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Surface distortion decomposition for vector controlled source audio-magnetotelluric data

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RESUMEN

Los datos audiomagnetotelúricos de fuente controlada (CSAMT) utilizando un solo dipolo eléctrico, también conocidos como datos CSAMT vectoriales, son procesados normalmente usando un modelo de inducción bidimensional el cual no incorpora efectos por distorsión galvánica. El número limitado de datos disponibles en CSAMT vectorial no permite modelos que incluyan simultáneamente efectos de inducción 2-D y distorsion galvánica local. En este artículo proponemos dos modelos unidimensionales de distorsión, los cuales son casos especiales del método general de descomposición tensorial MT de Groom y Bailey (1989). Uno de estos asume que el parámetro de distorsión llamado *splitting* (*s*) puede hacerse cero escogiendo un sistema coordenado apropiado. El otro, desarrollado en términos de elipses de polarización, supone que el parámetro de distorsión *shear* (*e*) puede hacerse cero rotando los ejes principales a los ejes de medición. El primero de los modelos genera la resistividad, la fase y los parámetros *twist* (*t*) y *shear*, mientras que el segundo genera la resistividad, la fase y los parámetros *twist* y *splitting*.

PALABRAS CLAVE: CSAMT, datos tensoriales, datos vectoriales, distorsión galvánica, parámetros de distorsión, inducción 2-D.

ABSTRACT

Single-source controlled audio-magnetotelluric (CSAMT) data, also known as *vector* CSAMT data, has been routinely collected and processed using the conventional 2-D induction model which does not incorporate galvanic distortion effects. However, the small number of data available in vector CSAMT does not allow models that simultaneously incorporate 2-D induction and local galvanic distortion. We propose two 1-D distortion models that are specialized cases of the general Groom and Bailey's (1989) MT decomposition method: one that allows the distortion parameter *splitting* can go to zero by choosing an appropriate coordinate system, and the other that assumes the distortion parameter *shear* can be zeroed by rotating the principal axes. The former produces the resistivity, phase and distortion parameters *twist* and *shear* while the latter generates resistivity, phase and distortion parameters *twist* and *splitting*.

KEY WORDS: Tensor and vector CSAMT, galvanic distortion, distortion parameters, 2-D induction.

INTRODUCTION

The controlled source audio-magnetotelluric (CSAMT) technique was first proposed by Goldstein and Strangway (1975) for shallow geophysical exploration. The method was presented as an alternative to the audio-magnetotelluric (AMT) method that takes advantage of natural electric and magnetic fields. However, often natural signals are either weak or totally absent and it is necessary to wait until good signals arrive with the inconvenience of wasting valuable survey time.

When the distance from the source and observation point in the CSAMT method is more than three skin depths δ with

$$\delta \approx 503\sqrt{\frac{\rho_{app}}{f}} , \qquad (1)$$

where f is the frequency and ρ_{app} is the apparent resistivity at the survey point, the magnetotelluric (MT) plane wave approximation is valid and the apparent resistivity and phase expressions derived for MT observations can be used. This is known as the far-field condition. *Tensor* CSAMT (see, for example Qian, 1994; Li and Pedersen, 1994; Boerner *et al.*, 1993) is based on measuring two horizontal electrical field component and their corresponding orthogonal magnetic components with two or more simultaneous sources (Hughes and Carlson, 1987). Tensor data is expensive to collect and very often only one source is used. If this is the case we refer to the method as *vector* CSAMT. Vector CSAMT data has been routinely collected in recent years and is processed using the conventional 2-D induction model which does not incorporate galvanic distortion effects. However, the small number of data available in vector CSAMT does not allow models that simultaneously incorporate both 2-D induction and local galvanic distortion. Alternatively, a 1-D model with up to two distortion parameters can be used. We present two 1-D distortion models that are specialized cases of the general Groom and Bailey's (1989) MT decomposition method. The first assumes that the distortion parameter *splitting* can be made zero by choosing an appropriate coordinate system, and the other is developed in terms of polarization ellipses, where the distortion parameter *shear* can be zeroed by rotating the principal axes to the measurement axes. The former produces the distortion parameters *twist* and *shear* while the latter generates the parameters *twist* and *splitting*.

