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Stochastic analysis of air and sea surface temperature time series in the North Atlantic

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RESUMEN

Se consideran unos modelos paramétricos lineales de la temperatura superficial del mar (SST) y del aire (AST) en el Atlántico Norte. Se discuten los aspectos teóricos de la aplicación de modelos autoregresivos estacionarios lineales (ARM) para las series de tiempo escalares consideradas. Se examinan algunos resultados de aplicar dicho método a los datos climáticos. Se obtiene un enfriamiento considerable al norte del paralelo 44 para SST y AST durante el presente siglo. Las series de los promedios anuales de AST y SST calzan mejor con ARM de bajo orden, predominando el primer orden. Las series de los promedios estacionales calzan mejor con órdenes superiores a uno.

PALABRAS CLAVE: Variable climática, Atlántico Norte, temperatura superficial del mar, temperatura del aire, tendencias de la temperatura

ABSTRACT

Linear parametric models of sea surface temperature (SST) and air surface temperatures (AST) in the North Atlantic are considered.Theoretical aspects of the application of linear stationary Autoregressive Models (ARM) of observed scalar time series are discussed. Some results of the application of the above method to the climatic data are examined. Considerable cooling of SST and AST in the XXth century is marked for the region north to 44°N. Annually averaged series of AST and SST are best fitted with low-order ARM and the first order prevails. Seasonally averaged series are best fitted with orders higher than 1.

KEY WORDS: Climatic variability, North Atlantic, sea surface temperature, air temperature, temperature trends.

I. INTRODUCTION

Autoregressive time series analysis is successfully used in economics and engineering, and also in geosciences. Traditional non-parametric methods for the analysis of climatic, meteorologic, hydrologic or oceanographic time series have two peculiarities making them ineffective in many cases (Privalsky *et al.*, 1992).

Firstly, it is hard to interpret the results of the analysis from the physical point of view. Secondly, climatic or hydrologic time series may not be long enough for the estimates obtained by traditional methods to be reliable.

Parametric methods overcome these shortcomings. The maximum entropy method provides detailed spectra in comparison with non-parametric methods. Moreover, this method enables one to obtain statistically acceptable results from rather short original data series.

The results of an analysis of air surface temperature (AST) and sea surface temperature (SST) scalar time series are presented in this paper. Original data and techniques are considered in section 2. Statistical parameters, parameters of autoregressive models (ARM), linear trends, spectra and statistical predictability are discussed in section 3. Conclusions are given in section 4.

II. DATA AND METHODS OF ANALYSIS

An advanced global climate archive, COADS (Comprehensive Ocean-Atmosphere Data Set) has now become available. A general description of COADS may be found for example in Oort *et al.*, (1987) and Pan and Oort (1990).

Though COADS contains records from 1854, only the period 1899-1979 is considered since there are many gaps in the preceding period. Since there are also numerous blanks in the data north of 52°N, especially in winter, the region of the research is limited to 30°N-52°N.

Monthly mean SST ($\overline{T}_w$) and AST ($\overline{T}_a$) at 2° x2° grid points from the relative section of COADS were used in this research. Data were averaged over 2°-latitude strips across the Atlantic Ocean. Two types of time series were prepared from the original data.

1. "Short" time series, of annually averaged original monthly means. The length of these series is 81.
2. "Long" time series, of seasonally averaged original monthly means. The length of these series is 324. The removal of seasonal averages preceded the analysis.

The technique of time series autoregressive (AR) analysis is discussed in detail in Box and Jenkins (1970).

If x_t is a stationary random linear regular process in discrete time ($t=...,-1,0,1,...$), we may express it as

$$x_t = \mu + \sum_{j=0}^{\infty} \psi_j a_{t-j} \tag{1}$$

where μ is the expectation of x_t, $a_{t-j}, j = 0,1,2...$ is a sequence of independent Gaussian random values with zero mean and variance σ_a^2 ("white noise"), and ψ_j is a weighting function which satisfies the condition $\sum_{j=0}^{\infty}\psi_j^2<\infty$. Equation (1) represents the process x_t as a white noise transformed by the weighting function ψ_j:

$$\psi(B)=1+\psi_1 B+\psi_2 B^2+\ \dots \tag{2}$$

where B is displacement operator $Bx_t=x_{t-1}$.
Writing (1) in the operational form one obtains

$$x_t=\psi(B)a_t\ . \tag{3}$$

After some transformations the autoregressive process of order p may be written:

$$x_t-\phi_1 x_{t-1}-\dots-\phi_p x_{t-p}=a_t, \tag{4}$$

where

$$\phi(B)=1+\phi_1 B+\phi_2 B^2+\dots+\phi_p B^p \tag{5}$$

is the autoregression operator and ϕ_j are the autoregression coefficients.

The most common criteria used for selecting the best-fitting ARM order are the *AIC* criterion (Akaike, 1976)

$$AIC(p)=\ln\left[(n-p)\hat\sigma_a^2\right]+2p/n, \tag{6}$$

and the *CAT* criterion (Parzen, 1977):

$$CAT(p)=\frac{1}{n}\sum_{j=1}^{p}\frac{n-j}{n}\hat\sigma_a^{-2}(j,\theta)-\frac{n-p}{n}\hat\sigma_a^{-2}p \tag{7}$$

Here $\hat\sigma_a^2$ is estimated on the white noise sequence $\{a_t, t=0,1,2\dots\}$.

A spectral density estimate obtained by the maximum entropy method (Smylie *et al.*, 1973) may be written

$$s(f)=\frac{2\sigma_a^2}{\left|1-\sum_{j=1}^{p}\phi_j e^{-i2\pi j}\right|^2} \qquad 0\le f\le 0.5, \tag{8}$$

where σ_a^2 is the white noise variance and f is the frequency.

Linear trends should be removed from the original time series before estimating the ARM parameters. A linear filter with the following weighting function was used for this purpose (Roden, 1963):

$$h(n-1)=\frac{6}{n^2}\left(\frac{2t}{n}-1\right), \qquad t=1,2,\dots,n \tag{9}$$

The mean square error (MSE) of the estimate $\hat\alpha$ of the inclination angle α is defined as:

$$\sigma_\alpha=\left(\frac{12\alpha_t^2}{n^3}\right)^{1/2} \tag{10}$$

Here α_t^2 is the time series variance after the removal of the trend. If $|\hat\alpha|$ exceeds $3\sigma_\alpha$ the trend is considered to be statistically significant.

Statistical predictability is characterised by the least-square relative prediction error (RPE). Low errors correspond to high predictability and vice versa. The RPE at unit lead time is most commonly used:

$$d_p(1)=D_p(1)/\sigma_\xi^2=\sigma_a^2/\sigma_\xi^2 \tag{11}$$

where σ_ξ^2 is the variance of a predicted process. The RPE satisfies the condition $0\le d_p(1)\le 1$. For a deterministic process $d_p(1)=0$ and for white noise $d_p(1)=1$.

Another useful parameter is the limit of statistical predictability τ_p. It is defined as the limit prediction lead time for which the MSE stays below the variance of the predicted process. In accordance with this definition the level of predictability is reached when the RPE attains a certain value γ (error level):

$$d_p(1)\ge\gamma \text{ when } \tau\ge\tau_p \tag{12}$$

III. RESULTS AND DISCUSSION

Almost all time series (both SST and AST) featured relatively warm periods separated by cold "gaps" (1916-18, 1929-30, 1941-43). Such periods of warming and cooling coincide to a great extent for different latitudes (see Figures 1 and 2).

