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Propagation of coupled Rayleigh-gravity waves on the ocean floor

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RESUMEN

El fondo del mar puede propagar ondas acopladas de gravedad, acústicas y de Rayleigh. Se presentan expresiones para la dispersión y el flujo de energía, y se demustra que las ondas se dividen en dos ramas. La rama de lata velocidad tiene estructura multimodal y un corte de alta frecuencia.

PALABRAS CLAVE: Sismo, onda Rayleigh, onda de gravedad-acústica.

ABSTRACT

It is shown that the Ocean-Earth crust interface can propagate gravity-sound Rayleigh waves. Dispersion properties of waves and flux of energy are derived. It is shown that the waves split into low and fast velocity branches. The fast branch has a multimode structure and has a cutoff in frequency and wave number. Numerical solutions are discussed.

KEY WORDS: Earthquake, Rayleigh wave, gravity-sound wave.

1. THEORY

Consider a liquid layer of thickness h (Ocean) over a solid halfspace (Figure 1). A surface Rayleigh wave propagates along x due to a seismic excitation. The properties of the Rayleigh wave are strongly modified because of contact with the water layer. In general the problem requires the simultaneous joint solution of equations from hydrodynamics and the theory of elasticity. The basic factors are gravity, compressibility of the liquid displacement of the free surface and rigitity of the elastic halfspace. Simplifying assumptions include incompressibility in the liquid or absolute rigidity.

However, in a real situation such approximations may not be valid. Thus, if the phase velocity of the Rayleigh wave approaches the velocity of the acoustic wave in the liquid, it is not possible to consider the liquid as incompressible. This may be seen from the numerical estimations below. The elastic properties of the liquid layer and the solid halfspace must be taken into account.

Of special interest is the neighborhood of the intersection of the dispersion curve of the Rayleigh wave with the strongly dispersed branches of volume modes of sound waves in the liquid layer. At these intersections the waves become strongly coupled and lose their individuality.

The equations of motion consist of coupled equations of elasticity and of hydrodynamics for vectors of mechanical displacement $\boldsymbol{U} = \{U_x, 0, U_z)\}$ and of velocity in the liquid $\upsilon = \{\upsilon_x, 0, \upsilon_z)\}$. In the y direction the system is considered as uniform (Figure 1).

The elastic equations of motion $\boldsymbol{U}$ are:

$$\gamma \frac{\partial^2 \boldsymbol{U}}{\partial t^2} = \lambda \nabla div \boldsymbol{U} + \mu \nabla^2 \boldsymbol{U} \; . \tag{1}$$

The equations of hydrodynamics are

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v}\right) = -\nabla p + \rho \boldsymbol{g} \tag{2}$$

$$\frac{\partial \rho}{\partial t} + div \rho \boldsymbol{v} = 0, p = p(\rho, t), \tag{3}$$

where γ and ρ are the mass densities in the solid and the water, $p = (\rho/\rho_0)^A$ is the pressure in the liquid, $\rho \boldsymbol{g}$ is the gravi-

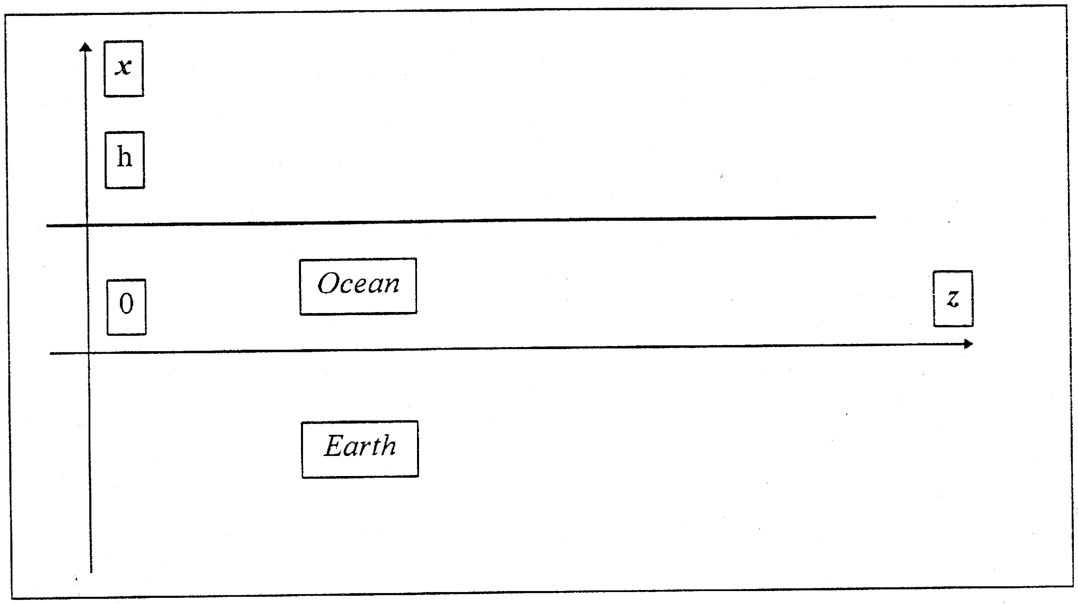

Fig. 1. Geometry of the Ocean-Earth system.

tational force, and μ, λ are Lame's constants. For small displacements and after linearization of equations (1) and (2) all variables may be assumed proportional to exp $(i(\omega t\text{-}kx))$.

