

# Magnitude determination of Mexican earthquakes

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## RESUMEN

Se estudian dos escalas de magnitud alternativas,  $M_A$  y  $M_E$ , para estimar, en quasi-tiempo real, la magnitud de sismos moderados y grandes ocurridos en el territorio mexicano y registrados por la estación de banda ancha en la Ciudad Universitaria (CU), México, D.F.,  $M_A$  está basada en la amplitud de las trazas de velocidad filtradas con un paso de banda de 15 a 30 seg y  $M_E$  en la estimación de la energía sísmica radiada. Ambas escalas son calibradas con la magnitud de momento sísmico,  $M_w$ .  $M_A$  es más adecuada para medir la magnitud de eventos superficiales moderados; sin embargo, sufre de saturación para grandes eventos.  $M_E$  no presenta problemas de saturación y puede utilizarse para medir la magnitud de eventos profundos. Debido a que  $M_A$  mide características del evento a período largo, mientras que las contribuciones a  $M_E$  provienen esencialmente de frecuencias cercanas a la frecuencia de esquina, la magnitud medida con ambas escalas podría diferir, aún cuando la magnitud  $M_A$  no esté saturada. La disparidad en ambas magnitudes puede indicar la naturaleza anómala de un evento por lo que recomendamos el uso de ambas escalas, cuando sea posible.

**PALABRAS CLAVE:** Escala de magnitud, energía sísmica, amplitud.

## ABSTRACT

We explore two alternative magnitude scales,  $M_A$  and  $M_E$ , for quasi-real time estimate of magnitude of moderate and large Mexican earthquakes using broadband recordings at Ciudad Universitaria (CU), Mexico D.F., Mexico.  $M_A$  and  $M_E$  are based on amplitude of band-pass filtered (between 15 and 30 sec) velocity traces and estimate of radiated seismic energy, respectively. Both scales are tied to the moment magnitude,  $M_w$ .  $M_A$  is adequate for shallow, moderate, and large earthquakes, but appears to saturate for major and great earthquakes.  $M_E$ , on the other hand, does not suffer from saturation and should be valid for events of all depths. In as much as  $M_A$  measures long-period characteristics of an event while the contribution to  $M_E$  mostly comes from frequencies near the corner frequency of the event, the magnitude on the two scales may differ for the same earthquake, even if  $M_A$  has not saturated. Since a large disparity in the two magnitudes may be indicative of anomalous nature of an earthquake, we recommend the use of both scales whenever possible.

**KEY WORDS:** Magnitude scale, seismic energy, amplitude.

## INTRODUCTION

While the need for a quick and reliable estimate of magnitude of local and regional Mexican earthquakes can hardly be overemphasized, it often present difficulties. This is because most of the conventional seismographs are not well calibrated and many of the seismograms saturate during moderate and large earthquakes. For this reason, magnitudes of local and regional earthquakes in Mexico have been estimated on an inconsistent basis, which has changed with time. A summary of some of the methods used in computing magnitudes in the past and especially those reported in the catalog of Figueroa (1970) is given by Singh *et al.* (1984). Presently the magnitude is estimated from coda duration using a formula given by Havskov and Macías (1983). To derive this formula, the coda duration was calibrated against the body-wave magnitude ( $m_b$ ). The scale, established for a few seismic stations, is useful for small to moderate events. In as much as  $m_b$  is not a reliable measure of the earthquake size a coda magnitude tied to  $m_b$  is unsatisfactory, especially for moderate to large earthquakes.

Since 1985 digital accelerographs are in operation in Ciudad Universitaria (CU), UNAM, Mexico, D.F. A

broadband seismograph was installed in CU in April 1991. Thus it is now possible to have quasi-real time access to on-scale digital records of acceleration and/or velocity. This offers an opportunity for a quick and reliable estimate of magnitude. In this paper we explore two alternative methods to estimate magnitudes of moderate and large Mexican earthquakes, one based on amplitudes of filtered (15 to 30 sec) velocity traces, and the second based on calculation of radiated seismic energy.

## MAGNITUDE, $M_A$ , BASED ON AMPLITUDES OF FILTERED VELOCITY TRACES

There are advantages in developing a magnitude scale based on amplitudes of very long-period waves, since such a scale measures the average, static nature of the source, it does not saturate, and it is less sensitive to details of the Earth's structure. For most moderate Mexican events ( $M \leq 6$ ), however, the signal is lost in the noise at CU at periods greater than about 30 sec. For this reason we develop a magnitude scale based on amplitudes of 15 to 30 sec waves. The data comes from broadband seismograms recorded at CU since April 1991. Except for large earthquakes, the presently available accelerograms at CU are generally not suitable for extracting 15 to 30 sec waves.

DATA

The broadband station at CU (named UNM), consists of Streckeisen STS-1 seismometers and Geoscope recording system (Romanowicz *et al.*, 1991) modified to acquire 20 samples/sec data for regional and local events. The response of the system is flat for velocity from 300 to 0.2 sec. For this study we selected all shallow events ( $h \leq 50$  km) within 1200 km from UNM, which occurred since April 1991, when the station became operational, and whose centroid moment tensor solution (CMT) has been reported by the Harvard group (Dziewonski *et al.*, 1981). We also included 6 events whose CMT solutions were not available, but whose seismic moments and mechanisms could be independently estimated. Table 1 lists these events, and Figure 1 shows their locations and best double-couple mechanisms. For these events we filtered the broadband velocity seismograms between 15 and 30 sec, using a 3-pole, zero-phase Butterworth filter. From these filtered seismograms we computed the amplitude as the square root of the sum of the squares of the maximum amplitude of each one of the three components. We normalized the amplitude of each of these events to a standard seismic moment ( $M_0$ ) of  $10^{23}$  dyne-cm. Figure 2 shows the distribution of normalized amplitudes,  $A_0$ , as a function of hypocenter distance,  $R$ . Note that there is a cluster of events for  $R$  between 250 and 400 km, and lack of events