The zonal means (T_w and T_a), the variances before and after removal of the linear trends $\left(\sigma_{T_w}^2,\sigma_{T_a}^2,\tilde\sigma_{T_w}^2\text{ and }\tilde\sigma_{T_a}^2\right)$ and the parameters of the linear trends (inclination angles $\hat\alpha_{T_a}$ and $\hat\alpha_{T_w}$, and their variances σ_a and σ_w) are given in Tables 1 and 2. The parameters of ARM for both SST and AST are found in Tables 5 and 6, and $d_p(1)$, $d_p(p)$, $\tau_p(1)$, in Table 7. Zonal means for seasonally averaged data are displayed in Figures 3 and 4.

The variability of AST and SST is not uniform in the region; the highest values of $\tilde\sigma_{T_w}^2$ and $\tilde\sigma_{T_a}^2$ (0.33 and larger) are found in 38°N - 44°N (see Figures 5 and 6).

Linear trends

The procedure of extracting linear trends from the original time series preceded the construction of ARM. Trends

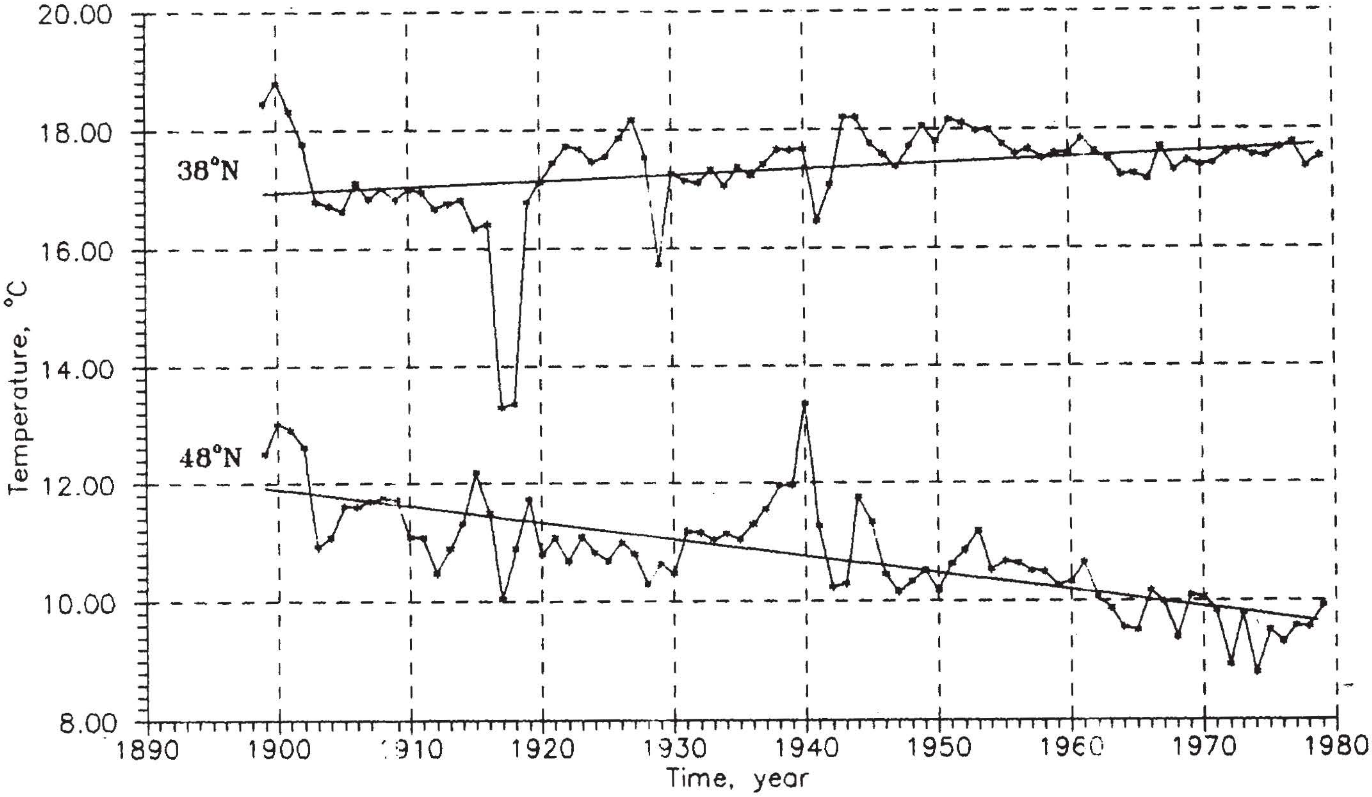

Fig. 1. Distribution of air surface temperature annual means in the North Atlantic and linear trends at 38°N and at 48°N (1899-1979).

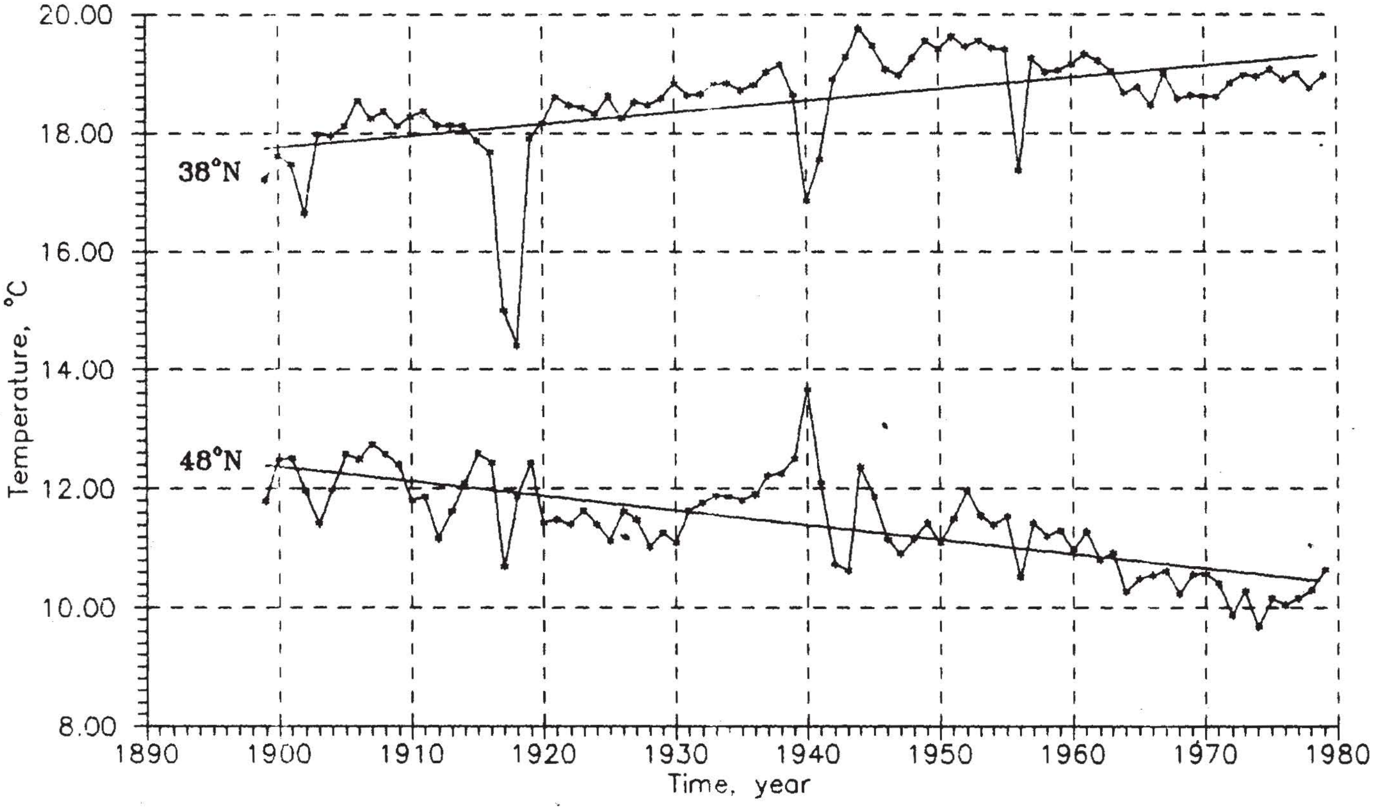

Fig. 2. Distribution of sea surface temperature annual means in the North Atlantic and linear trends at 38°N and 48°N (1899-1979).

