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Backus-Gilbert inversion of potential field data in the frequency domain and its application to real and synthetic data

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RESUMEN

Se presenta un algoritmo para la inversión de datos del campo potencial en el dominio de las frecuencias utilizando la transformación de Backus-Gilbert. Se describe uno de los problemas fundamentales en todo proceso de interpretación geofísica como es la solución del problema directo y del problema inverso. La comparación de los resultados en el dominio del espacio y de las frecuencias muestra las ventajas del algoritmo aquí presentado. Se demuestra la efectividad del algoritmo solucionando tareas de geofísica ambiental como la detección de depósitos antiguos de desechos industriales. Esta técnica interpretativa es aplicable también a la interpretación de investigaciones geofísicas en sitios arqueológicos.

PALABRAS CLAVE: Inversión Backus-Gilbert, exploración geofísica, métodos de potencial.

ABSTRACT

An algorithm for the inversion of potential field data in the frequency domain using the Backus-Gilbert method is represented. This leads to an underdetermined system of linear algebraic equations. It can be easily solved because the matrix has non-zero elements only on its main diagonal. Since the solution represents a harmonic function, its extremes are located at the boundary of the model. This leads to unacceptable distribution functions of physical parameters. Therefore the concept of weighted minimum length was introduced. Advantages and drawbacks of weighting in the frequency domain are discussed. Theoretical as well as practical examples suggest that the algorithm may be applied in practice. A comparison of the Backus-Gilbert inversion in space domain and frequency domain from a numerical point of view shows the advantages of the proposed algorithm.

KEY WORDS: Backus-Gilbert inversion, geophysical exploration, potential field methods.

INTRODUCTION

The aim of potential field interpretation in geophysical exploration or environmental studies is to derive a meaningful model of subsurface structure from data measured at the surface. As is well known, any potential field anomaly can be generated by an infinite number of distributions of physical parameters. This leads to an inverse problem which, in general, is nonlinear and ill-posed.

In order to treat this problem numerically, discretization and orthogonal decomposition of the distribution function of physical parameters must be applied. By fixing the geometry of the model, the nonlinearity can be avoided. One commonly used method is subdividing the region under the profile or area of investigation into a number of cells, where it is assumed that the source of the anomalous potential field lies within this region.

Even if the problem were linearized in the described manner and the number of measurements were much greater than the number of cells, the derived system of equations would still be poorly conditioned, and using conventional least squares estimation of the model parameters would fail. One way to overcome this problem is to add some information not contained in the data or in the equations and to introduce additional constrains.

Mottl and Mottlova (1972) used integer linear programming to optimize a so-called "shape preference" function. Green (1975) and Chávez and Garland (1983) found a model with minimum distance from an initial guess using the *Backus-Gilbert method* (Backus and Gilbert, 1967). Safon *et al.* (1977) used linear programming and constraints on static moments of the mass distribution. Last and Kubik (1983) searched for a solution with maximum compactness to fit the anomaly in a weighted least squares sense. This method was later broadened by Guillen and Menichetti (1984), who used a minimum constraint on the moment of inertia of the mass distribution with respect to a given line passing through the center of mass.

Especially from the viewpoint of numerical realization, the Backus-Gilbert approach seems to be very promising. It leads to the minimization of the distance of an acceptable model from a prior estimate of model parameters. In the sense of matrix algebra this leads to a simple minimum length or weighted minimum length solution. The system of equations can be solved using standard numerical procedures. One advantage is the simplicity of the algorithm, another that data variances do not influence the estimate but only the *a posteriori* variances of model parameters (Parker, 1971).

Inversion of potential field data can also be understood from the viewpoint of linear inverse filtering techniques. These techniques rest on the concept of an equivalent layer (Grant and West, 1965). Assuming the field to be known on a horizontal plane, the density distribution of a thin layer of prescribed depth can be uniquely determined. As the number of observations is limited in real cases, only a finite number of density values within the layer can be determined. Gunn (1975) used multichannel Wiener filters to detect bodies with specified density-magnetization ratios. Kiss (1976) described the application of single channel filtering method to magnetic data measured on profiles. An equivalent concept was used by Tsokas and Papazachos (1992) for magnetic field data measured on a finite horizontal plane. The main disadvantage of the method is that one obtains only a projection of localization and horizontal extension of the inhomogenities into a single horizontal layer. The depth dependency of the source strength (density, magnetization) cannot be resolved.

The method described in this paper tries to combine these two concepts. Essentially it is based on the concept of the Backus-Gilbert inversion algorithm. To speed up the computations and to allow weighting under global viewpoints, the concept of linear filtering in the frequency domain will be introduced.