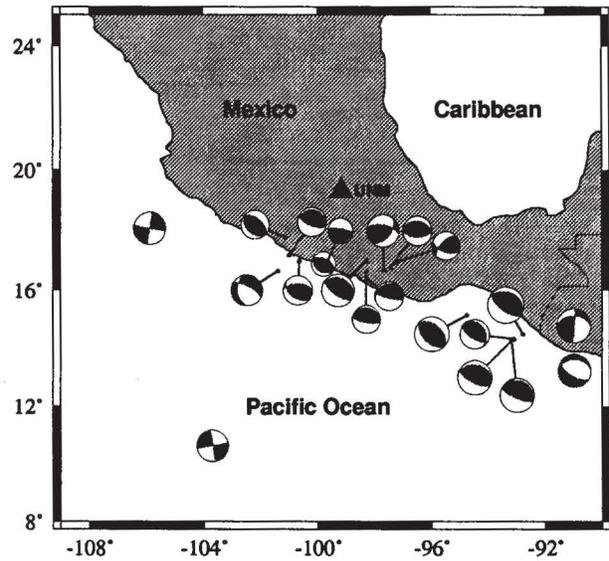


Fig. 1. Best double-couple solutions of earthquakes located within 1200 km from the UNM station (triangle), occurring between April 1991 and September 1993, whose records were analyzed in this study to establish a magnitude scale,  $M_A$ , based on the amplitude of 15 to 30 sec waves.

Table 1

Events used in developing a magnitude scale based on amplitude

Date	Time	Distance (km)	Azi (°)	Depth (km)	Moment ( $10^{16}$ Nm)	Amplitude ( $\mu$ /sec)
19910401	734	376	331	40 <sup>†</sup>	63.00 <sup>†</sup>	87.12
19910407	939	332	323	47 <sup>†</sup>	5.50 <sup>†</sup>	4.90
19910528	56	276	14	27 <sup>‡</sup>	0.81 <sup>‡</sup>	1.56
19910725	1546	380	302	15 <sup>†</sup>	53.00 <sup>†</sup>	48.80
19910918	948	1003	301	15 <sup>†</sup>	220.00 <sup>†</sup>	98.21
19911111	1746	720	302	15 <sup>†</sup>	93.00 <sup>†</sup>	51.96
19911124	347	337	327	15 <sup>†</sup>	12.00 <sup>†</sup>	23.96
19920109	403	264	11	30 <sup>‡</sup>	1.20 <sup>‡</sup>	2.19
19920212	1156	269	50	34 <sup>†</sup>	7.00 <sup>†</sup>	7.67
19920317	643	851	312	30 <sup>†</sup>	17.00 <sup>†</sup>	8.86
19920331	2056	317	31	15 <sup>†</sup>	16.00 <sup>†</sup>	28.58
19920530	1630	851	312	29 <sup>†</sup>	350.00 <sup>†</sup>	71.71
19920607	901	339	344	15 <sup>†</sup>	12.00 <sup>†</sup>	26.02
19920607	1741	313	343	15 <sup>†</sup>	7.90 <sup>†</sup>	21.77
19920928	741	1104	309	15 <sup>†</sup>	130.00 <sup>†</sup>	25.81
19921208	628	1081	26	15 <sup>†</sup>	41.00 <sup>†</sup>	13.35
19930331	1018	311	40	12 <sup>‡</sup>	13.00 <sup>‡</sup>	27.22
19930515	309	300	350	20 <sup>‡</sup>	100.00 <sup>‡</sup>	147.76
19930903	1235	850	309	22 <sup>‡</sup>	1500.00 <sup>‡</sup>	477.00
19930919	1410	822	311	16 <sup>‡</sup>	540.00 <sup>‡</sup>	219.62
19930930	1827	677	320	19 <sup>‡</sup>	510.00 <sup>†</sup>	567.37

<sup>†</sup> CMT catalog

<sup>‡</sup> This study

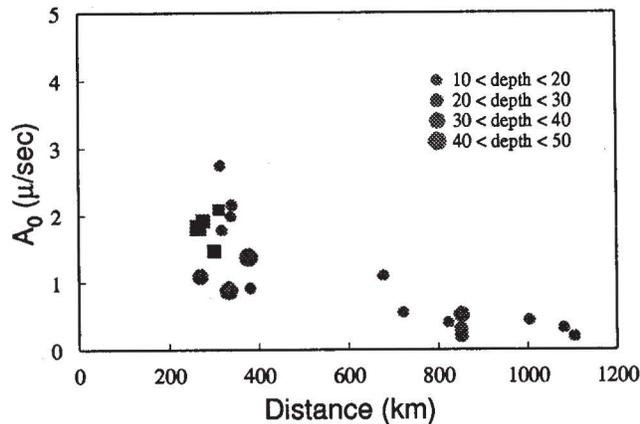


Fig. 2. Observed amplitudes as a function of hypocentral distance. The amplitudes are normalized to correspond to a seismic moment,  $M_0$ , of  $10^{23}$  dyne-cm. Filled circles: seismic moments and depth from CMT catalog; filled squares: this study. Symbol size is scaled to event depth.

between 400 and 700 km. The scatter in the amplitude data in the range of 250 and 400 km is probably due to real differences in the depths of events as well as due to errors in the reported  $M_0$  values. It is known that the depths of moderate, shallow Mexican earthquakes reported in the CMT catalog are, generally, greater than the depths estimated from high quality local data (e.g., Singh and Ordaz, 1993) or from synthetic modeling of teleseismic body waves (Pacheco *et al.*, 1993; M. Pardo, personal communication, 1993). It is possible that the greater depths result in a bias towards higher  $M_0$  values in the CMT catalog, although the issue is yet to be resolved (G. Ekström, personal communication, 1993). From the above we conclude that there are uncertainties in the reported  $M_0$  values and part of the scatter of the amplitude data in Figure 2 may be due to these uncertainties.