Table 1

Mean values ($\bar{T}_a$), variances before the removal of a linear trend ($\sigma^2_{T_a}$) and after the removal ($\tilde{\sigma}^2_{T_a}$) and parameters of the latter ($\hat{\alpha}_{T_a}, \sigma_a$) of the annually averaged air surface temperature time series (1899-1979).

Latitude °N	$\bar{T}_a$, °C	$\sigma^2_{T_a}$, °C2	$\tilde{\sigma}^2_{T_a}$, °C2	$\hat{\alpha}_{T_a}$, °C/yr	σ_a, °C/yr
52	8.87	0.42	0.24	-0.009	0.003
50	10.46	0.58	0.22	-0.022	0.002
48	10.79	0.82	0.28	-0.026	0.003
46	11.43	0.52	0.30	-0.015	0.003
44	12.27	0.55	0.35	-0.007	0.003
42	13.40	0.57	0.36	0.003	0.004
40	15.46	0.57	0.33	0.008	0.003
38	17.33	0.67	0.35	0.009	0.004
36	18.81	0.34	0.12	0.009	0.003
34	19.96	0.15	0.10	0.004	0.002
32	20.94	0.14	0.09	0.004	0.002
30	21.75	0.14	0.08	0.004	0.002

Table 2

Mean values ($\bar{T}_w$), variances before the removal of a linear trend ($\sigma^2_{T_w}$) and after the removal ($\tilde{\sigma}^2_{T_w}$) and parameters of the latter ($\hat{\alpha}_{T_w}, \sigma_w$) of the annually averaged sea surface temperature time series (1899-1979).

Latitude °N	$\bar{T}_w$, °C	$\sigma^2_{T_w}$, °C2	$\tilde{\sigma}^2_{T_w}$, °C2	$\hat{\alpha}_{T_w}$, °C/yr	σ_w, °C/yr
52	9.58	0.26	0.20	-0.004	0.002
50	11.22	0.34	0.12	-0.017	0.002
48	11.43	0.62	0.22	-0.022	0.003
46	11.90	0.49	0.31	-0.012	0.003
44	12.80	0.45	0.35	-0.004	0.003
42	14.06	0.48	0.36	0.006	0.003
40	16.54	0.68	0.39	0.014	0.003
38	18.51	0.78	0.39	0.018	0.004
36	19.75	0.47	0.19	0.018	0.002
34	20.81	0.26	0.13	0.012	0.002
32	21.65	0.24	0.10	0.012	0.002
30	22.32	0.27	0.10	0.013	0.002

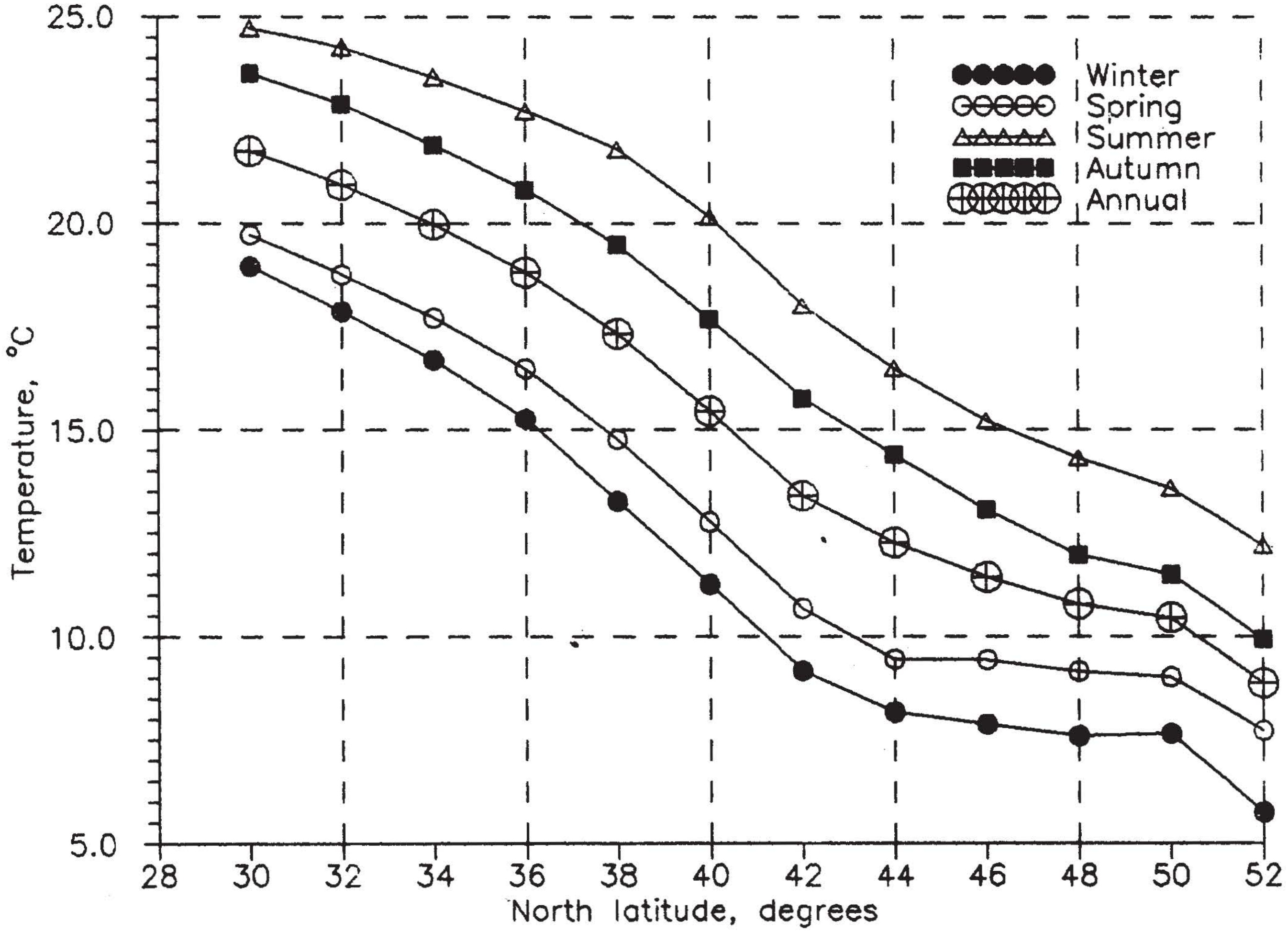

Fig. 3. Zonally averaged air surface temperature for all seasons of the year and annual in the North Atlantic (1899-1979).