DECOMPOSITIONS FOR VECTOR CSAMT

Although the decomposition of the impedance tensor method was developed originally for MT, its use can be extended to frequency domain CSAMT. An important difference is the size of the electric scatterers. Shallow electrical inhomogeneities (from tens to a few hundred meters) are abundant in the CSAMT frequency range more than in MT. This makes CSAMT data more prone to be distorted. The assessment of the distortion parameters associated with conductive bodies represents additional information over the apparent resistivity $\rho_a(\omega)$ and phase angle $\phi(\omega)$ in CSAMT. These can be related to targets and may reduce the ambiguity in the interpretation.

The CSAMT field is analogous to the MT plane wave survey when the electric and magnetic field components are measured separately in the far field zone using at least two orthogonal sources. Tensor decomposition methods can be still applied. Thus, the electromagnetic fields $\overline{E}=(E_x,E_y)$ and $\overline{H}=(H_x,H_y)$ are related through

$$\overline{E}=Z\overline{H} \tag{2}$$

where **Z** is the ground impedance. As in magnetotellurics, we have an ensemble of events (e,h) from which all the elements of **Z** can be evaluated. This technique is known as *Tensor* CSAMT. The Groom and Bailey two-dimensional (2-D) impedance tensor given by

$$\boldsymbol{Z}=\boldsymbol{RTSAZ_2R^T} \tag{3}$$

can be used in (2) to solve for the three distortion parameters called *twist* (t), *shear* (e), and *split* (s), due to inhomogeneities in the ground. Here, the impedance **Z** is decomposed in terms of the matrix operators $\boldsymbol{R}$, $\boldsymbol{T}$, $\boldsymbol{S}$, and $\boldsymbol{A}$ which represent the rotation, twist, shear, and anisotropy matrix operators respectively (Groom and Bailey, 1989).

Multi-source CSAMT surveys are expensive to carry out. Single-source surveys have been frequently used instead, with the disadvantage that only one coherent excitation mode of $\overline{H}$ is available at a given site. Thus there are not enough equations to solve (2) for all elements of the complex impedance tensor **Z**. Some simplifying assumptions are needed in order to apply distortion analysis to vector CSAMT data. The alternative simplification is to assume a one-dimensional (1-D) inductive structure whose impedance tensor $\boldsymbol{Z_1}$ is of the form

$$Z_1=\begin{pmatrix}0 & a\\ -a & 0\end{pmatrix} \tag{4}$$

where a is complex. In the general distortion model given by equation (3) applied to the 1-D case, there are six parameters to solve for, namely, Re a, Im a, t, e, s, and the regional strike θ. We cannot solve for all of them because equation (2) defines only four independent equations. For a 1-D earth model the regional strike is undefined, and can be chosen arbitrarily. A reasonable question to ask is whether one may find a strike θ for which one of the distortion parameters t, e or s can be made zero. This seems plausible because the same distortion applied to regional models with a different strike angle yields different local distortion parameters.

Another question is whether any of the distortion parameters can be made to vanish by choosing an appropriate coordinate system. The answer seems to be no. A pure twist, for example, is conserved even when the coordinate system is rotated (Figure 1). Thus a twist cannot be made to vanish no matter what regional strike is used. Any 1-D distortion factorization must include it. Hence, either splitting or shear has to be caused to vanish using an appropriate azimuth angle θ. A zero shear factorization can easily be constructed which has a particularly simple visualization in terms of polarization ellipses. The construction of such factorizations is discussed in the following sections.

ZERO-SPLITTING 1-D FACTORIZATION

Assume that there is a rotation angle to the coordinate system of measurements such that the splitting vanishes. In this case, the Groom and Bailey impedance tensor decomposition is given by

$$\boldsymbol{Z}=\boldsymbol{RTSZ_1R^T}, \tag{5}$$

or more explicitly by

$$Z=K\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1 & t\\ -t & 1\end{pmatrix}\begin{pmatrix}1 & e\\ e & 1\end{pmatrix}\begin{pmatrix}0 & a\\ -a & 0\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \tag{6}$$