### THEORETICAL PREDICTION

Because of the lack of data between 400 and 700 km and generally sparse data set for  $R$  larger than 700 km (Figure 2), it is difficult to establish the shape of the attenuation curve,  $A_0(R)$ , with confidence. For this reason we computed theoretical amplitudes as a function of distance. In this computation, a crustal structure (Table 2), which corresponds to the group velocity curve observed at UNM, was used and synthetic seismograms were computed by summing normal modes (Herrmann, 1993). Table 2 also gives general  $Q$  values used for computing the synthetic seismograms. These values of  $Q$  are reasonable at periods of 15 to 30 sec. (e.g., Kovach, 1978; Aki, 1980). These synthetic seismograms were processed in the same fashion as the observed data. Synthetics were computed for three types of focal mechanisms: shallow dip-slip, steep dip-slip, and strike-slip. For a given depth, distance, and a fixed seismic moment of  $10^{23}$  dyne-cm, the synthetic seis-

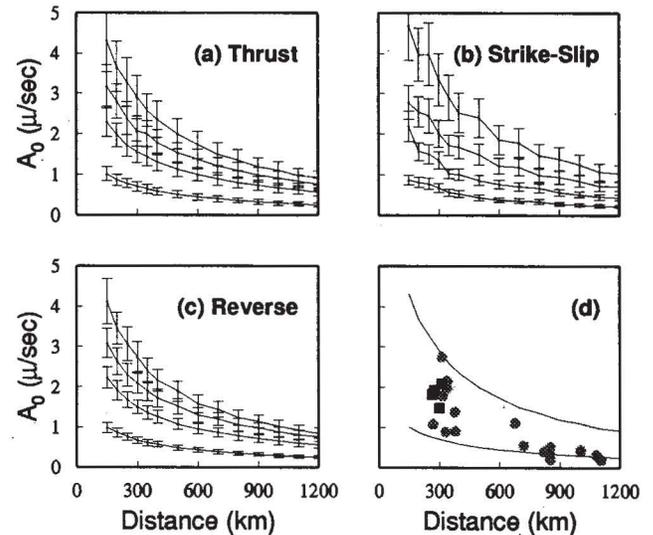


Fig. 3. Theoretical normalized amplitudes (corresponding to  $M_0 = 10^{23}$  dyne-cm) as a function of distance for depths between 15 km (upper curve) to 45 km (lower curve), spaced every 10 km. Bars represent the standard deviation. (a) Thrust events. (b) Strike-slip events. (c) Reverse events. (d) Observed normalized amplitudes. The upper and lower lines represent theoretical curves for thrust events of depths 15 and 45 km, respectively.

mograms were computed for a suite of reasonable strike, dip, and rake values and for a range of  $180^\circ$  in azimuth. The average values and standard deviations of the computed and then filtered amplitudes are shown in Figure 3. As can be seen from this figure, the amplitudes are much less sensitive to focal mechanism than to depth.

Table 2

Crustal structure and  $Q$  values

Thickness (km)	P velocity (km/sec)	S velocity (km/sec)	Density ( $g/cm^3$ )	$Q_p$	$Q_s$
5	5.0	3.0	2.5	400	200
10	6.2	3.6	2.8	400	200
10	6.5	3.7	2.8	400	200
15	6.7	3.8	2.9	800	400
	8	4.7	3.3	800	400

### DETERMINATION OF THE MAGNITUDE

Most moderate and large earthquakes along the coast of Mexico occur at depths of  $20 \pm 5$  km (e.g., Singh and Mortera, 1991). For this reason we have based our determination of magnitude on an attenuation curve corresponding to a source depth of 20 km and shallow-dip thrust mechanism. However, the theoretical attenuation curve for this depth was consistently  $0.75 \mu/sec$  above the observed amplitude data. Figure 4 shows the normalized amplitude data and the theoretical attenuation curve for this depth after being shifted down by  $0.75 \mu/sec$ . This curve provides the basis for the estimation of the seismic moment and the magnitude.

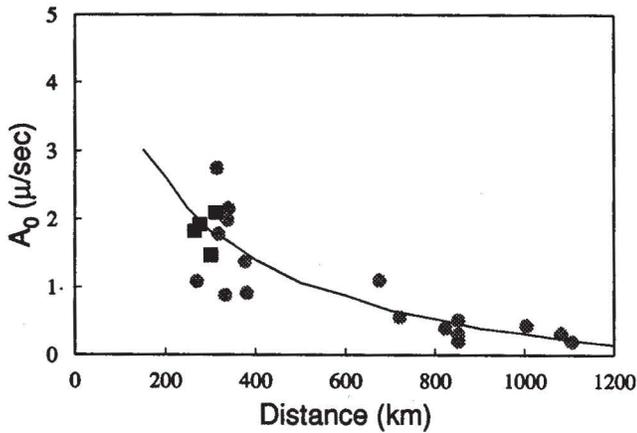


Fig. 4. Standard curve for  $h = 20$  km. This curve, along with equations 2 and 3, defines  $M_A$  scale. Squares: data from this study; circles: data from CMT catalog.

The procedure for determining the magnitude  $M_A$  consists of: (a) Filtering broadband seismograms between 15 and 30 sec; (b) Computing amplitude ( $A$ ) from:

$$A = \sqrt{A_E^2 + A_N^2 + A_Z^2} \tag{1}$$

where  $A_Z$ ,  $A_N$ , and  $A_E$  are the maximum amplitudes, in  $\mu/\text{sec}$ , measured on vertical, north-south and east-west components, respectively. (c) Estimating distance from S-P time, (d) Reading the value of the amplitude ( $A_0$ ) from the standard curve (Figure 4) corresponding to that distance. (e) Computing the moment ( $M_0$ ) from the relation:

$$M_0 = (A / A_0) \times 10^{23} \text{ dyne-cm} \tag{2}$$

(f) Computing the magnitude ( $M_A$ ) using the following relation:

$$M_A = (\log_{10}(M_0) - 16.1) / 1.5 \tag{3}$$

We note that equation (3) is the definition of the moment magnitude,  $M_w$  (Kanamori, 1977).

Because we intend to determine the magnitude before the earthquake is precisely located, it is assumed in the calculation that the depth is 20 km. The scatter in the data in Figure 4 suggests that the computed  $M_A$  may differ from  $M_w$  by about 0.2 units. This also can be seen from Figure 5. In this figure we compare  $M_A$  with other magnitude determinations (Tables 1 and 4). In dots and filled squares we plot all events with  $M_w$  determined from the seismic moment, most of which were used in the magnitude calibration (Table 1). Most points, except for the large events and two anomalous events, lie within the  $\pm 0.2$  magnitude units range given by the dotted lines.