FORMULATION OF THE DIRECT PROBLEM

A potential field anomaly $g(\vec{r})$ can be expressed as the 3-D convolution

$$g(\vec{r}) = \iiint F(\vec{r}-\vec{r}')\,\rho(\vec{r}')d^3\vec{r}' \tag{1}$$

of a Green's function F (representing the effect of a point source) with the distribution function of physical parameters within the source region, the source strength ρ. In practice, there is a finite number of discrete observation points $\vec{r}_i$, $i=1,\ldots,N$. The continuous problem may be discretized with the assumption that the source strength can be represented by a finite number M of coefficientes, i.e.

$$\rho(\vec{r}) = \sum_{j=1}^{M} \rho_j \varphi_j(\vec{r}) \tag{2}$$

A widely used assumption is that the source is homogeneous within a number of subregions; thus

$$\varphi_j(\vec{r}) = \begin{cases} 1 : & \vec{r} \in V_j \\ 0 : & \vec{r} \notin V_j \end{cases}, \quad j = 1, \ldots, M. \tag{3}$$

Inserting (2) into (1) leads to

$$g_i = \iiint F(r_i - \vec{r}') \sum_{j=1}^{M} \rho_j \varphi_j(\vec{r}') d^3\vec{r}' \tag{4}$$

$$= \sum_{j=1}^{M} F_{i-j}\,\rho_j \qquad i=1,\ldots,N, \tag{5}$$

where

$$F_{i-j} = \iiint F(\vec{r}_i - \vec{r}')\varphi_j(\vec{r}')d^3\vec{r}' = \iiint_{vj} F(\vec{r}_i - \vec{r}')d^3\vec{r}' \tag{6}$$

denoting a discrete Green's function which represents the impulse response of a linear filter. Thus, equation (5) can be considered as a finite discrete convolution. In the special case of a source presumed to be composed of equal subregions (e.g., rectangular prisms) regulary distributed within several horizontal layers of arbitrary thickness as shown in Figure 1, equation (5) changes to

$$g_i = \sum_{k=1}^{K}\left(\sum_{j=1}^{N} F_{i-j,k}\,\rho_{j,k}\right), \quad i = 1,\ldots, N. \tag{7}$$

Herein K and N denote the number of layers and of subregions within the layer.

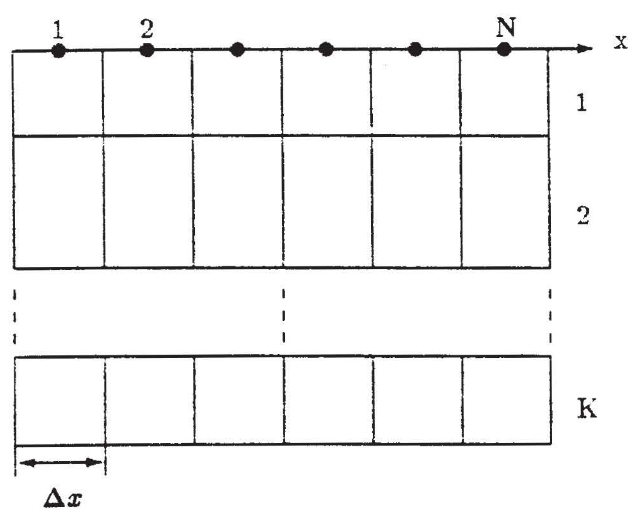

Fig. 1. 2-D model for computation and inversion of gridded potential field data (• denotes nodes of the grid).

Assuming further that the observation points are nodes of a regular grid with constant elevation corresponding to discretization of the source region in the horizontal plane, $\{F_{i-j,k}\}$ becomes a number of shift-invariant filters. Application of the discrete convolution theorem (see e.g. Elliott, 1987) leads to

$$G_n = \sum_{k=1}^{K} \Phi_{n,k} \cdot R_{n,k} \qquad n = 1,\ldots, N. \tag{8}$$

where G_n, $n=1,\ldots,N$ denotes the discrete Fourier transform (DFT) of the set of potential field data g_i, $i = 1,\ldots, N$, and $\Phi_{n,k}$, $n=1,\ldots, N$, denote the DFT of the Green's function and the source strength for the k-th layer.

FORMULATION OF THE INVERSE PROBLEM

Let an *a priori* model be represented by its spectral coef-ficients $R^o_{n,k}$, $n=1,\ldots, N$, $k= 1, \ldots, K$. The potential field generated by the model and its discrete spectrum $\vec{G}^0$ can be computed using (8). Assuming the measured data to be perfectly accurate, we may write

$$G_n = \sum_{k=1}^{K} \Phi_{n,k}\left(R^0_{n,k} + \Delta R_{n,k}\right), \quad n= 1,\ldots,N. \tag{9}$$

Here $\Delta R_{n,k}$ denotes the correction of the model parameters. For the misfit between the measured and calculated data $\Delta \vec{G}$ it is found that

$$\Delta G_n = G_n - \sum_{k=1}^{K} \Phi_{n,k}\, R^0_{n,k} = \sum_{k=1}^{K} \Phi_{n,k}\, \Delta R_{n,k}, \quad n=1,\ldots,N. \tag{10}$$