with K the normalization coefficient given by

$$K=\frac{1}{\sqrt{1+e^2}\sqrt{1+t^2}} \quad . \tag{7}$$

Substituting this form of impedance tensor into (2) and making θ =0 we are left with four equations to solve for the real parameters (e,t,a_r,a_i), where a_r and a_i are the magnitudes of the real and imaginary parts of a. The explicit expressions are given by

$$\begin{aligned}
E_{xr}&=K\left[(t-e)(a_rH_{xr}-a_iH_{xi})+(1-et)(a_rH_{yr}-a_iH_{yi})\right]\\
E_{xi}&=K\left[(t-e)(a_iH_{xr}+a_rH_{xi})+(1-et)(a_iH_{yr}+a_rH_{yi})\right]\\
E_{yr}&=K\left[-(1+et)(a_rH_{xr}-a_iH_{xi})+(e+t)(a_rH_{yr}-a_iH_{yi})\right]\\
E_{yi}&=K\left[-(1+et)(a_iH_{xr}+a_rH_{xi})+(e+t)(a_iH_{yr}+a_rH_{yi})\right]\,.
\end{aligned} \tag{8}$$

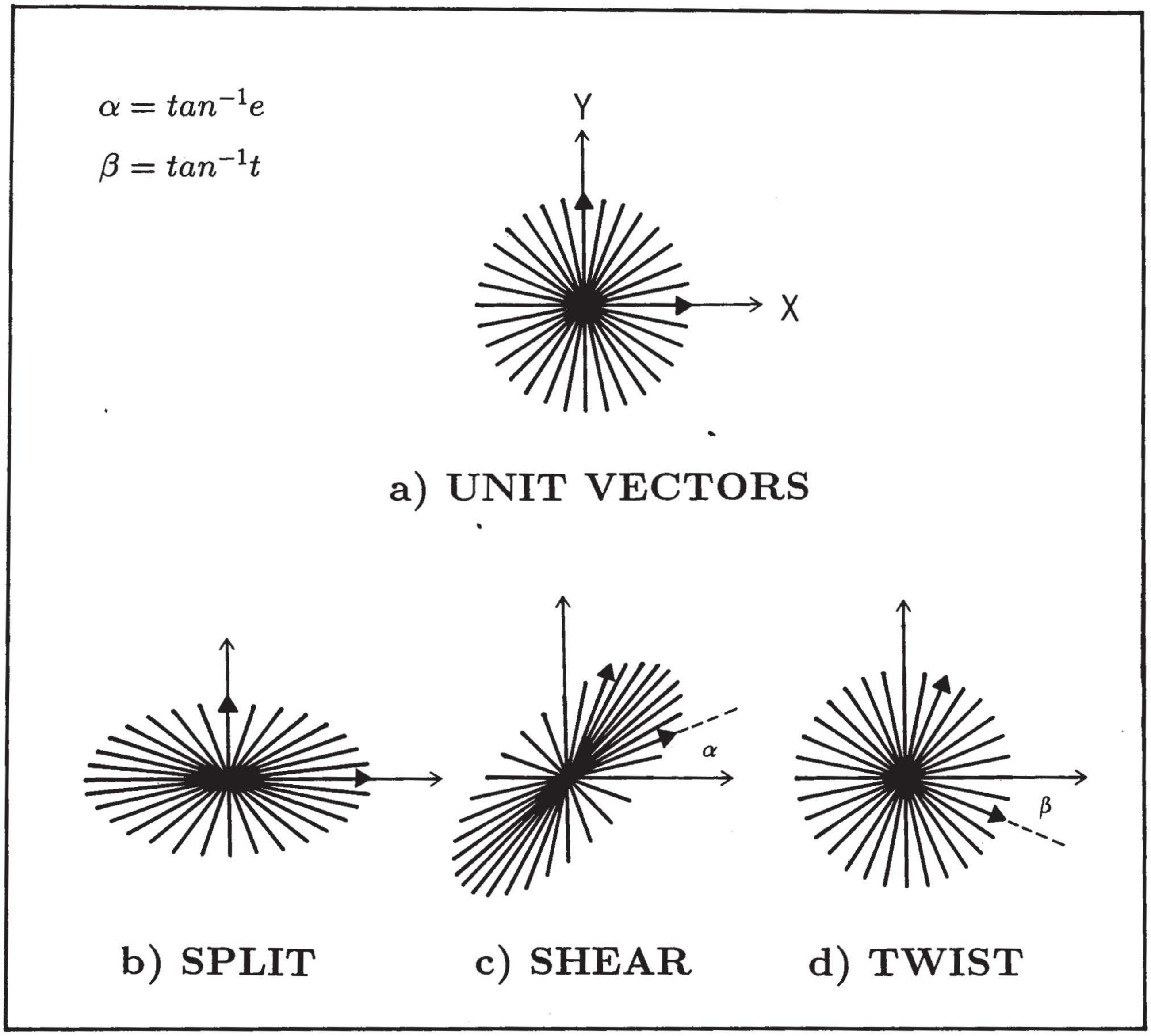

Fig. 1. The effects of b) the anisotropy operator A, c) the shear operator S, and d) the twist operator T on a family of unit vectors a) (modified after Groom, 1988).

Renaming a_r, a_i, e and t as x_1, x_2, x_3 and x_4 respectively, the functional of these non-linear relations is represented by

$$f_i(x_1, x_2, x_3, x_4) = 0 \tag{9}$$

with i=1...4. To find the zeroes of this function we proceed to linearize this system. Each f_i is expanded in a Taylor series and the resulting matrix equation can be solved using standard minimization techniques (see Appendix A).

ZERO-SHEAR 1-D FACTORIZATION

In the impedance decomposition (3), if we choose θ such that the shear e is zero, it takes the form

$$\boldsymbol{Z} = \boldsymbol{RTAZ}_1\boldsymbol{R}^T \ , \tag{10}$$

where the site gain is absorbed by matrix $\boldsymbol{A}$. Then the distortion model can be written after equation (2) as

$$\overline{E} = \boldsymbol{RTAZ}_1\boldsymbol{R}^T\overline{H} \ . \tag{11}$$

