.0058	0.138
45	113.2	0.0091	0.0045	0.135
60	113.9	0.0089	0.0036	0.156

Discussion

Some authors (Blanch *et al*., 1995; Xu and McMechan, 1998) guess that the constant value Q over some interval of spectral frequency, which one can usually see, is equal to a near constant value Q on an average graphic consisting of the separate theoretical graphics Q (as by Liu *et*

Table 7. Estimated values of Q by (32) for f_r=45.

τ \ $3/(f_r\tau_\sigma)$	10	20	40	80	160	320
0.05	189	90.7	56.1	60.3	106	228
0.1	118	56.9	35.4	37.4	64.3	136
0.2	73.6	36.1	22.6	23.3	38.5	79.5
0.3	56.2	27.9	17.6	17.7	28.3	57.1
0.4	46.5	23.3	14.9	14.7	22.6	44.7
0.5	40.2	20.4	13.1	12.7	19.0	36.7
0.6	35.8	18.3	11.9	11.4	16.4	31.0
0.7	32.4	16.8	10.9	10.4	14.5	26.7
0.8	29.8	15.5	10.2	9.58	13.0	23.4
0.9	27.7	14.6	9.66	8.97	11.8	20.7
0.95	26.8	14.1	9.42	8.71	11.3	19.5

al., 1976) for N>1 dissipation mechanisms with different relaxation times. As one can see from (25), this method of obtaining constant Q fails in the case of one dissipation mechanism. Contrary, the experimental formula (32) does not depend on the number of dissipation mechanisms.

Although the formulas (25) – (27) of methods for estimating factor Q are simple, they have several sources for errors. At first, it is a non-linear form of the "line" part of the logarithm of spectral ratio which can be clearly seen in field data. It is necessary smoothing seismograms with noise, and developing new methods of Q-estimation for synthetic seismograms obtained at large τ_σ.

Second, insignificantly different sizes and positions of time windows can give significantly different spectra what is caused by increased role of noise at the edges of impulses. Suitable automatic algorithms for generating the time windows are needed.

Third, a correct estimation of the travel time is needed as one can see from Figure 3.

Finally, a good estimation for the usually unknown impulses reflected from the top of target layer in the point x of Fig.2 is necessary.

Errors from these four sources can give significantly incorrect values Q, up to 50% and more.

I have solved these problems for synthetic seismograms in case of fine spacing the receivers and not large values τ_σ. As the result, Q values calculated here do not depend on offset what must be theoretically for isotropic media, because factor Q is a property of medium only. This is a good criterion for correctness of methods for estimating Q. If one sees a factor Q depending on offset (see for example Dasgupta and Clark, 1998), it means that the medium is anisotropic or there are the errors in algorithm of estimating Q.

As known, PML absorbing boundary does not exclude completely reflections from the boundaries of area for finite-difference problems. The modification of PML presented here decreases the reflections in comparison with classical PML by Collino and Tsogka (2001) due to lucky choice of exponent functions for approximation (23).

Errors of the approximated formula (32) for dependence Q on relaxation times are in agreement with errors of estimating Q. For example, if it is excluded the first and ultimate columns from Table 2, then the error of estimation by (32) becomes twice less.

Conclusion

A numerical 3D model for seismic wave propagation in viscoelastic media is developed, which differs from the previous (Robertsson *et al*. 1994) by modification of the finite-difference scheme, and by including the improved variant of PML absorbing boundary.

It is shown that the numerical model for one dissipation mechanism can be directly applied to three simple schemes of interaction of *N* dissipation mechanisms by using average relaxation times.

The synthetic seismograms are obtained for 3D, and 2D variants of a reservoir which show close values of factor Q in the viscoelastic layer.

The methodology of estimation of factor Q for surface reflection data is developed which differs from previous by more exact calculation of travel times for reflected waves, and by manual choice of spectral windows.