The system of equations (10) can now be written as a matrix equation

$$\Delta\vec{G} = \hat{\Phi} \cdot \Delta\vec{R} \tag{11}$$

For clarity and to show the properties of the matrix $\hat{\Phi}$, equation (11) can be written in full matrix notation as

$$\begin{pmatrix} \Delta G_1 \\ \Delta G_2 \\ \vdots \\ \Delta G_N \end{pmatrix} = \begin{pmatrix} \Phi_{1,1}\ldots\Phi_{1,K} & 0 \ldots 0 & \ldots & 0 \ldots 0 \\ 0 \ldots 0 & \ldots\ \Phi_{2,1}\Phi_{2,K} & \ldots 0 & \ldots\ 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 \ldots 0 & 0 \ldots 0 & \ldots & \Phi_{N,1}\ldots\Phi_{N,K} \end{pmatrix} \cdot \begin{pmatrix} \Delta R_{1,1} \\ \vdots \\ \Delta R_{1,K} \\ \Delta R_{2,1} \\ \vdots \\ \Delta R_{2,K} \\ \vdots \\ \Delta R_{N,K} \end{pmatrix} \tag{12}$$

System (11) represents an underdetermined system of linear algebraic equations. It has an infinite number of solutions and the question arises which of them should be preferred. We propose to select the minimum norm solution, such that the variation of model parameters differs minimally (in a least-square sense) from the reference model. The objective function can be written as follows

$$\Psi = \Delta\vec{R}^T \cdot \Delta\vec{R} + \vec{\mu}^T \cdot \left(\Delta\vec{G} - \hat{\Phi}\Delta\vec{R}\right) \tag{13}$$

Here $\vec{\mu}$ denotes a vector of Lagrange multipliers. $\Delta\vec{R}^T$ is the conjugate transpose of $\Delta\vec{R}^T$. Differentiating Ψ with respect to $\Delta\vec{R}$ one obtains for the new estimate of the model parameters (MENKE, 1987)

$$\vec{R} = \vec{R}^0 + \hat{\Phi}^T\left(\hat{\Phi}\,\hat{\Phi}^T\right)^{-1} \Delta\vec{G} \tag{14}$$

Application of the inverse DFT yields the desired physical model parameters $\rho_{j,k}$, $j = 1, \ldots N$, $k = 1, \ldots, K$.

This solution has two main shortcomings. It can be shown (see, e.g., Ballani and Strommeyer, 1982; Sansó *et al.*, 1986) that the solution is harmonic and, consequently, its extremes lie at the boundaries of the region under consideration. This property is not acceptable for real subsurface structures. As the resolution of potential field data decreases exponentially with increasing depth, this approach leads to dispersion of the calculated anomalous distribution of physical parameters at the bottom of the model (Chávez and Garland, 1983).

To avoid these features, a weighted measure of length may be introduced. The weights are arbitrary, and therefore, they can be chosen to fulfil a *priori* conditions. This offers one way to include *a priori* information into the inversion procedure. These measures may be particularly appropriate when the model parameters represent a discretized continuous function. The estimates of model parameters must satisfy the minimum condition for the objective function

$$\Psi_{\hat{\Lambda}} = \Delta\vec{R}^T \hat{\Lambda} \Delta\vec{R} + \vec{\mu}^T \cdot \left(\Delta\vec{G} - \hat{\Phi}\Delta\vec{R}\right) \quad , \tag{15}$$

where $\hat{\Lambda}$ denotes a diagonal weighting matrix with elements

$$\hat{\Lambda}_{i,j} = \lambda_i \delta_{i,j} = \lambda_{i(n,k)}\delta_{i,j}, \quad i,j = 1, \ldots, N \cdot K \tag{16}$$

and $\delta_{i,j}$ denotes the Kronecker delta. The weights λ_i are related to any frequency index n and layer index k in the following manner

$$i(n,k) = K(n-1) + k, \quad n=1, \ldots, N, \; k = 1, \ldots, K \tag{17}$$

The corrections to model parameters are obtained from (see, e.g., Tarantola, 1987)

$$\Delta\vec{R} = \hat{\Lambda}^{-1}\hat{\Phi}^T\left(\hat{\Phi}\hat{\Lambda}^{-1}\hat{\Phi}^T\right)^{-1} \Delta\vec{G} \quad . \tag{18}$$

If the weighting matrix equals the identify matrix and $\vec{R}^0$ is set to zero, equation (18) reduces to (14).

MINIMUM LENGTH SOLUTION IN THE FREQUENCY DOMAIN

Now consider the system (18) to be solved. $\hat{\Phi}\hat{\Lambda}^{-1}\hat{\Phi}^T$ denotes the symmetrical matrix of the weighted inner products of the row vectors of matrix $\hat{\Phi}$. From (12) it can be seen that the individual rows of $\hat{\Phi}$ are orthogonal. This implies

$$\left(\hat{\Phi}\hat{\Lambda}^{-1}\hat{\Phi}^T\right)_{i,j} = F_i\delta_{i,j}, \quad i,j=1,\ ...,\ N, \tag{19}$$

where

$$F_i = \sum_{k=1}^{K} \frac{\Phi_{i,k}\Phi^*_{i,k}}{\lambda_{K(i-1)+k}} \tag{20}$$