This is a useful factorization that has a direct geometrical visualization in terms of polarization ellipses. First rotate the equation to match the principal axis system of $\overline{H}$, *i.e.*

$$\boldsymbol{R}^T\overline{E} = \boldsymbol{TAZ}_1\boldsymbol{R}^T\overline{H} \ , \tag{12}$$

so that the electric field in the new reference axes is

$$\overline{E}'=\boldsymbol{R}^T\overline{E}=\boldsymbol{TAZ}_1\overline{H}' \quad , \tag{13}$$

where $\overline{H}'=\boldsymbol{R}^T\overline{H}$. The rotation angle of **R** in terms of the field components is given by the expression (see Appendix B):

$$\theta_h=\frac{1}{2}\tan^{-1}\frac{2(H_{xr}H_{yr}+H_{xi}H_{yi})}{\left(H_{yr}^2+H_{yi}^2-H_{xr}^2-H_{xi}^2\right)} \quad . \tag{14}$$

Thus $\theta=\theta_h$ is the required azimuth. The next step is to rotate $\overline{E}$ so that the major axis of its polarization ellipse is normal to the major axis of the H-polarization ellipse:

$$\theta_t=\frac{1}{2}\tan^{-1}\frac{2(E'_{xr}E'_{yr}+E'_{xi}E'_{yi})}{\left(E'^2_{yr}+E'^2_{yi}-E'^2_{xr}-E'^2_{xi}\right)} \quad . \tag{15}$$

This rotation is the twist angle. It may be computed also by

$$\theta_t=\frac{\pi}{2}-\left|\theta_e-\theta_h\right| \quad , \tag{16}$$

where θ_e is the angle required to rotate the electric field polarization axis into the original measurement system (see Appendix B). Once θ and θ_t are known we are left with a decomposition that includes the splitting matrix A which stretches or elongates the field components by adjusting the E and H polarization ellipses. The decomposition has the form

$$\begin{pmatrix}E_1\\E_2\end{pmatrix}=\frac{1}{\sqrt{1\ \ s^2}}\begin{pmatrix}1+s & 0\\0 & 1\text{-}s\end{pmatrix}\begin{pmatrix}0 & a\\-a & 0\end{pmatrix}\begin{pmatrix}H_1\\H_2\end{pmatrix} . \tag{17}$$

or, explicitly,

$$E_1=\frac{a}{\sqrt{1+s^2}}(1+s)H_1$$
$$E_2=\frac{a}{\sqrt{1+s^2}}(1-s)H_2 \quad , \tag{18}$$

which can easily be solved for s and a.