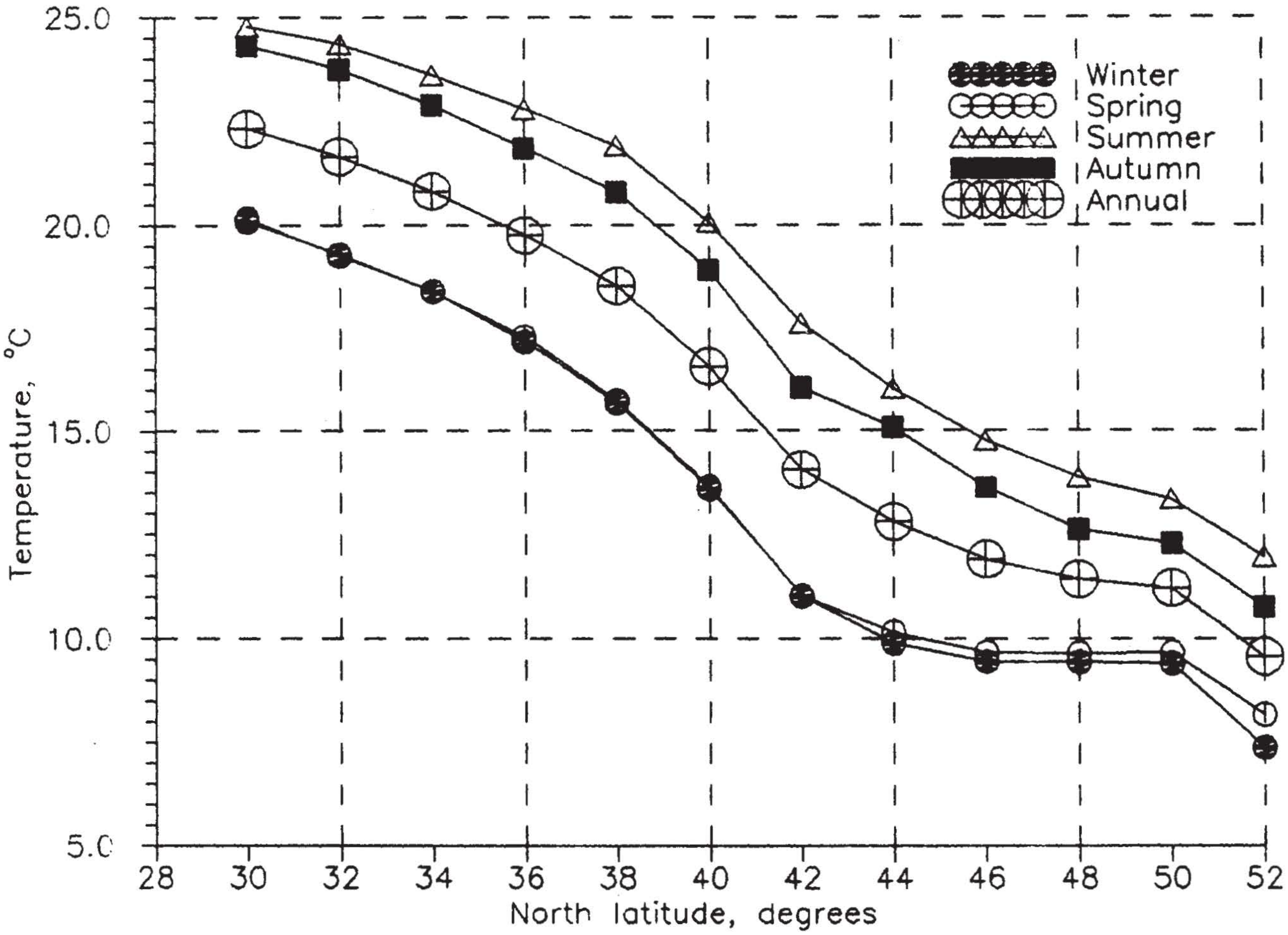

Fig. 4. Zonally averaged sea surface temperature for all seasons of the year and annual in the North Atlantic (1899-1979).

were computed for both "short" and "long" series (after the removal of seasonal averages).

Negative trends, with $|\hat{\alpha}_{T_w}|$ and $|\hat{\alpha}_{T_a}|$ equal to 0.01-0.02, prevail. The maxima are located at 48°N (-0.022 and -0.026 for AST and SST respectively). Cooling in the North Atlantic agrees with the results of the simulations (Karol, 1989; Kellog, 1990) and with the observations (Trenberth, 1990).

Some interesting results are found by comparing trends for different seasons of the year. The strongest cooling in the AST is found for 44°-50° N in winter and spring where the integral trend exceeds 1.6°C. At the same time the SST was cooling strongly in spring and the summer. Statistically significant AST warming does not exceed 0.81°C. Summer and autumn are the seasons when the strongest temperature rise (0.65°-0.81°C) is observed for 36°-40°N. Considerable warming of SST occurs basically in winter (1.6°C at 30°-42°N), but the temperature rise is significant at all seasons (see Tables 3 and 4).

Stochastic models

The Burg-Levinson algorithm described in detail by Smylie *et al.* (1973) was used to estimate the parameters of ARM. An optimum model order was chosen by the Akaike and Parzen criteria. The optimum order corresponds to the minimum criteria values. Note that both criteria point to the same models order in most cases.

ARM of "short" series

About one half of the time series was fitted by an ARM of order M=1. In 30% of the cases the SST and AST optimum orders agree. We failed to find any regularity in the distribution of M over the region. Thus, M varies from 1 (simple Markov chain) to 5 (AST), and from 1 to 4 (SST). In this sense, the stochastic models of SST and AST in neighbouring 2°-strips may differ essentially from each other (see Tables 5 and 6).

ARM of "long" series

The ARM estimated for the time series of seasonal means show considerable dispersion of p. It is interesting that the ARM with $p>1$ fit better both for the SST and the AST series. Since high orders of p prevail, it might not be necessary to list all the autoregression coefficients.

The autoregressive coefficient ϕ_1 is just the correlation coefficient between two neighbouring seasons. For the SST, ϕ_1=0.19 - 0.64 and for the AST ϕ_1= 0.21 - 0.46. Thus