If all weights are greater than zero, the matrix inverse exists since

$$F_i = \sum_k \frac{|\Phi_{i,j}|^2}{\lambda_{K(i-1)+k}} \geq \sum_k \frac{|\Phi_{i,k}|^2}{\lambda_{max}} = \frac{1}{\lambda_{max}} \sum_k |\Phi_{i,k}|^2 > 0, \quad i=1,...,N, \tag{21}$$

where λ_{max} is the maximum weight. Thus

$$\left(\hat{\Phi}\Lambda^{-1}\hat{\Phi}^T\right)^{-1}_{i,j} = \frac{1}{F_i}\delta_{i,j} \tag{22}$$

The solution of system (18) can be given explicitly as

$$\Delta R_{n,k} = \frac{\Phi^*_{n,k}}{\lambda_{i(n,k)}F_n} \cdot \Delta G_n, \quad n=1,...,N, k=1,...,K \tag{23}$$

From the viewpoint of the theory of digital filtering, (23) represents an inverse digital filtering in the frequency domain. Thus

$$\Xi_n^{(k)} = \frac{\Phi^*_{n,k}}{\lambda_{i(n,k)}F_n} \quad n=1,...,N, k=1,...,K \tag{24}$$

is a set of K finite discrete frequency response functions. They are mapping the potential field data onto the discrete source strength distribution within the K layers of the model.

Let us discuss the choice of weights. There are three main possibilities. Initially we may choose

$$\lambda_n = \lambda_{K(n-1)+1} = ... = \lambda_{K(n-1)+K}, \quad n=1,...,N \tag{25}$$

This is equivalent to pure frequency weighting independent of the individual layer. But from (20) and (24) it can be seen that the λ's will be reduced and, consequently, the final results are not affected by them. This follows originally from the condition of perfect data fitting (see the second term of the objective function (15)). Another way is to choose

$$\lambda^{(k)} = \lambda_{K(1-1)+k} = ... = \lambda_{K(N-1)+k}, \quad K=1,...,K \tag{26}$$

This corresponds to pure depth-dependent weighting. It represents the easiest way of introducing *a priori* information about the depth of burial. This information can be obtained, e.g., from other geophysical data (refraction seismic data, geoelectrical measurements, etc.). The most universal way of weighting is to choose all weights independently. This corresponds to individual frequency dependent weighting for separate layers. But since weighting occurs in the frequency domain, it has to be kept in mind that it can only be done globally. For local weighting in the space domain see Chávez and Garland (1983).

For a single layer ($K=1$), system (11) reduces to an even-determined system of linear algebraic equations. As pointed out, no weighting can be applied. The model parameters are obtained from

$$\Delta R_n = \frac{\Phi^*_n}{|\Phi_n|^2} \cdot \Delta G_n, \quad n=1,...,N\ . \tag{27}$$

This leads to the concept of the equivalent layer. Based on autocorrelation analysis, it has been adapted to geophysical prospection for buried archeological remains by Karousova and Karous (1989). The main aim of this effort is to transform the data into an easily readable format and, also, visually to enhance the maps in order to resolve subsurface structures that are fairly close together. It may be particularly suited to the processing of magnetic maps.

APPLICATION TO REAL AND SYNTHETIC DATA

In order to test the algorithm two examples will be given. In the first example the algorithm is applied to gravity data, in the second example to magnetic data.

To allow a comparison of results, a synthetic model similarly to the model used by Chávez and Garland (1983) was initially chosen. In Figure 2 the model obtained from inversion without weighting is shown. This illustrates the shortcomings mentioned above: the dispersion of the anomaly in the lower part of the model and the poor correlation between the actual and the computed density anomalies because the solution is harmonic. Much better results are obtained by weighting in the frequency domain (Figure 3). Here only depth-dependent weighting was employed. No assumptions about the lateral extension of the anomaly are made. For the same result with minimum length inversion in the space domain much more *a priori* information is required. In addition to depth-dependent weighting, weighting dependent on lateral co-ordinates must be used.