Since there is by construction, a shear-free decomposition, it is clear that shear is not needed to model a 1-D earth that includes galvanic distortion effects. This model generates the distortion parameters splitting (s) and twist (t) and a single impedance (a) calculated as the ratio of the major axes of the $\overline{E}$ and $\overline{H}$ polarization ellipses. The parameters obtained using the factorization (10) depend on the source field and would be different if a different source location were used. This method of evaluating an apparent resistivity from CSAMT data was originally suggested by Yamashita (personal communication) as an empirical way of dealing with distortion effects. Here we have shown that it is based on a valid distortion model.

CONCLUSIONS

We have adapted the Groom and Bailey factorization approach of the magnetotelluric tensor with far-field vector CSAMT data in the presence of electrical inhomogeneities. Two distortion decomposition models are proposed: one that assumes that the distortion parameter *splitting* can be caused to vanish by choosing an appropriate coordinate system, and the other, in terms of polarization ellipses, that causes the distortion parameter *shear* to vanish by rotating the principal axes into the measurement axes. The former produces the distortion parameters *twist* and *shear* while the latter generates the parameters *twist* and *splitting*.

Both models appear to be physically valid, but it is necessary to carry out numerical modeling with synthetic and real data to test their performance. The **zero splitting** factorization model requires numerical techniques to find appropriate solutions for the resistivity, phase, and the two distortion parameters shear (e) and twist (t). It may yield unstable or multiple solutions when the initial model is inappropriate, and may be difficult to interpret in terms of physical parameters. By contrast, the parameters using the **zero shear** factorization method are found analytically and may be more appropriate in the absence of an *a priori* conductivity model. Both approaches depend on the location of the energy source: results do not have an absolute meaning regarding to their magnitude. However, the effectiveness of the methods in locating good conductors can be improved by using a source-sensors configuration that enhances distortion of the electric field, making use of the geological knowledge of the survey area.

The conventional 2-D model, that does not consider distortion effects may be useful for qualitatively locating the conductive anomalies because small misalignments of the electric axes in the presence of high resistivity ratios leads to practically undistinguishable polarization modes. Even if the undistorted model were appropriate, under this circumstance it can only resolve 1-D worth of information. Thus, unless the measurement direction coincides with the principal induction directions and there is no distortion present, we will not get results which are quantitatively useful.

APPENDIX A

The expansion in Taylor series of

$$f_i(x_1,x_2,x_3,x_4)=0 \tag{19}$$

is (*e.g* Press *et al.*, 1988)

$$f_i(\overline{X}+\delta\overline{X})=f_i(\overline{X})+\sum\frac{\partial f_i}{\partial x_i}\delta x_j+O_i(\delta\overline{X}^2) \quad , \tag{20}$$

where $\overline{X}$ denote the entire vector solution. Neglecting quadratic and higher order terms $\delta\overline{X}^2$ we obtain a set of linear equations for the correction $\delta\overline{X}$ such that each function approaches to zero simultaneously. The new system is expressed as

$$\beta_i=\sum\alpha_{ij}\delta x_j \quad , \tag{21}$$

where $\beta_i = -f_i$ and $\alpha_{ij} = \frac{\partial f_i}{\partial x_j}$.

This matrix equation can be solved using LU decomposition. The corrections are added to the solution and tested for convergence using the damped Newton-Rapson method (Conte and de Boor, 1980). Thus, for the *i*th attempt,

$$X_i^{new} = X_i^{old} + \delta X_i \frac{\beta_i}{2^n} \quad , \tag{22}$$

with n=1,2,3,... . The X_i^{new} solution is accepted if it leads to a reduction in the residual error, *i.e.*, if

$$|e_i(X_{n+1})| < |e_i(X_n)| \quad , \tag{23}$$

where

$$|e_i(X_{n+1})| = \sum (f_i(X) - \beta_j(X))^{1/2} \quad . \tag{24}$$

The process is iterated to convergence.