The viscoelastic model and the method of Q-estimation are applied to obtaining experimental dependences of factor Q on relaxation times. The approximate formula is suggested for such dependence.

Appendix "A"
Creep functions for N dissipation mechanims

From the theory of viscoelasticity, for one mechanism of the standard linear solid, the stress response can be written as $G = [k_2 + k_1 \exp(-k_1 t/\eta)]\theta(t)$, where k and η are coefficients in stress expressions for elastic and viscose elements: $\sigma = k\varepsilon$, and $\sigma = \eta\dot{\varepsilon}$ (see Fig. A1).

For N mechanisms interacting in parallel, one can write:

$$G = \sum_{n=1}^{N} [k_{2n} + k_{1n} \exp(-k_{1n} t/\eta_n)]\theta(t) .$$

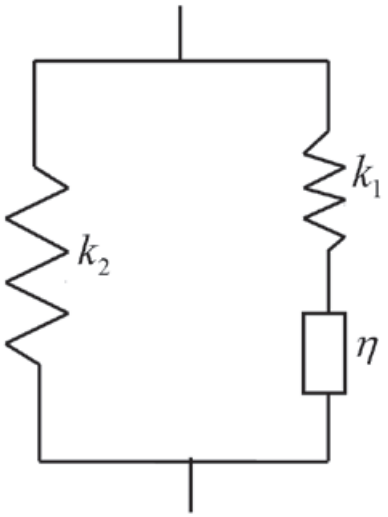

Figure A1. Scheme of standard linear solid.

In this case, the scheme of interactions of mechanisms is equivalent to a parallel scheme with equal values $k_2 = \frac{1}{N}\sum_{n=1}^{N} k_{2n}$ for elastic elements.

Therefore, defining the relaxation times for each mechanism as

$$\tau_{\sigma n} = \eta_n / k_{1n} , \quad \tau_{\varepsilon n} = \eta_n (\frac{1}{k_{1n}} + \frac{1}{k_2}) ,$$

one obtains $k_{1n} = k_2 (\frac{\tau_{\varepsilon n}}{\tau_{\sigma n}} - 1)$, and

$$G = k_2 N[1 + \tfrac{1}{N}\sum_{n=1}^{N} (\frac{\tau_{\varepsilon n}}{\tau_{\sigma n}} - 1)\exp(-t/\tau_{\sigma n})] .$$

This definition is equivalent to formula (18) for the creep function.

For N mechanisms interacting sequentialy, one can write by analogy:

$$J = \sum_{n=1}^{N} [\frac{1}{k_{2n}} + \frac{1}{k_{1n}} - \frac{1}{k_{1n}} \exp(-k_{1n} t/\eta_n)]\theta(t) .$$

The scheme of interactions is equivalent to a sequential scheme with equal values $2/k = \frac{1}{N}\sum_{n=1}^{N} (1/k_{1n} + 1/k_{2n})$ for elastic elements.

Defining the relaxation times for each mechanism as

$$\tau_{\varepsilon n} = \eta_n / k_{1n} , \quad \tau_{\sigma n} = \eta_n / (k_{1n} + k_{2n}) ,$$

one obtains $\frac{1}{k_{1n}} = \frac{2}{k}(1 - \frac{\tau_{\sigma n}}{\tau_{\varepsilon n}})$, and

$$J = \frac{2N}{k}[1 - \tfrac{1}{N}\sum_{n=1}^{N} (1 - \frac{\tau_{\sigma n}}{\tau_{\varepsilon n}})\exp(-t/\tau_{\varepsilon n})]$$

what gives formula (14) for the creep function.

Appendix "B"
Model equations for N dissipation mechanisms

By expanding the time derivative in the convolution in (5), with using the formula (14), one can get:

$$\dot{\sigma}_{ij} = L_{ij} - \tfrac{1}{N}\sum_{n=1}^{N} \tau_n (m_n - \dot{\sigma}_{ij}) , \qquad \text{(B1)}$$

where $m_n = \frac{1}{\tau_{\varepsilon n}} \exp(-t/\tau_{\varepsilon n})\theta(t) * \dot{\sigma}_{ij}$.