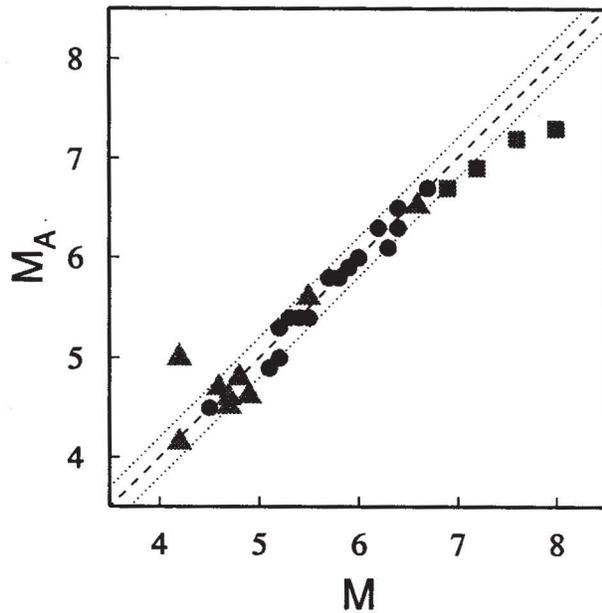


Fig. 5. Comparison of  $M_A$  with  $M$  for events listed in Tables 1 and 4. Squares: very large events ( $M=M_w$ ), dots: moderate to large events ( $M=M_w$ ), triangles: small to moderate events ( $M=m_b$ ). Dotted lines represent  $\pm 0.2$  magnitude range.

#### LIMITATION OF THE $M_A$ SCALE

$M_A$  as defined in equation 3 should not saturate for events whose source displacement spectra are flat at periods shorter than 15 to 30 sec. This is likely to be true for magnitudes less than 7.0 or so. We tested this assumption using digital accelerograms recorded at UNM during the 25 April, 1989 ( $M_s = 6.9$ ), San Marcos earthquake. The accelerograms were integrated to obtain velocity and then filtered in the same way as the broadband data. Figure 6a shows the vertical component of the velocity. Amplitude  $A$  (equation 1) for this earthquake is 2466  $\mu/\text{sec}$ . Using equation 2, we estimate a moment of  $1.45 \times 10^{26}$  dyne-cm, which from equation 3 corresponds to an  $M_A = 6.71$ . For comparison, CMT reports a moment of  $2.5 \times 10^{26}$  dyne-cm ( $M_w = 6.86$ ). This suggests that  $M_A$  may not saturate for  $M_w < 7.0$ .

We also computed filtered velocities from digital accelerograms of the 19 and 21 September, 1985 Michoacan earthquakes, recorded at Tacubaya, Mexico, D.F. The vertical component is shown in Figure 6b,c. The values of  $A$  are 14800 and 11400  $\mu/\text{sec}$  for the 19 and 21 September events, yielding seismic moments of  $1.1 \times 10^{27}$  ( $M_A = 7.3$ ) and  $6.9 \times 10^{26}$  dyne-cm ( $M_A = 7.2$ ), respectively. These values are extremely low in comparison with the reported values of  $M_w = 8.1$  and 7.6 for the two events. For such major and great earthquakes the waves with 15 to 30 sec period generated from different parts of the fault plane may not add coherently at CU, resulting in saturation of

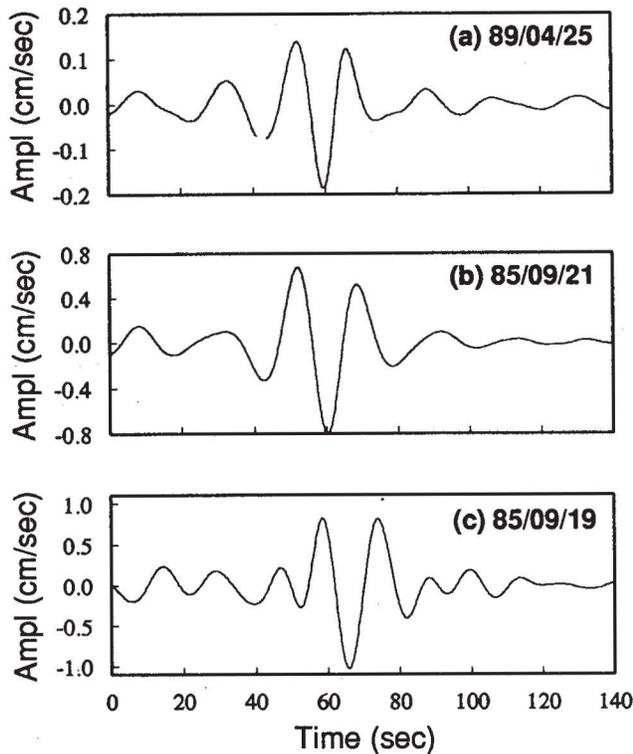


Fig. 6. Vertical velocity records obtained from integrating accelerograms from Tacubaya and then band-pass filtering between 15 and 30 sec. (a) April 25, 1989, (b) September 21, 1985, and (c) September 19, 1985.

the proposed magnitude scale beyond about  $M_w = 7$ . These three earthquakes and the recent September 10, 1993 Chiapas earthquake ( $M_w = 7.2$ ) are represented on Figure 5 as squares. Although the San Marcos and the Chiapas earthquakes are within the  $\pm 0.2$  error range, the trend towards the saturation has already developed at  $M_w = 7.0$ .

Figure 6 shows that 15 to 30 sec waves energy can be extracted from digital accelerograms recorded at hill sites in Mexico City, equipped with 12 bit A/D converters, for magnitude greater or equal 6.9 events. However, STS-1 seismometers may saturate for events with  $M$  greater than about 6.0. It raises the question whether useful data can be obtained for  $6 < M < 6.9$  events. Figure 7 compares the filtered velocity records from the broadband seismograms and accelerograms recorded at UNM for the second of the 15 May, 1993 doublet ( $M = 6.0$ ). The accelerograph which recorded this event has a 12 bit A/D converter connected to Kinometrics FBA-23 sensors with full scale range of  $\pm 100$  gals (Quaas *et al.*, 1993). Note that the amplitudes obtained from broadband seismograms and accelerograms are very similar. Recently the UNM station has been upgraded to a Quanterra 24-bit digitizer connected to STS-2 seismometers and FBA-23 accelerometers. This would permit on-scale recording of either velocity and/or acceleration from all moderate and large events.