APPENDIX B

Let $\overline{H}$ be the horizontal magnetic field. In general, $\overline{H}$ is a complex vector which can be expressed in terms of its x and y components as:

$$\begin{aligned} H_x &= H_{xr} + iH_{xi} \\ H_y &= H_{yr} + iH_{yi} \end{aligned} \tag{25}$$

where the subscripts r and i stand for the real and imaginary parts. In terms of the phase and amplitude these equations are

$$\begin{aligned} H_x &= He^{i\phi_x} \\ H_y &= He^{i\phi_y} \quad , \end{aligned} \tag{26}$$

with

$$\begin{aligned} H &= \sqrt{H_x^2 + H_y^2} \\ \phi_x &= \tan^{-1}\frac{H_{xi}}{H_{xr}} \\ \phi_y &= \tan^{-1}\frac{H_{yi}}{H_{yr}} \quad . \end{aligned} \tag{27}$$

In order to transform $\overline{H}_x$ and $\overline{H}_y$ to a new coordinate system such that they are 90° out of phase and such that the plane of the resulting polarization ellipse lies in the new system, we apply the rotation operator $\mathbf{R}^{-1}$ to $\overline{H}$, *i.e.*

$$H_x' = \mathbf{R}^{-1}H = \begin{pmatrix} \cos\theta_h & -\sin\theta_h \\ \sin\theta_h & \cos\theta_h \end{pmatrix} \tag{28}$$

or

$$\begin{aligned} H_x' &= H_x\cos\theta_h - H_y\sin\theta_h \\ &= H_{xr}\cos\theta_h + iH_{xi}\cos\theta_h - H_{yr}\sin\theta_h - iH_{yi}\sin\theta_h \\ &= H_{xr}\cos\theta_h - H_{yr}\sin\theta_h + i(H_{xi}\cos\theta_h - H_{yi}\sin\theta_h) \end{aligned} \tag{29}$$

and similarly

$$H_y' = H_{xr}\sin\theta_h + H_{yr}\cos\theta_h + i(H_{xi}\sin\theta_h - H_{yi}\cos\theta_h) \quad . \tag{30}$$

As H_x' and H_y' are related through

$$H_y' = i\alpha H_x' \quad , \tag{31}$$

with α a scalar, then

$$\begin{aligned} &H_{xr}\sin\theta_h + H_{yr}\cos\theta_h + i(H_{xi}\sin\theta_h - H_{yi}\cos\theta_h) = \\ &i\alpha(H_{xr}\cos\theta_h - H_{yr}\sin\theta_h + i(H_{xi}\cos\theta_h - H_{yi}\sin\theta_h) . \end{aligned} \tag{32}$$

Thus, we obtain the following expressions

$$\begin{aligned} H_{xr}\sin\theta_h + H_{yr}\cos\theta_h &= -\alpha(H_{xi}\cos\theta_h - H_{yi}\sin\theta_h) \\ H_{xi}\sin\theta_h + H_{yi}\cos\theta_h &= \alpha(H_{xr}\cos\theta_h - H_{yr}\sin\theta_h) \quad . \end{aligned} \tag{33}$$

Dividing the first of these two equations by the second we get

$$\frac{H_{xr}\sin\theta_h + H_{yr}\cos\theta_h}{H_{xi}\sin\theta_h + H_{yi}\cos\theta_h} = \frac{-H_{xi}\cos\theta_h - H_{yi}\sin\theta_h}{-H_{xr}\cos\theta_h - H_{yr}\sin\theta_h}, \tag{34}$$

which leads to the equation

$$\theta_h = \frac{1}{2}\tan^{-1}\frac{2(H_{xr}H_{yr} + H_{xi}H_{yi})}{H_{yr}^2 + H_{yi}^2 - H_{xr}^2 - H_{xi}^2} \quad . \tag{35}$$

This is the angle needed to rotate $\overline{H}$ to the new reference system. In a similar way, we may find the angle θ_e required to perform a transformation of the polarization ellipse defined by the electrical horizontal field $\overline{E}$.

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