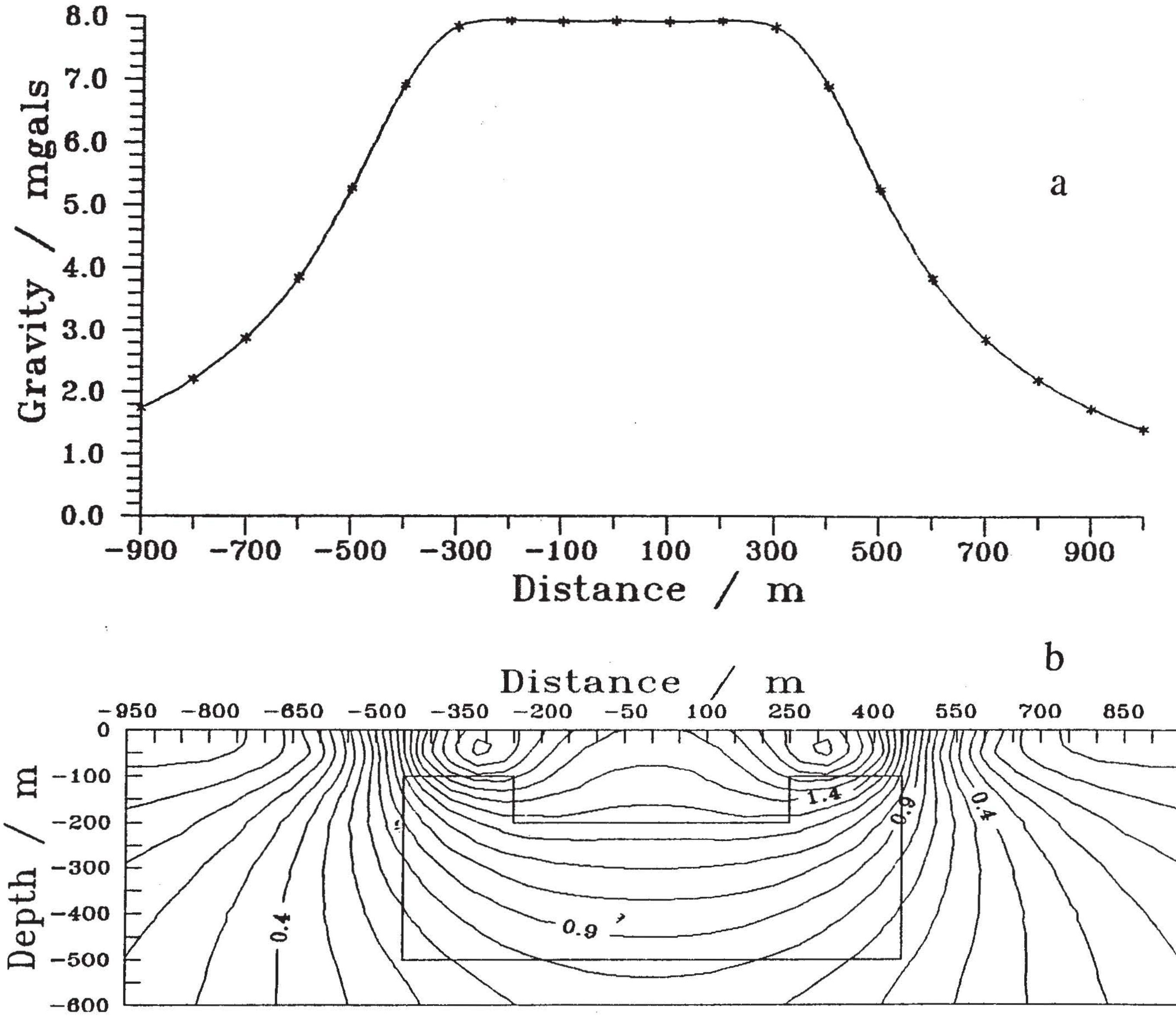

Fig. 2. Inversion of gravity data using minimum length inversion in the frequency domain. (a) gravitational attraction of the model; (b) synthetic model in comparison with the result of inversion.

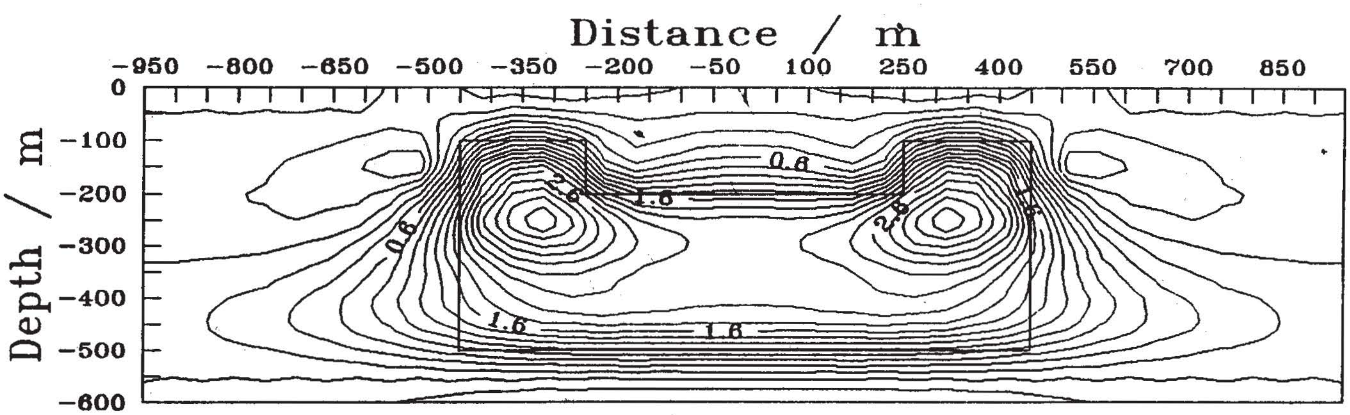

Fig. 3. Inversion of gravity data using weighted minimum length inversion in the frequency domain. A much improved correlation between synthetic model and computed density distribution is achieved.