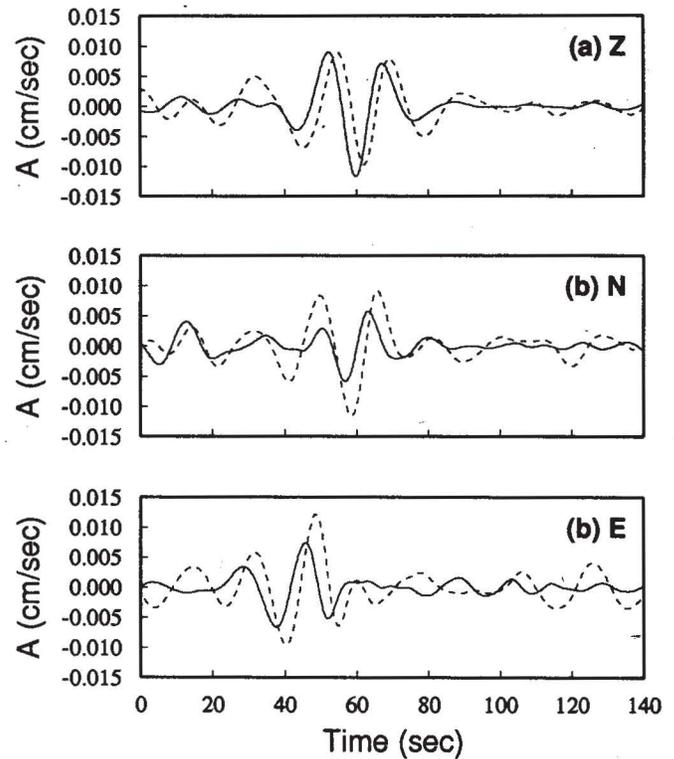


Fig. 7. Comparison of the broadband velocity records (solid line) with the integrated accelerograms of May 15, 1993 event ( $M_w = 5.9$ ) from CU (dashed line), both band-pass filtered between 15 and 30 sec. (a) Z, (b) NS, (c) EW.

Apart from the saturation problem,  $M_A$  is only appropriate for shallow events located at  $R \geq 200$  km from CU since 15 to 30 sec surface waves are well developed only for such events. It is for these reasons that we now explore another magnitude scale.

#### MAGNITUDE SCALE, $M_E$ , BASED ON RADIATED SEISMIC ENERGY

A measure of the size of an earthquake is provided by the radiated seismic energy,  $E_s$ . Recently the following relationship between  $E_s$  and  $M_w$  for Mexican earthquakes was developed by Singh and Ordaz (1993):

$$\log_{10}(E_s) = 1.5 M_w + 11.95 \quad (E_s \text{ in ergs}) \quad (4)$$

In this relationship  $E_s$  was estimated from coastal and inland digital accelerograms recorded at local and regional distances ( $R \leq 150$  km). As seismic waves are known to be amplified at CU (Singh *et al.*, 1988; Ordaz and Singh, 1992), equation (4) is not expected to be valid for this site. In the following we develop an appropriate  $E_s$ - $M_w$  relationship for CU.

**DATA AND ESTIMATION OF RADIATED ENERGY**

The data set used in deriving the  $E_s$ - $M_w$  relation at CU is given in Table 3. It includes the events listed in Table 1, shallow events which have given rise to good quality digital accelerograms and intermediate depth events recorded on VBB (Figure 8).

$E_s$  was computed using the following expression (see Singh and Ordaz, 1993)

$$E_s = \frac{4 \pi R^2 [G^2(R) / R^2] \rho \beta}{F_s^2} \left[ 2 \int_0^\infty [v_N^2(f) + v_E^2(f) + v_Z^2(f)] e^{2\pi f R / \beta Q(f)} df \right] \quad (5)$$

where  $R$  = hypocentral distance,  $\rho$  = density,  $\beta$  = shear-wave velocity,  $Q(f)$  = quality factor,  $F_s$  = free-surface amplification,  $v_i(f)$  = velocity amplitude spectrum of the  $i$ -th component and  $G(R)$  is the geometrical spreading term, which may be written as:

$$G(R) = \begin{cases} R & \text{for } R \leq R_0 \\ \sqrt{R_0 R} & \text{for } R > R_0 \end{cases} \quad (6)$$

In accordance with Singh and Ordaz (1993) we took  $\rho=2.8 \text{ gm/cm}^3$ ,  $\beta = 3.5 \text{ km/sec}$ ,  $Q(f) = 273 f^{0.66}$  (Ordaz and Singh, 1992),  $F_s = 2$ , and  $R_0 = 100 \text{ km}$ . The intense part of the ground motion was included in computing the spectra.

A plot of  $E_s$  against  $M_w$  is shown in Figure 9. It is known that the amplification at C.U. is frequency dependent (e.g., Ordaz and Singh, 1992), reaching a maximum of about 10 between 3 and 5 seconds and falling to 1 at longer and shorter periods. For this reason  $\log(E_s)$ - $M_w$  relation is not expected to be linear as in equation (4). Also,  $E_s$  in CU is expected to be greater than  $E_s$  given by equation (4) (see Appendix A). In view of the scatter in the data (Figure 9), however, a linear  $\log(E_s)$ - $M_w$  relation seems reasonable. With the slope fixed to 1.5 we obtain:

$$\log_{10}(E_s) = 1.5 M_w + 12.68 \quad (7)$$

In Figure 9 solid line shows the relation (7), dashed lines give  $\pm$  one standard deviation of  $\log_{10} E_s$  (0.42), and dotted line is the relation (4) from Singh and Ordaz (1993). From equation (7) we define the magnitude scale,  $M_E$ , by:

$$M_E = \frac{2}{3} \log_{10}(E_s) - 8.45 \quad (8)$$

Table 3  
Events whose records at CU have been used to establish the  $E_s$ - $M_w$  relationship