By taking the time derivative of m_n, one can get:

$$\tau_{\varepsilon n} \dot{m}_n = -m_n + \dot{\sigma}_{ij} . \qquad \text{(B2)}$$

Substituting this equation into (B1) gives:

$$\dot{M}_{ij} = \dot{\sigma}_{ij} - L_{ij} , \qquad \text{(B3)}$$

where $M_{ij} = \frac{1}{N}\sum_{n=1}^{N}(\tau_{\varepsilon n} - \tau_{\sigma n})m_n$.

The equation (B1) can be rewritten in the form

$$\frac{1}{N}\sum_{n=1}^{N}[(1-\tau_n)\dot{\sigma}_{ij} = L_{ij} - \tau_n m_n] \quad \text{(B4)}$$

As it follows from (6), the equality in brackets inside the sum of (B4) is valid for each dissipation mechanism, i.e., for each value of index n. Therefore, one can multiply it by $\tau_{\varepsilon n}$ inside the sum what gives the equation:

$$\overline{\tau}_{\sigma}\dot{\sigma}_{ij} = \overline{\tau}_{\varepsilon}L_{ij} - M_{ij} \,. \quad \text{(B5)}$$

Defining $R_{ij} = M_{ij}/(\overline{\tau\tau}_{\varepsilon})$, one obtains equations (15) and (16) from (B5) and (B3).

In the case of parallel mechanisms, the time derivative in the convolution in (17) is expanded with using the formula (18) that gives:

$$\dot{\sigma}_{ij} = L_{ij} - \frac{1}{N}\sum_{n=1}^{N}\tau_{1n}(m_{1n} - L_{ij}) \,, \quad \text{(B6)}$$

where $m_{1n} = \dfrac{1}{\tau_{\sigma n}}\exp(-t/\tau_{\sigma n})\theta(t) * L_{ij}$.

By taking the time derivative of m_{1n}, one can get:

$$\tau_{\sigma n}\dot{m}_{1n} = -m_{1n} + L_{ij} \,. \quad \text{(B7)}$$

Substituting this equation into (B6) gives equation (B3) in which $M_{ij} = \frac{1}{N}\sum_{n=1}^{N}(\tau_{\varepsilon n} - \tau_{\sigma n})m_{1n}$.

The equation (B6) can be rewritten in the form

$$\frac{1}{N}\sum_{n=1}^{N}[\dot{\sigma}_{ij} = L_{ij} + \tau_{1n}L_{ij} - \tau_{1n}m_n] \,. \quad \text{(B8)}$$

The equality in brackets inside the sum of (B8) is valid for each dissipation mechanism, i.e., for each value of index n. Therefore, one can multiply it by $\tau_{\sigma n}$ inside the sum what gives the equation (B5). As above, (B3) and (B5) lead to (15) and (16) by defining $R_{ij} = M/(\overline{\tau\tau}_{\varepsilon})$.

In the case 3), by the same way, one can obtain the equation (B3) with the definition

$$M_{ij} = \frac{1}{N}\left[\sum_{n=1}^{N_1}(\tau_{\varepsilon n} - \tau_{\sigma n})m_n + \sum_{n=1}^{N_2}(\tau_{\varepsilon n} - \tau_{\sigma n})m_{1n}\right] .$$

For the first group, the equation

$$\sum_{n=1}^{N_1}[(1-\tau_n)\dot{\sigma}_{ij} = L_{ij} - \tau_n m_n] \quad \text{(B9)}$$

is valid which can be multiplied by $\tau_{\varepsilon n}$ inside the sum, and for the second group, the equation

$$\sum_{n=1}^{N_2}[\dot{\sigma}_{ij} = L_{ij} + \tau_{1n}L_{ij} - \tau_{1n}m_{1n}] \quad \text{(B10)}$$

is valid which can be multiplied by $\tau_{\sigma n}$ inside the sum. After these multiplications, the sum of (B9), and (B10), divided by N, gives again the equation (B5).