In order to detect possible soil pollution in the urban area of Leipzig, environmental studies initiated by the Department of the Environment were carried out over the last two years. With the aid of historical research and interpretation of old aerial photographs several localities with increased potential hazard were selected. The Eichberg, a hill in the NE part of the town, was among them. It is a sand lens of 2 m to 5 m thickness over a layer of boulder clay of about 10 m thickness. During 1914-1936 sand was obtained from three pits, which were later filled. The task of the geophysical investigation was to detect the pits, determine their depths and estimate the material of the filling.

Geoelectrical profiling using the Wenner configuration and magnetic measurements of total field anomaly were carried out to detect the pits and to determine their lateral extension (for theoretical background see Arnaud Gerkens, 1989). The two results showed a good agreement. Since only the largest pit, in the north part of the area, showed a significant magnetic anomaly, it was further investigated. The depth of the pits was estimated using geoelectrical depth sounding with Schlumberger configuration. For the largest pit a bottom depth of 3.5±0.5 m was estimated. Because the anomaly had a significant strike in the EW direction, 2-D inversion techniques could be applied to several profiles in NS direction. One of them is shown in Figure 4.

The results of the depth estimation were used to select depth-dependent weights for the inversion of the magnetic data. The weights were chosen to be proportional to the mean depth of the layer. The uppermost layer was weighted by unity. Figure 5 shows the result of the weighted minimum length inversion in the frequency domain. The estimated lateral extension agrees with those found from geoelectrical data and with estimates using reduction to the pole of the total magnetic field anomaly. The shape of the pit can be recovered very well, but the distribution of susceptibilities cannot be recovered exactly. This is mainly due to nonuniqueness of the inverse problem in potential field interpretations.

To estimate the composition of the filling, computed values of susceptibility are of limited value for they are only averages of the real physical parameters. Moreover, the susceptibility of anthropogenic deposits is not definable. Mauritsch and Walach (1990) suggested a statistical method using horizontal gradient data of total field anomaly to estimate the filling o pits. Application of this method indicated that the pit probably contains trash and rubble. It was probably used by the population as a garbage dump. The existence of oil drums within the pit, which had been suspected, may be excluded.

DISCUSSION

The concept of minimum length inversion in the generalized spectral domain is nothing new in principle. However, this is the first proposed application to potential field interpretation for the purposes of exploration geophysics, archeometry and environmental studies.

Our approach is similar to that of Pěč and Martinec (1984) for computing density models of the Earth's mantle using spherical harmonic expansions of the Earth's external gravity field. As a reference model they used the seismic model PREM (Dziewonski and Anderson, 1981). The spherical harmonic coefficients of the lateral density variations were computed using the minimum length inversion method. The existence, uniqueness and stability of the solution were proven by Matyska (1987). Martinec and Pěč (1990) discussed the effect of the solution being harmonic. For the computed density model of the Earth's mantle this meant that gravitational effects of the core-mantle boundary would be virtually compensated near the Earth's surface, which makes no sense from a physical point of view. The autors suggested using additional constraints to overcome this drawback. From the viewpoint of the theory of orthogonal descomposition of continuous functions, the concept of spherical harmonics and the concept of Fourier series are equivalent.

Backus-Gilbert inversion in the space domain and in the frequency domain are equivalent due to the isomorphism between Hilbert spaces. As shown in Figure 4, the observed and calculated data differ slightly at both ends of the profile, although a perfect fit was claimed. This is due to violating the assumptions of the discrete convolution theorem. The signal and filter are not periodic; but because of the latent periodicity of the DFT they are treated as if they were. In linear filtering this would be called *aliasing* or *edge effect.* It can be minimized by lateral extrapolation of the potential field data before running the inversion procedure (see, e.g., Bracewell, 1965).

The main advantage of the minimum length method in the frequency domain over the space domain is that the computational effort is much smaller. For the misfit between calculated and observed data we find for the prism model (see, e.g., Green, 1975).

$$\Delta\vec{g} = \hat{\Gamma}\Delta\vec{\rho}, \tag{28}$$

where $\hat{\Gamma}$ denotes the *Green's matrix.* This is equivalent to the system of equations (5). The corrections of the physical model parameters can be computed by applying the minimum length method

$$\Delta\vec{\rho} = \hat{\Gamma}^T\left(\hat{\Gamma}\hat{\Gamma}^T\right)^{-1}\Delta\vec{g} \tag{29}$$

Since the *Gram* matrix $\hat{\Gamma}\hat{\Gamma}^T$ is symmetric, only $\frac{N}{2}$ $(N+1)$ elements have to be actually computed instead of N^2. But the Green's functions are not orthogonal and therefore all elements of the matrix are nonzero. As a matrix inversion can be computed with $O(cN^3)$ algebraic operations, the calculation of $\Delta\vec{\rho}$ with the aid of (29) requires $O(cN^3 + d(M+1)N^2)$ operations, where $M = K.N$. c and d are constants which depend on the chosen algorithm.