Date	Time	Distance (km)	Azi (°)	Depth (km)	Moment (Nm)	Energy (erg)	Comments
850919	1317	336	61	17	1.1x10 <sup>21</sup>	4.5x10 <sup>24</sup>	s.m. tac and cu
850921	0137	306	50	22	2.6x10 <sup>20</sup>	4.2x10 <sup>23</sup>	s.m. tac and cu
890425	1429	278	358	15	2.4x10 <sup>19</sup>	4.4x10 <sup>23</sup>	s.m. tac and cu
910401	0734	376	331	40	6.3x10 <sup>17</sup>	4.4x10 <sup>21</sup>	bb
910407	0939	332	323	47	5.5x10 <sup>16</sup>	1.8x10 <sup>20</sup>	bb
910528	0056	276	14	27	8.1x10 <sup>15</sup>	2.2x10 <sup>19</sup>	bb
910725	1546	380	38	15	5.3x10 <sup>17</sup>	5.7x10 <sup>20</sup>	vbb
910918	0948	1000	301	15	2.2x10 <sup>18</sup>	8.3x10 <sup>21</sup>	bb
911027	1857	114	4	56	8.9x10 <sup>14</sup>	1.5x10 <sup>18</sup>	vbb and sm
911124	0347	337	327	15	1.2x10 <sup>17</sup>	2.6x10 <sup>20</sup>	bb
920109	0403	254	8	30	1.2x10 <sup>18</sup>	2.1x10 <sup>19</sup>	vbb
920212	1156	269	50	34	7.0x10 <sup>16</sup>	3.5x10 <sup>20</sup>	vbb
920317	0643	850	312	30	1.7x10 <sup>17</sup>	4.3x10 <sup>20</sup>	vbb
920331	2056	322	43	15	1.6x10 <sup>17</sup>	7.3x10 <sup>20</sup>	vbb
920530	1630	851	312	29	3.5x10 <sup>18</sup>	3.6x10 <sup>21</sup>	vbb
920607	0901	339	344	15	1.2x10 <sup>17</sup>	6.8x10 <sup>20</sup>	vbb
920607	1741	313	343	15	7.9x10 <sup>16</sup>	9.7x10 <sup>20</sup>	vbb
920928	0741	1100	309	15	1.3x10 <sup>18</sup>	2.6x10 <sup>21</sup>	vbb
930331	1018	311	40	26	1.9x10 <sup>17</sup>	8.0x10 <sup>20</sup>	vbb
930515	0309	300	350	20	5.8x10 <sup>17</sup>	3.0x10 <sup>21</sup>	vbb and sm
930515	0319	300	350	20	1.0x10 <sup>18</sup>	1.5x10 <sup>22</sup>	sm
930903	1235	850	309	22	1.5x10 <sup>19</sup>	7.3x10 <sup>22</sup>	vbb
930910	1912	851	308	22	8.0x10 <sup>19</sup>	2.1x10 <sup>23</sup>	vbb
930919	1410	822	311	16	5.4x10 <sup>18</sup>	5.8x10 <sup>21</sup>	vbb

bb: broad-band data (5 sps)  
vbb: very broad-band data (20 sps)

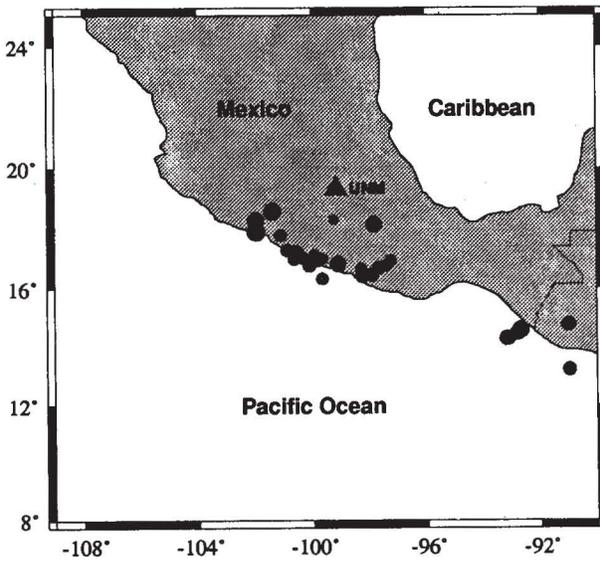


Fig. 8. Map showing the location for the events listed in Table 3 and used in obtaining the energy-magnitude relation at CU. Symbol size is proportional to  $M_w$ .

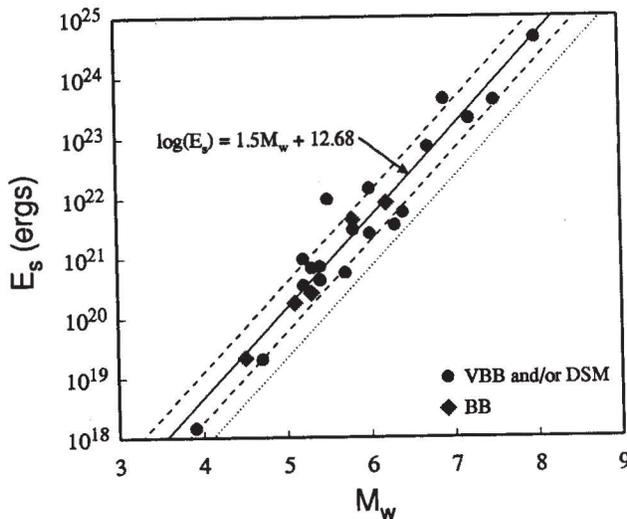


Fig. 9.  $E_s$  at CU versus  $M_w$  for events listed in Table 3. Solid line: best fit with fixed slope of 1.5, dashed line: standard deviation, dotted line: equation 4.

Because of the site effect, the estimated radiated seismic energy,  $E_s$ , at CU is, on average, five times greater than that estimated from coastal data (compare equations 4 and 8). Clearly  $E_s$  at CU from equation (5) should not be taken as an estimate of radiated seismic energy from an earthquake. Equation (8), however, may be used to determine the energy magnitude,  $M_E$ , with a standard error of 0.28.