Unfortunately, in the case of N parallel mechanisms, some authors use instead of (17) and (18) the widely known formulae (Carcione *et al*, 1988; Carcione, 1993; Robertsson *et al*., 1994; Xu and McMechan, 1998; Komatitsch and Tromp, 2002, Mikhailenko *et al*., 2003):

$$\dot{\sigma}_{ij} = \dot{\phi}_2 * L_{ij}$$

$$\phi_2(t) = [1 + \sum_{n=1}^{N}\tau_{1n}\exp(-t/\tau_{\sigma n})]\theta(t) \,,$$

In this case, one will obtain the equation (B3) with definition $M_{ij} = \frac{1}{A}\sum_{n=1}^{N}\tau_{1n}m_{1n}$, where $A = \frac{1}{N}\sum_{n=1}^{N}\dfrac{1}{\tau_{\sigma n}}$.

Instead of (B5), one will obtain the similar equation:

$$\dot{\sigma}_{ij} = (1+NB)L_{ij} - AM_{ij} \,, \quad \text{(B11)}$$

where $B = \frac{1}{N}\sum_{n=1}^{N}\tau_{1n}$. In essence, A and B are average values.

Scarcely likely, (B11) looks better than (B5). Equation (B11) depends explicitly on number of relaxation mechanisms N, but (B5) does not. For example, it is easily seen in the case of proportional values $\tau_{\varepsilon n} = 2\tau_{\sigma n}$ for all relaxation mechanisms.

Independence on N of (B5) is more correct from the point of view of Physics. Therefore, I use and recommend the creep functions ϕ, and ϕ_1, but not ϕ_2.

Appendix "C"
Travel time estimation for multilayered media

Suppose that there are n layers above the target one. Then (see Fig. 2),

$$x_0 = \sum_{i=1}^{n} z_i tg(\beta_i)\,, \qquad \text{(C1)}$$

where z_i is the thickness of i-th layer, and β_i is the incidence angle in it.

From Snell's law, $a_0 = \dfrac{\sin(\beta_i)}{V_i}$ is a constant, and from (C1), one can derive a nonlinear equation for obtaining it (V_i is the ray velocity):

$$x_0 = a_0 \sum_{i=1}^{n} \frac{z_i V_i}{\sqrt{1 - a_0^2 V_i^2}} \qquad \text{(C2)}$$

Consequently, knowing a_0 from (C2), one can calculate a half of the travel time for upper ray:

$$t_0 = \sum_{i=1}^{n} \frac{z_i}{V_i \sqrt{1 - a_0^2 V_i^2}} \qquad \text{(C3)}$$

By analogy, for the down ray, one can write:

$$x = a \sum_{i=1}^{n} \frac{z_i V_i}{\sqrt{1 - a^2 V_i^2}} \qquad \text{(C4)}$$

where $a = \dfrac{\sin(\alpha_i)}{V_i}$, and α_i is the incidence angle for it.

The half of the travel time for down ray in the n upper layers

$$t_1 = \sum_{i=1}^{n} \frac{z_i}{V_i \sqrt{1 - a^2 V_i^2}} \qquad \text{(C5)}$$

From definitions,

$$0.5\Delta t = t_1 - t_0 + \frac{z}{V \cos(\alpha)} \qquad \text{(C6)}$$

where z, V, and α without indexes belong to the target layer.

From (C6) with (C4), using substitution $w=x_0-x$, one finds out a nonlinear equation for calculating value a:

$$t_0 + 0.5\Delta t = t_1 + \frac{a(z^2 + w^2)}{w}$$

Finally, knowing a, one calculates t_1 from (C5), and the travel time in the target layer

$$\tau_0 = 2(t_0 - t_1) + \Delta t$$

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