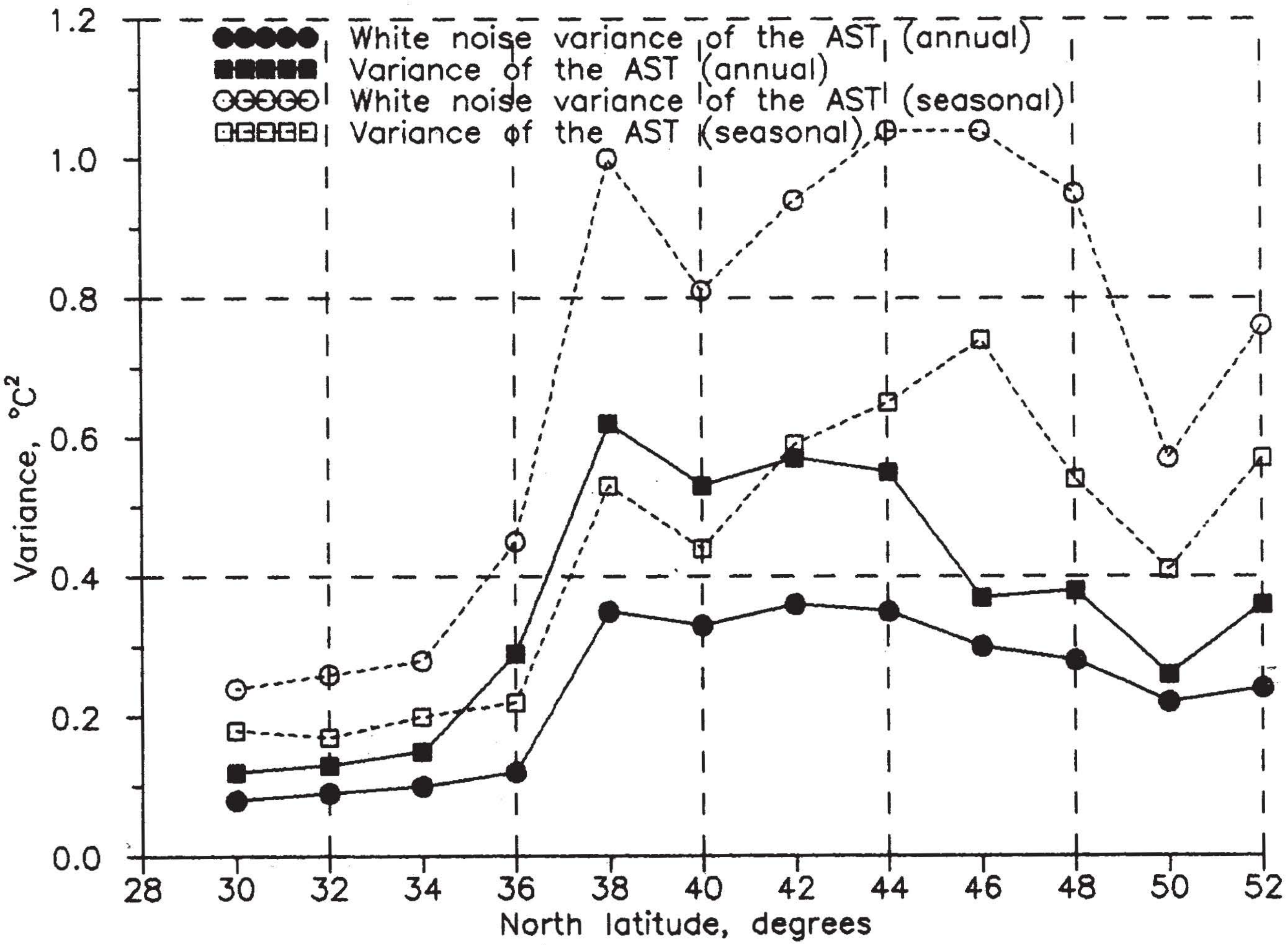

Fig. 5. Variance and white noise variance of air surface temperature computed from the annual and seasonal series in the North Atlantic (1899-1979).

Table 3

Statistically significant integral linear trends in zonally averaged air surface temperature time series in the North Atlantic (1899-1979)

Latitude °N	Winter	Spring	Summer	Autumn	Annual
52	-0.81	-0.81	-0.81		-0.81
50	-2.88	-1.60	-0.81	-0.81	-1.60
48	-3.24	-2.91		-0.81	-2.40
46	-2.00	-2.40			-0.81
44		-2.40			
42					
40			0.73	0.73	0.81
38		0.57	0.73	0.73	0.81
36		0.57	0.65	0.81	0.81
34				0.40	
32		0.40			0.32
30		0.40	0.40	0.32	0.32

Table 4

Statistically significant integral linear trends in zonally averaged sea surface temperature time series in the North Atlantic (1899-1979)

Latitude °N	Winter	Spring	Summer	Autumn	Annual
52			-0.81		
50	0.24	-1.60	-1.60	-0.81	-1.60
48	0.36	-2.88		-0.81	-1.60
46	-0.81	-2.40			-0.81
44		-1.60			
42	1.60		0.57	0.81	
40	1.60		0.81	1.60	0.81
38	1.60	1.60	0.81	1.60	0.81
36	1.60	1.60	0.81	1.60	0.81
34	1.60	0.81	0.65	0.81	0.81
32	1.60	0.81	0.48	0.81	0.81
30	1.60	0.81	0.81	0.81	0.81

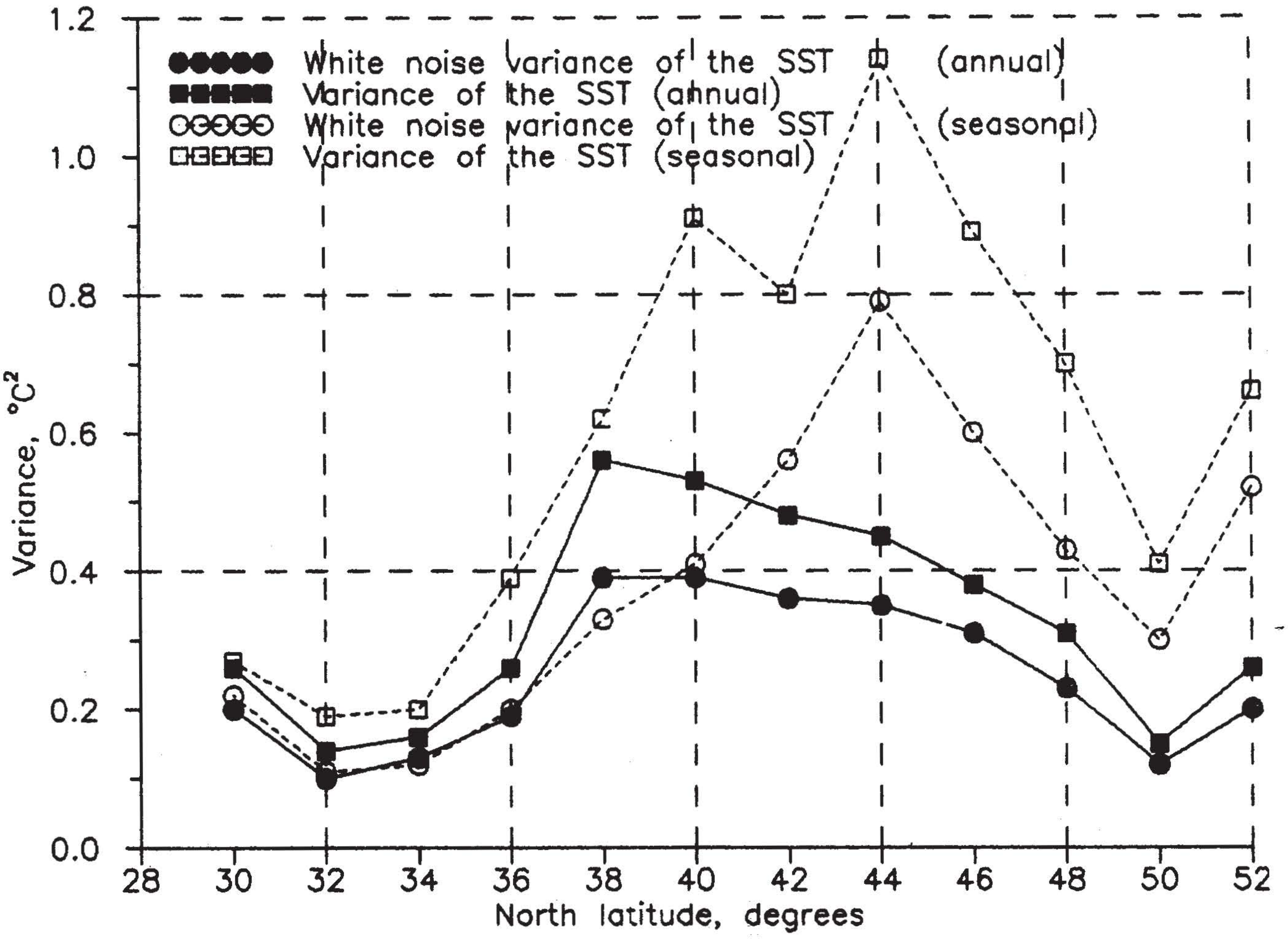

Fig. 6. Variance and white noise variance of the sea surface temperature computed from the annual and seasonal series in the North Atlantic (1899-1979).