In Figure 10 we performed a test on the validity of equation (8) comparing  $M_E$  with  $M_w$  as given in Table 3 and two other data sets: (1) Poor CU strong motion records

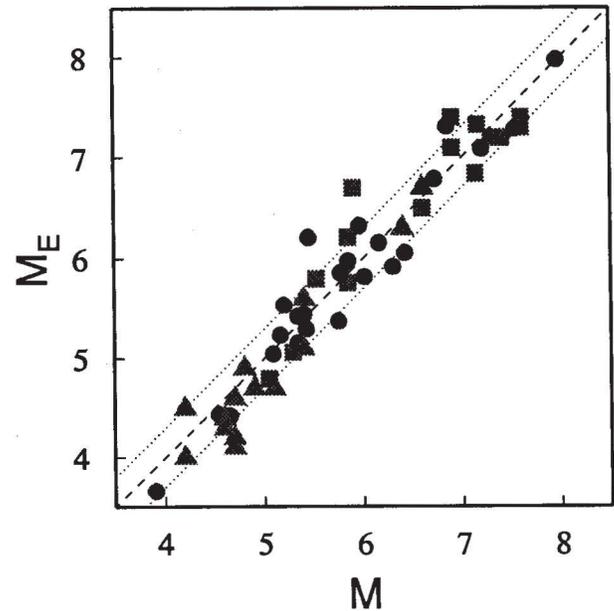


Fig. 10. Comparison of  $M_E$  with  $M$  for events listed in Tables 3 and 4. Squares: large earthquakes whose energy was computed from poor quality strong motion records ( $M = M_w$ ), dots: small to large events with good strong motion, VBB or BB recordings ( $M = M_w$ ), and triangles: small events with VBB recordings ( $M = m_b/M_s$ ). Dotted lines represent  $\pm 0.3$  magnitude range.

of past events with known  $M_w$ , (2) Recent broadband CU recordings with no estimate of  $M_w$  but reported values of  $m_b$  or  $M_s$  (Table 4). From Figure 9 and 10 we conclude that  $M_E$ , defined in equation (8), is a versatile magnitude scale tied to  $M_w$ , which does not suffer from saturation and may be valid for events of all depths.

#### POSSIBLE LIMITATIONS OF THE $M_E$ SCALE

The estimation of  $E_s$  from equation (5) requires broadband velocity amplitude spectrum. A major part of the contribution to  $E_s$ , however, comes from frequencies near the corner frequency,  $f_c$ , of the earthquake. This frequency,  $f_c$ , lies between about 0.025 and 2 Hz for events with magnitude between 8 and 4.5, respectively. It follows that reliable estimates of  $E_s$  of large events requires well-recorded long-period data whereas higher frequencies need to be well recorded for  $E_s$  estimation of moderate and small events. Singh and Ordaz (1993) show that the integration in equation 5 should be carried out until 6 times  $f_c$  to account for 80% of the total  $E_s$ . It would appear, then, that it may not be possible to reliably estimate  $E_s$  for the events listed in Table 3 since for larger events (19 and 21 September, 1985; 25 April 1989) only accelerograms are available which may not resolve that long period spectra and for other moderate events only VBB (20 samples/sec; cut off

Table 4

Events with either poor quality accelerograms or with VBB data but with only reported  $m_b$  or  $M_s$  values

Date	Time	Distance (km)	Azi (°)	Depth (km)	M	$M_E$	$M_A$
19640706	0722	179	51	96	7.3 <sup>1</sup>	7.2	-
19650823	1946	500	318	16	7.4 <sup>2</sup>	7.2	-
19680203	0536	327	6	29	5.9 <sup>3</sup>	6.7	-
19680802	1406	362	340	16	7.3 <sup>2</sup>	7.2	-
19780319	0139	290	14	27	6.6 <sup>4</sup>	6.5	-
19781129	1052	443	322	18	7.6 <sup>2</sup>	7.3	-
19790314	1107	288	53	27	7.6 <sup>2</sup>	7.4	-
19801024	1453	191	314	63	7.1 <sup>4</sup>	6.8	-
19811025	0322	319	68	32	7.2 <sup>4</sup>	7.3	-
19820607	0652	300	342	11	6.9 <sup>5</sup>	7.1	-
19820607	1059	281	341	18	6.9 <sup>5</sup>	7.4	-
19880208	1351	284	26	48	5.8 <sup>4</sup>	5.8	-
19891008	2232	258	18	35	5.1 <sup>4</sup>	4.8	-
19900113	0207	337	9	34	5.3 <sup>4</sup>	5.1	-
19900511	2343	274	32	15	5.5 <sup>4</sup>	5.8	-
19900531	0735	301	19	26	5.9 <sup>4</sup>	6.2	-
19930122	0637	339	344	100	4.2 <sup>6</sup>	3.4	-
19930311	2043	309	67	71	5.4 <sup>6</sup>	5.1	-
19930318	1851	300	48	43	4.7 <sup>6</sup>	4.6	4.6
19930320	1113	445	59	80	5.1 <sup>6</sup>	4.7	-
19930324	1118	326	350	33	4.6 <sup>6</sup>	4.3	4.7
19930525	0623	273	346	51	4.2 <sup>6</sup>	4.0	4.2
19930719	1511	172	49	74	4.7 <sup>6</sup>	4.2	-
19930805	0120	238	333	51	4.9 <sup>6</sup>	4.7	4.6
19930820	1306	388	66	33	4.2 <sup>6</sup>	4.5	5.0
19930826	1159	260	39	31	4.6 <sup>6</sup>	4.4	4.7
19930827	0145	251	40	33	4.7 <sup>6</sup>	4.2	4.5
19930829	0840	199	52	87	4.7 <sup>6</sup>	4.2	-
19930910	1050	287	348	27	4.8 <sup>6</sup>	4.9	4.8
19930910	1728	866	308	28	5.4 <sup>7</sup>	5.5	5.6
19930930	1827	567	319	20	6.4 <sup>7</sup>	6.3	6.5
19931024	0752	310	356	29	6.6 <sup>7</sup>	6.7	6.5

1:  $M_w$  from Gonzales-Ruiz (personal communication, 1986)2:  $M_w$  from Chael & Stewart (1982)3:  $M_s$  from ISC.4:  $M_w$  from CMT (Harvard Catalog)5:  $M_w$  from Astiz & Kanamori (1984)6:  $m_b$  from the weekly PDE reports.7:  $M_s$  from the weekly PDE reports.

frequency  $\sim 5$  Hz) and/or BB (5 samples/sec, cut off frequency  $\sim 1.5$  Hz) data are available. Two exceptions are the doublet of 15 May 1993 listed in Table 3 and an earthquake on 29 July, 1993 ( $M \sim 4$ ), which were recorded by sensitive accelerographs as well as by broadband seismographs in CU on VBB and BB channels.  $E_s$  of 15 May 1993 doublet ( $M = 5.7, 5.9$ ) estimated from different data sets are nearly identical. Surprisingly these estimates differ only by a factor of about 1.5 even for the  $M \sim 4$  event.