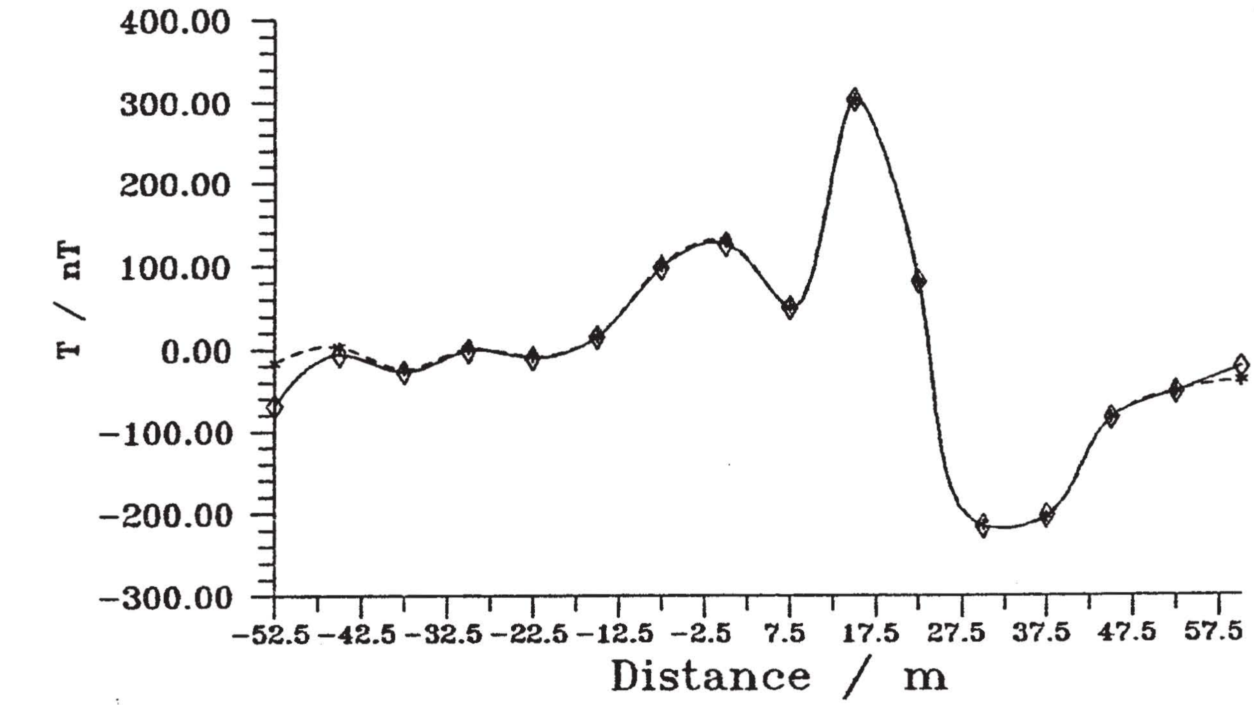

Fig. 4. Total magnetic field anomaly over a filled pit (solid line: measured anomaly, dashed line: computed anomaly). The computed anomaly differs at both edges slightly from the measured anomaly due to edge effects inherent in the numerical realization of inverse linear filtering.

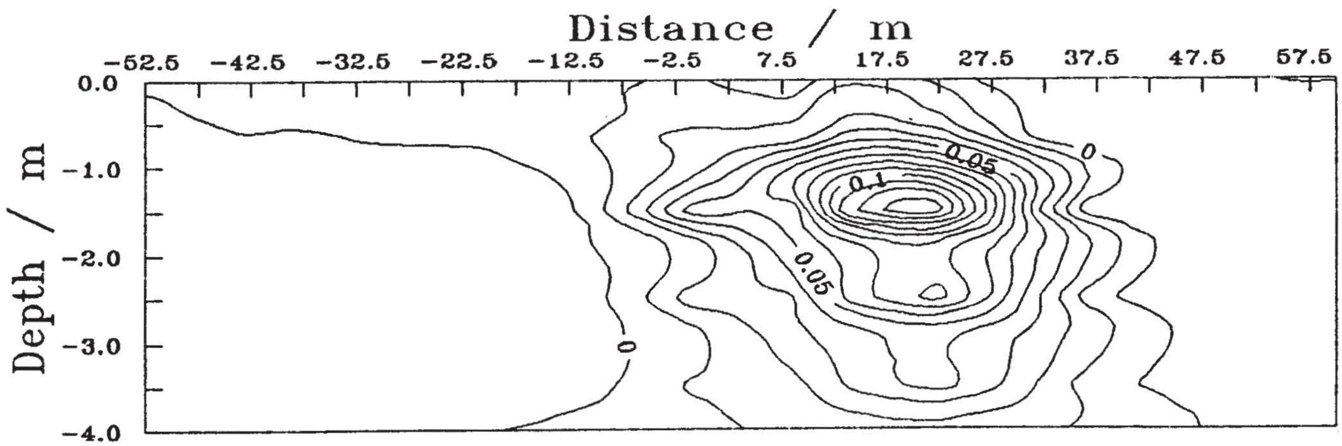

Fig. 5. Inversion of real magnetic data (see Fig. 4) using weighted minimum length inversion in the frequency domain. The map shows the computed distribution of susceptibilities within the model.