Table 5

Optimum order (p) and autoregression coefficients (ϕ_n, n=1,...,5) of the ARM of air surface temperature in the North Atlantic (1899-1979)

Latitude °N	p	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
52	3	0.48	0.31	-0.28		
50	1	0.41				
48	1	0.52				
46	1	0.45				
44	3	0.70	-0.30	0.23		
42	1	0.60				
40	1	0.63				
38	2	0.78	-0.20			
36	4	0.96	-0.26	-0.17	0.24	
34	1	0.58				
32	3	0.49	-0.03	0.23		
30	5	0.29	0.21	0.23	0.20	-0.20

Table 6

Optimum order (ρ) and autoregression coefficients (ϕ_n, n=1,...,4) of the ARM of sea surface temperature in the North Atlantic (1899-1979)

Latitude °N	p	ϕ_1	ϕ_2	ϕ_3	ϕ_4
52	1	0.54			
50	1	0.45			
48	4	0.52	-0.16	0.04	0.21
46	2	0.49	-0.17		
44	1	0.48			
42	1	0.49			
40	1	0.52			
38	1	0.56			
36	1	0.53			
34	1	0.42			
32	3	0.36	0.04	0.27	
30	4	0.37	0.12	0.34	-0.17

the dependence between neighbouring years is stronger than between neighbouring seasons. Note that there are some zones where ϕ_4 is comparable with ϕ_1 and ϕ_2. For example, ϕ_4 exceeds ϕ_2 at 36°N: $\phi_4 = 0.26$ and $\phi_2 = 0.20$ (SST), $\phi_4 = 0.35$ and $\phi_2 = 0.01$ (AST). This fact testifies to the influence of a season on the same season next year. This influence is not constant: it varies from latitude to latitude. No physical explanation is suggested. The preliminary extracting of seasonal trends excludes any overestimation of the influence of the regular annual cycle on the results.

Statistical predictability

An error level $\gamma = 0.8$ was used in this work. The distribution of the RPE at a unit lead time of order p, $d_p(1)$, and of order 1, $d_1(1)$, for "short" series is given in Table 7. The average of $d_p(1)$ is 76% (SST) and 66% (AST), both values being close to those obtained by Privalsky (1985, 1988). The limit of statistical predictability τ_p equals 1 year for the greater part of the series.

For "long" series $d_p(1)$ averages 64% for both SST and AST, though its oscillations over latitude is considerable (see Figures 7 and 8) The RPE of order 1 is always higher for 2-11% (AST) and for 1-9% (SST) than the RPE of order p.

Spectra

The behaviour of the spectral function may be used as an additional qualitative estimate of the predictability. High values of τ_p and low errors $d_p(1)$ indicate a concentration of energy in the high frequencies and a presence of energetically meaningful maxima at intermediate frequencies.

Table 7

Relative prediction error at the unit lead time of the order 1 and of the optimum order p ($d_1(1)$ and $d_p(1)$ respectively) and the limit of statistical predictability for annually averaged series of air surface temperature and sea surface temperature in the North Atlantic (1899-1979)

Lat. °N	$d_1(1)$,%	$d_p(1)$,%	$\tau_p(1)$,yr	$d_1(1)$,%	$d_p(1)$,%	$\tau_p(1)$,yr
	Air surface temperature			Sea surface temperature		
52	74	67	2	72	72	1
50	84	84	1	83	83	1
48	73	73	1	95	75	1
46	80	80	1	77	94	1
44	67	64	1	77	77	1
42	64	64	1	77	77	1
40	62	62	1	73	73	1
38	58	56	1	70	70	1
36	44	40	2	73	73	1
34	67	67	1	83	83	1
32	70	67	2	76	73	2
30	76	65	3	77	68	3

The characteristic feature of the oscillations in climate time series is irregularity. In general, behaviour in the frequency domain can be described by the monotonic rise of oscillation intensity with decreasing frequency ("red spectrum"). A typical spectrum is shown in Figure 9.

The SST and AST spectra at low frequency intervals are close to the "red spectrum", except those which feature statistically meaningful peaks at 0.21, 0.28, 0.34 and 0.5 cycles/year (see Figure 10).

IV. CONCLUSIONS

Summarising the results described above, we may formulate the following conclusions.

1. Climatic time series of SST and AST over most of the region between 44°N and 52°N feature negative trends (both annually and seasonally averaged). The latter are higher for AST than for SST. This agrees with the results of simulations and analyses. Thus there exists a region (or regions) where a decrease of temperature has been observed during the 20th century even though the globally averaged AST and SST was held to increase Wu *et al.*, 1990).

2. The optimum order of AR models for "short" time series does not exceed 5, and the simple Markov chain is the best approximation for about one half of all series.

3. "Long" series are described by AR models of order $p>1$. Use of a simple Markov chain leads to a substantial rise in RPE.

4. The RPE averages 64% for "long" series. Minima are observed at 40°N ($d_p(1)=45\%$)for SST and at 36°N ($d_p(1) = 49\%$) for AST. One-step prediction is effective in the strip 34°-40°N (SST) and 36°-40°N (AST), where $d_p(1) < 60\%$. For "short" series $d_p(1)$ averages 76% (SST) and 66% (AST).

5. The limit of statististical predictability for moderate latitudes in the North Atlantic does not exceed 1 year, as a rule. The RPE decrease does not entail an increase in the statistical predictability limit. This parameter is higher for "long" series ($\tau_p=1.5$)

6. SST and AST spectra generally feature a concentration of energy at low frequencies. Statistically significant peaks may be present at frequencies of 0.2-0.5 cycles/year in some series. This is the cause of the relative high predictability of SST and AST.