Much of the contribution to the integral in equation (5) comes from  $f \leq 5$  Hz even for  $M \sim 4$  events. The reason, no doubt, is the amplification of the seismic waves at CU, which essentially blurs the corner frequency.

## CONCLUSIONS

For a quick and reliable magnitude determination of moderate and large Mexican earthquakes, we have developed two magnitude scales,  $M_A$  and  $M_E$ , which use the broadband velocity and/or the acceleration records from CU. While  $M_A$  is based on maximum amplitude on velocity seismograms of 15 to 30 sec waves,  $M_E$  is defined in terms of radiated seismic energy. Both of these magnitude scales have been calibrated against moment magnitude,  $M_w$ .

For shallow, moderate to large events located at hypocentral distances,  $R$ , greater than about 200 km, the magnitude based on amplitude data,  $M_A$ , works well. It,

however, begins to saturate for  $M_w > 7$  events. This scale is also not appropriate for events with  $R < 200$  km and/or depths  $\geq 50$  km. The  $M_E$  scale is more versatile; it is valid for all moderate and large events. For shallow events we recommend determination of both magnitudes since a significant difference in the two values may indicate an anomalous nature of the earthquake.

The current instrumentation in CU provides on scale, quasi-real time, digital broadband velocity and/or acceleration data of all moderate and large events. It should now be possible to determine  $M_A$  and  $M_E$  within about 1/2 hour of the event.

With the proposed installation of many broadband stations in Mexico, and satellite transmission of the data, a quasi-real time determination of moment tensors may become possible. It, however, will require reasonably accurate location and origin time of the events. This is likely to remain a problem in Mexico for some time to come. In the mean time, the methods proposed in this paper can be used to compute magnitude in a quick and reliable fashion.

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#### APPENDIX A

Here we investigate the site amplification effect at CU on the  $E_s$ - $M_w$  relation. For an  $\omega^2$  source model the radiated energy  $E_s$  is given by

$$E_s = c \int_0^{\infty} \frac{f^2}{(1 + f^2/f_c^2)^2} df \quad (A1)$$

If the amplification is given by  $A(f)$  then

$$E_s = c \int_0^{\infty} \frac{f^2 A^2(f)}{(1 + f^2/f_c^2)^2} df \quad (A2)$$

In equations A1 and A2,  $c$  = constant, and  $f_c$  = corner frequency, which, for Brune's (1970) model, is given by

$$f_c = 4.91 \times 10^6 \beta (\Delta\sigma / M_0)^{1/3} \quad (A3)$$

where  $\beta$  = shear-wave velocity in km/sec,  $\Delta\sigma$  = stress-drop in bars, and  $M_0$  = seismic moment in dyne-cm. We take  $\beta = 3.5$  km/sec and  $\Delta\sigma = 100$  bars. Recalling that

$$\log_{10} M_0 = 1.5 M_w + 16.1 \quad (A4)$$

we compute  $E_s$  for different values of  $M_w$  through equations A4, A3, and A1. In the absence of site effect  $\log(E_s)$ - $M_w$  relation is given by (Singh and Ordaz, 1993)

$$\log_{10} E_s = 1.5 M_w + 11.95 \quad (A5)$$

We chose constant  $c$  in (A1) such that it gives the same  $E_s$ - $M_w$  relation as in equation (A5). We now compute  $\log(E_s)$ - $M_w$  where  $E_s$  is given by equation (A2).  $A(f)$  is taken from Ordaz and Singh (1992). Figure A1 shows the results as a dashed line. For comparison the figure also shows estimated  $E_s$  at CU versus  $M_w$  and the  $E_s$ - $M_w$  relation from coastal data. Both expected and observed  $E_s$  at CU are greater than the expected value of  $E_s$  from coastal data. However, observed  $E_s$  is less than the expected  $E_s$  from equation (A2). This suggests that either  $A(f)$  is somewhat overestimated or else some energy has been lost because of the band-limited nature of the data.

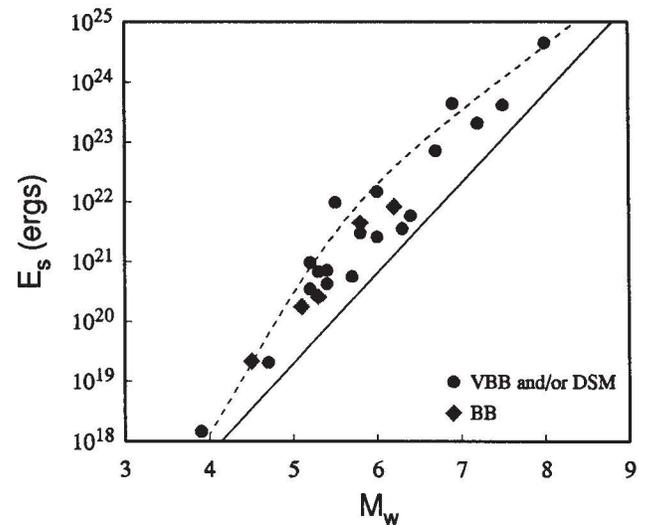


Fig. A1.  $E_s$  versus  $M_w$  plot. The data is from Table 3 (see also fig 9). Dashed line: predicted  $E_s$ - $M_w$  curve at CU including the amplification of seismic waves at the site. Solid line:  $E_s$ - $M_w$  relation based on coastal data. Note that the observed data generally lies between these two curves.

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