The calculation of $\Delta\bar{\rho}$ using (23) requires computation of the DFT of both Green's function and source strength for every layer; the multiplication of discrete spectra; plus, in order to obtain the correction of physical model parameters, the computation of the inverse DFT for everly layer. Because the DFT can be computed in $O(cN\log_2 N)$ operations with the Fast Fourier Transform (FFT), the whole inversion procedure requires only $O(3cM \log_2 N + dM)$ operations. Figure 6 shows the ratio of required operations for inversion in the frequency domain to the number of operations required for the inversion in the space domain. Constants c and d were set to 1.

CONCLUSIONS

In this paper a linear inversion method based on the Backus-Gilbert approach is proposed for application to 2-D potential field interpretations. As suggested by real and synthetic data examples, our algorithm is a good tool for inverting gravity as well as magnetic field data. Unlike conventional least-squares methods, which fit the data with a uniform source strength, this approach in general leads to a nonuniform source strength distribution. The calculated distribution function may be smoother than the real structure, but it reveals the inner structural pattern of the enbed-

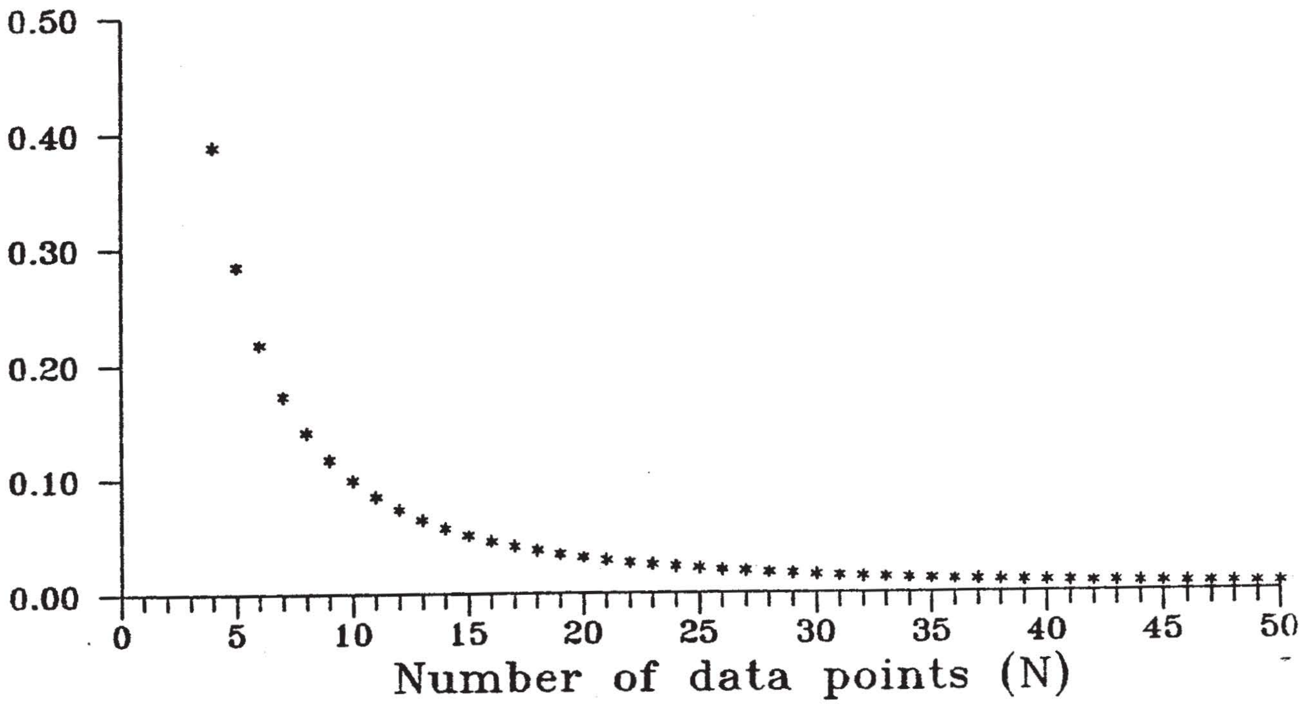

Fig. 6. Ratio of operations required for inversion in the frequency domain and in the space domain as a function of the number of data points N. The number of layers is set to $K = 10$.

ded body. This makes the method quite advantageous for interpretation of geophysical data in environmental studies and archeometry. Clearly, the choice of weights is often subjective. Yet, we believe that this allows a better incorporation of *a priori* information. In many cases a pure depth-dependent weighting may be sufficient. Utilizing Fast Fourier Transform (FFT) methods, the algorithm turns out to be very fast. As the data are assumed to be perfectly accurate, random noise and regional trend must be removed before running the inversion algorithm. The algorithm may be easily extend to 3-D problems.

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