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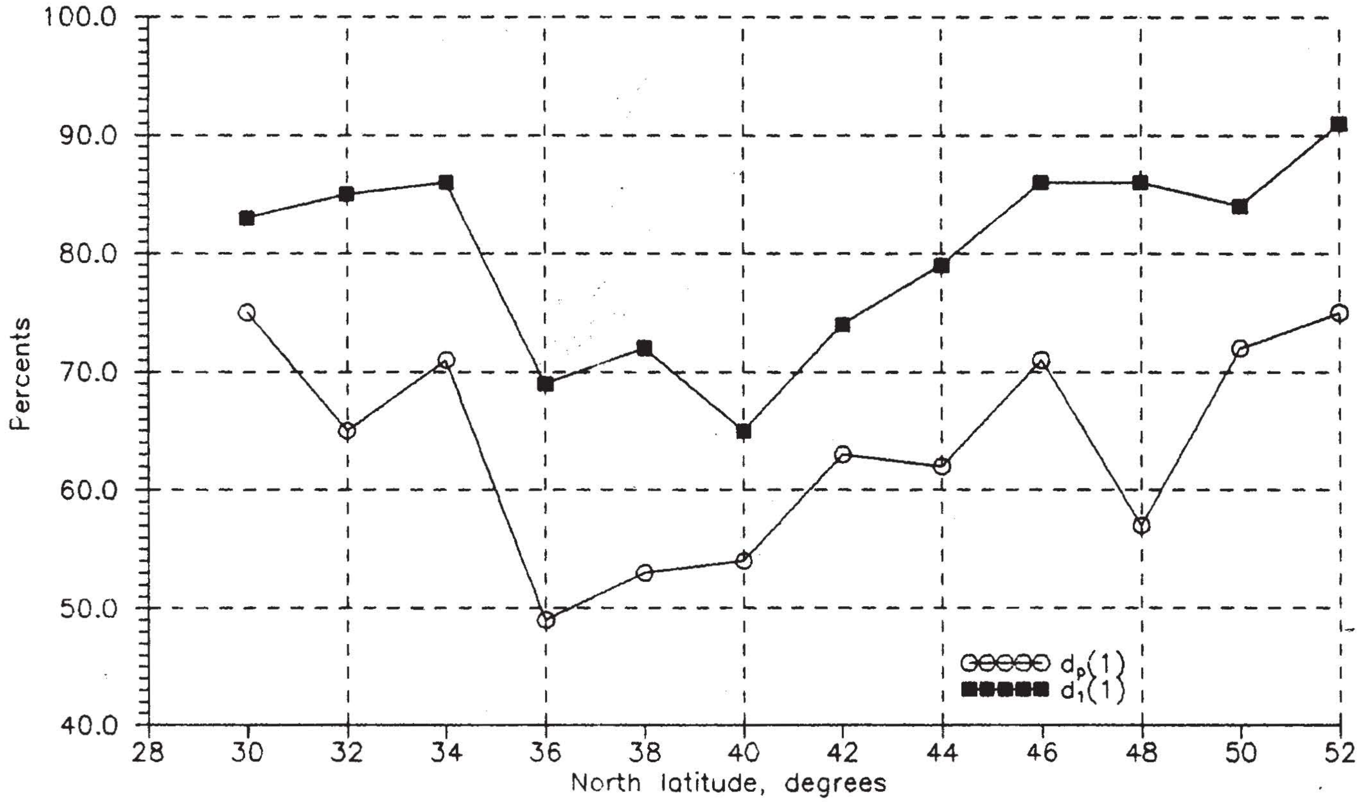

Fig. 7. Distribution of the relative prediction error at the unit lead time of zonally and seasonally averaged air surface temperature in the North Atlantic (1899-1979); $d_p(1)$ corresponds to the order p and $d_1(1)$ -to the order 1.

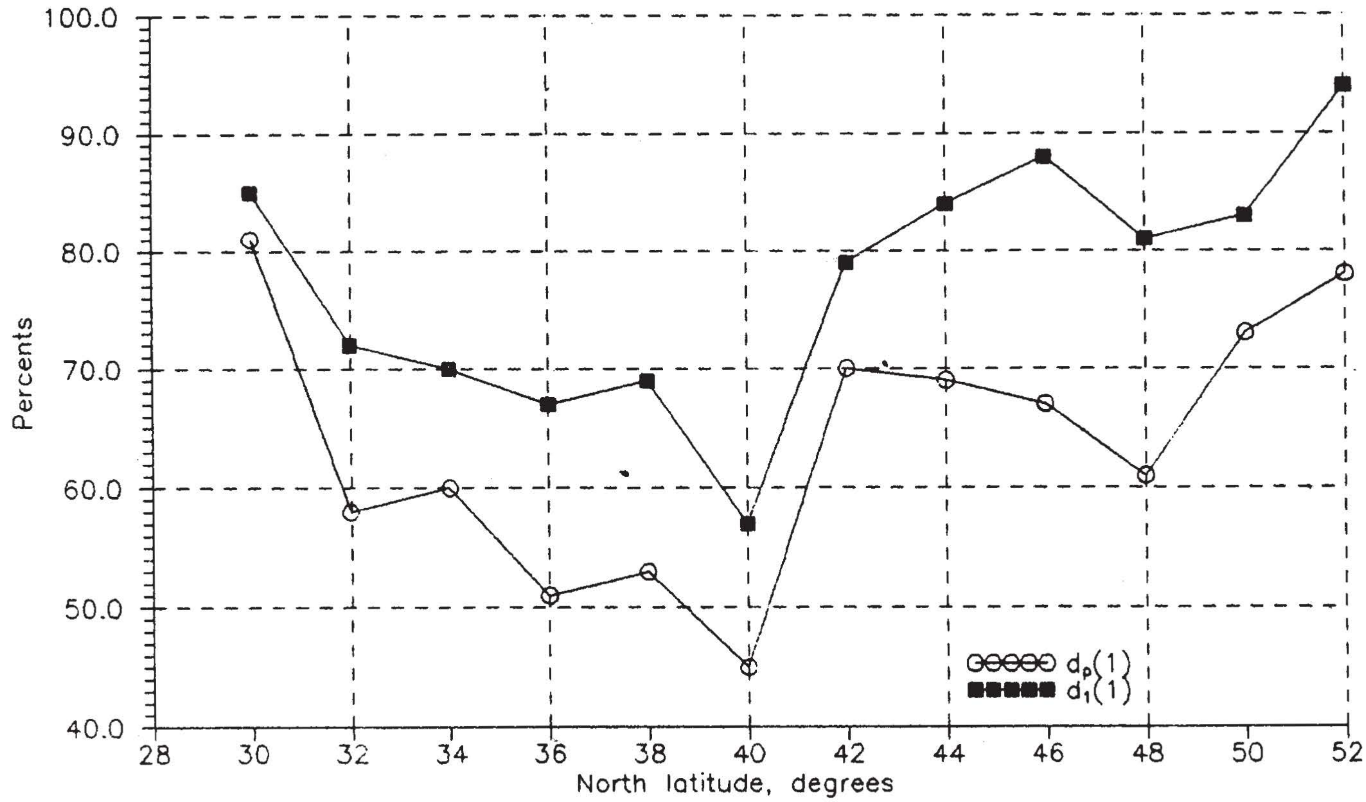

Fig. 8. Distribution of the relative prediction error at the unit lead time of zonally and seasonally averaged sea surface temperature in the North Atlantic (1899-1979); $d_p(1)$ corresponds to the order p and $d_1(1)$- to the order 1.

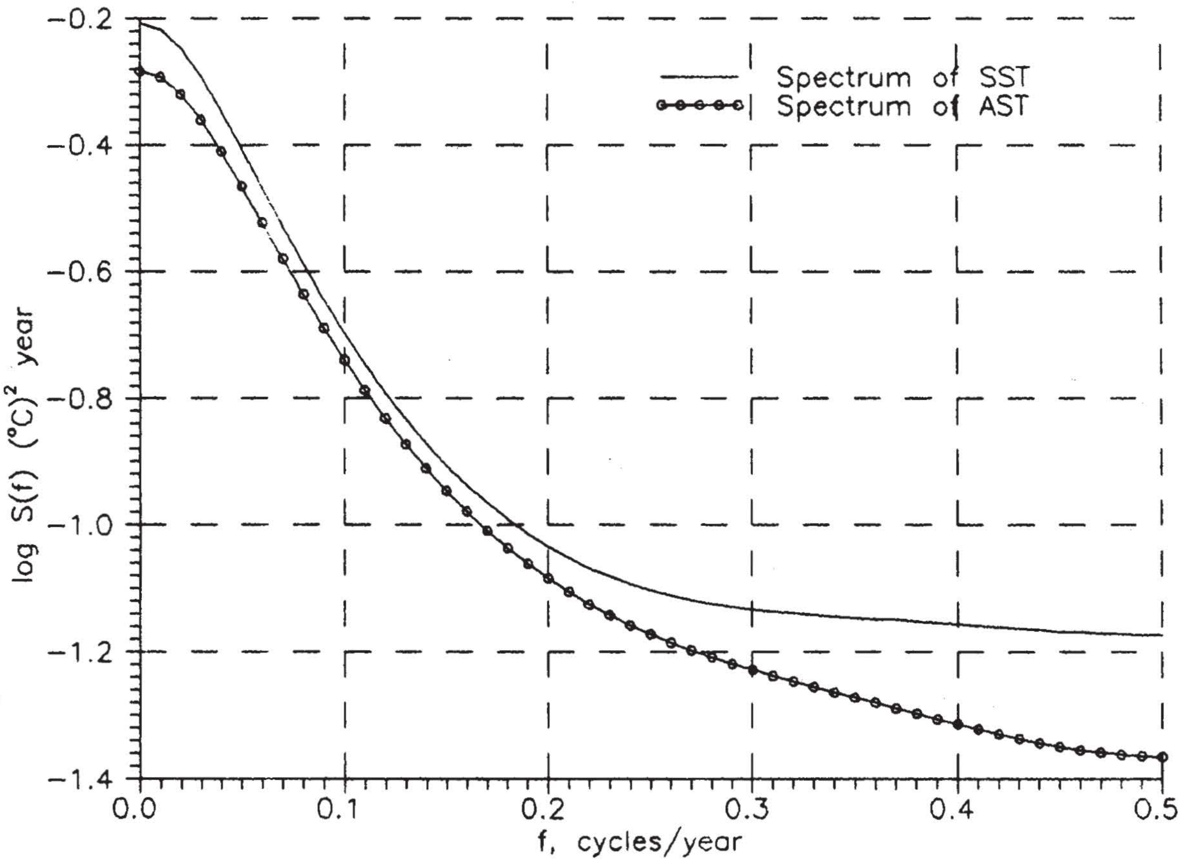

Fig. 9. Spectra of mean annual sea surface temperature and air surface temperature at 32°N and at 30°N in the North Atlantic.

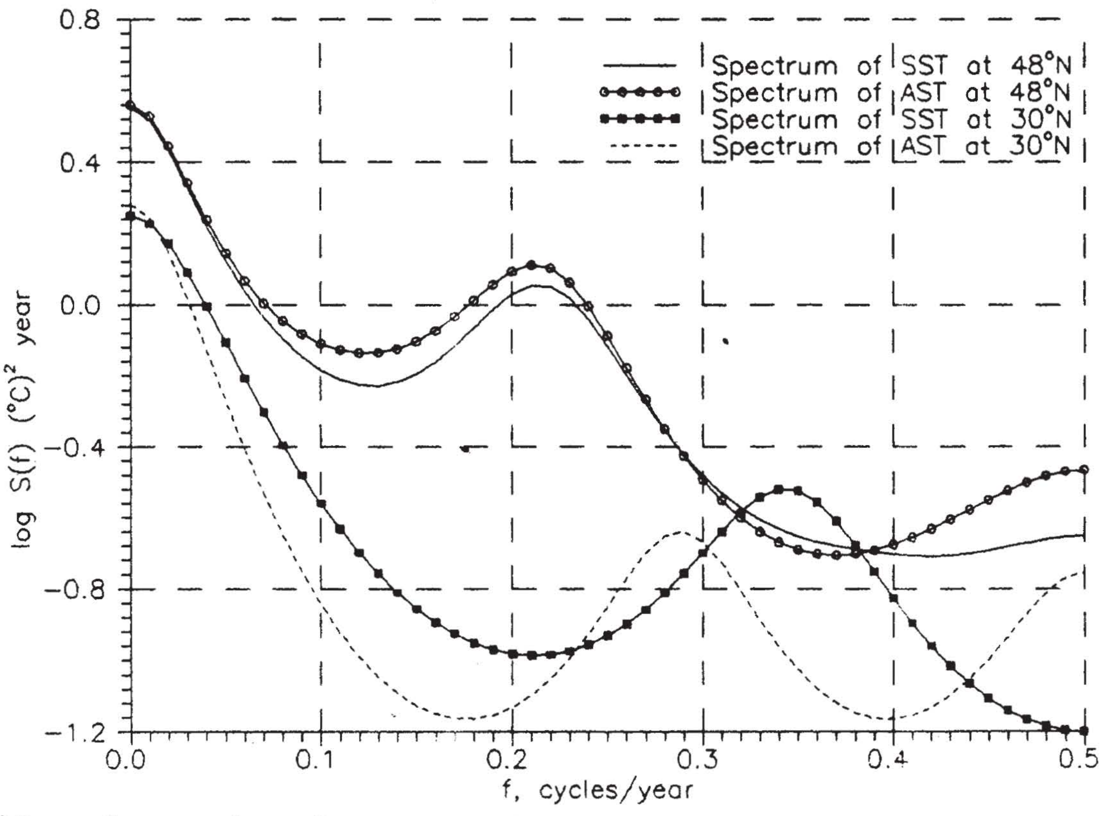

Fig. 10 Spectra of mean annual sea surface temperature and air surface temperature at 48°N and at 30°N in the North Atlantic.

BIBLIOGRAPHY

AKAIKE, H., 1976. On the entropy maximization principle. Proc. symp. on appl. of statist. Dayton (Ohio), 18.

BOX, G. E. P. and G. M. JENKINS, 1970. Time series analysis. Forecasting and control. Holden-Day, San Francisco.

KAROL, I. L., 1988. Introduction to the Earth's Climate Dynamics. Gridrometeoizdat, Leningrad, 216 p (in Russian).

KELLOG, W. W., 1991. Response to Skeptics of Global Warming. *Bull. Amer. Met. Soc., 74,* 499-511.

OORT, A. H., Y. H. PAN, R. W. REYNOLDS and C. F. ROPELEWSKI, 1987. Historical trends in the surface temperature over the oceans based on the COADS *Clim. Dyn., 2,* 29-38.

PAN, Y. H. and A. H. OORT, 1991. Correlation analyses between sea surface temperature anomalies in the eastern equatorial Pacific and the World Ocean. *Clim. Dyn., 12,* 1-188.

PARZEN, E, 1977. Multiple time series determining the order of approximating autoregressive schemes. *In: Multivariate analysis,* Amsterdam. North Holland, *4,* 238-295.

PRIVALSKY, V. E., 1985. Climatic variability. Stochastic models, variability, spectra. Nauka Publishers, Moscow, 184 p (in Russian).

PRIVALSKY, V. E., 1988. Stochastic models and spectra of interannual variability of mean annual sea surface temperature in the North Atlantic. *Dyn. Atmos. Oceans., 12,* 1-18.

PRIVALSKY, V. E., V. A. PANCHENKO and E. Y. ASARINA, 1992. Time Series Models with Applications to the Geosciences. Gidrometeoizdat, Saint-Petersburg, 222 p (in Russian).

RODEN, G., 1963. Sea level variations in Panama. *J. Geophys. Res., 68,* 5701-5710.

SMYLIE, D., G. CLARKE and T. ULRICH, 1973. Analysis of irregularities in the Earth's rotation. *Meth. Comput. Phys., 13,* 391-430.

TRENBERTH, K. E., 1990. Recent Observed Interdecadal Climate Changes in the Northern Hemisphere. *Bull. Amer. Met. Soc. 71,* 988-993.

WU, Z., R. E. NEWELL and J. HSIUNG, 1990. Possible factors controlling global marine temperature variations over the past century. *J. Geoph. Res. Ser. D, 95,* 11799-11810.

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