The solution of the linearized equations (1) and (2) may be written

$$v_z = c_s(A_1 e^{-\varphi\xi} + A_2 e^{\varphi\xi}), \quad 0 \le \xi \le h \tag{3}$$

$$p = i\rho_0 c_s^2(\alpha_1 A_1 e^{-\varphi\xi} + \alpha_2 A_2 e^{\varphi\xi})$$

$$u_z = ih\left(\frac{\tau_l}{y} e^{\tau_l\xi} C_1 + \frac{y}{\tau_t} e^{\tau_l\xi} C_2\right), \quad \xi \le 0 \tag{4}$$

$$u_x = h(e^{\tau_l\xi} C_1 + e^{\tau_l\xi} C_2)$$

where

$$\xi = \frac{z}{h}, \alpha_{1,2} = (r_4 \pm \varphi x^2)/(x^2 - \frac{y^2}{\eta_3^2})/x\eta_3 \tag{5}$$

$$r_4 = \eta_2(x^2 - 2y^2/\eta_3^2), \tag{6}$$

$A_{1,2}$ and $C_{1,2}$ are arbitrary constants, x=$\omega h/c_t$ and $y = kh$.

Introducing the boundary conditions of continuity of normal pressures and normal velocities across the boundary ξ=0, and the condition of free surface of the liquid at ξ=1, we obtain the general dispersion equation of coupled acoustic-gravity Rayleigh waves

$$D_R\left\{r_1 T(\varphi) - x^2 B\right\} + \frac{\rho_0}{\gamma} B\tau_1 x^2 (y^2 \frac{4}{B^2} - x^4) T(\varphi) = 0 \tag{7}$$

where

$$D_R = (\tau_t^2 + y^2)^2 - 4\tau_t\tau_l y^2, r_1 = 2y^2 - \eta_3^2 x^2 \tag{8}$$

$$\eta_1 = \frac{c_t}{c_l} \tag{9}$$

$$\eta_2 = \frac{c_t}{c_s}, T(\varphi) = \frac{\tanh\varphi}{\varphi}, \psi = i\varphi, \tag{10}$$

$$B = \frac{2c_t^2}{gh}, \eta_4 = \frac{(gh)^{1/2}}{c_s}, \varphi = (y^2 - x^2\eta_3^2 + \eta_2^2)^{1/2}, \eta_2 = \eta_4^2/2, \tag{11}$$

$$c_t = \left(\frac{\mu}{\gamma}\right)^{1/2}, c_l = \left(\frac{2\mu + \lambda}{\gamma}\right)^{1/2}, c_s = \left(\frac{p_0}{\rho_0}\right)^{1/2}. \tag{12}$$

As $B \to 0$ equation (7) describes the classical Rayleigh wave when the phase velocity is C_R, and the acoustic-gravity wave in the liquid. However, in general the value B is of order 10^2-10^3 and equation (5) becomes complicated. Consider some asymptotic cases admitting a simple analytical solution. For an incompressible liquid we may put $c_s \to \infty$ and equation (7) $\eta_{3,4}$=0, $\varphi = y$, $r_1 = 2y^2$. In the case of short waves $kh \to \infty$ and equation (7) can be rewritten more simply as

$$\Delta_1 \, \Delta_2 = 0 \tag{13}$$

where Δ_1=y-$x^2B/2$ and Δ_2=D_r+$(\rho/\gamma)2\tau_1 x^2/B$.

As seen from (13) two independent waves can propagate. One wave (with Δ_1=0) obeys ω^2=kg, and correspond to a surface gravity wave in an incompressible liquid (Whitham, 1974). The second branch with Δ_2=0 describes a Rayleigh wave on a solid halfspace with a modified dispersion law because of the presence of the water layer.

However, the real situation seems more complex for two reasons. First, the velocity of sound in the liquid is close to the velocity of Rayleigh waves $c_R \approx c_S$ and therefore we must take into account the sound waves in both materials. Second, because of the finite thickness of the liquid layer the waveguide P-modes can propagate even in incompressible water. These modes exist if the frequency of the wave exceeds a critical value which depends on the thickness of layer. The dispersed branches of these modes begin to intersect the Rayleigh wave dispersion curve (see Figure 2). This results in a significant complication of the physical picture of propagating waves in the system.

Some especially interesting phenomena arise at the points of intersection of the dispersed branches of P-modes with Rayleigh waves. However, in general it is impossible to obtain analytical solutions of the dispersion equation (7). A numerical analysis of equation (7) is shown in Figures 2 to 6. The numerical calculations are carried out for the following values of the parameters: c_l= 6 • 10^3*m/s*, c_t = 4 • 10^3*m/s*, c_S = 1.5 • 10^3*m/s*, γ= 3*g/cm*^{3}, ρ = 1*g/cm*^{3}. In this case we obtain η_1 = 0.67, η_3 = 2.7, η_2 = 0.04, η_4 = 0.15, B = 650. It is clear that parameter B is not small; therefore equation (7) must be solved numerically.

2. DISPERSION

The numerical solution of the dispersion equation (7) is shown in Figure 2. Two different waves can propagate.

(1) The gravity-acoustic branch shown as curve *a* corresponds to slow surface waves on a liquid layer.

(2) The next branches shown on Figure 2 (*b*,*c*), correspond to modified Rayleigh waves. For these branches an intersection of the Rayleigh wave with branches of P-

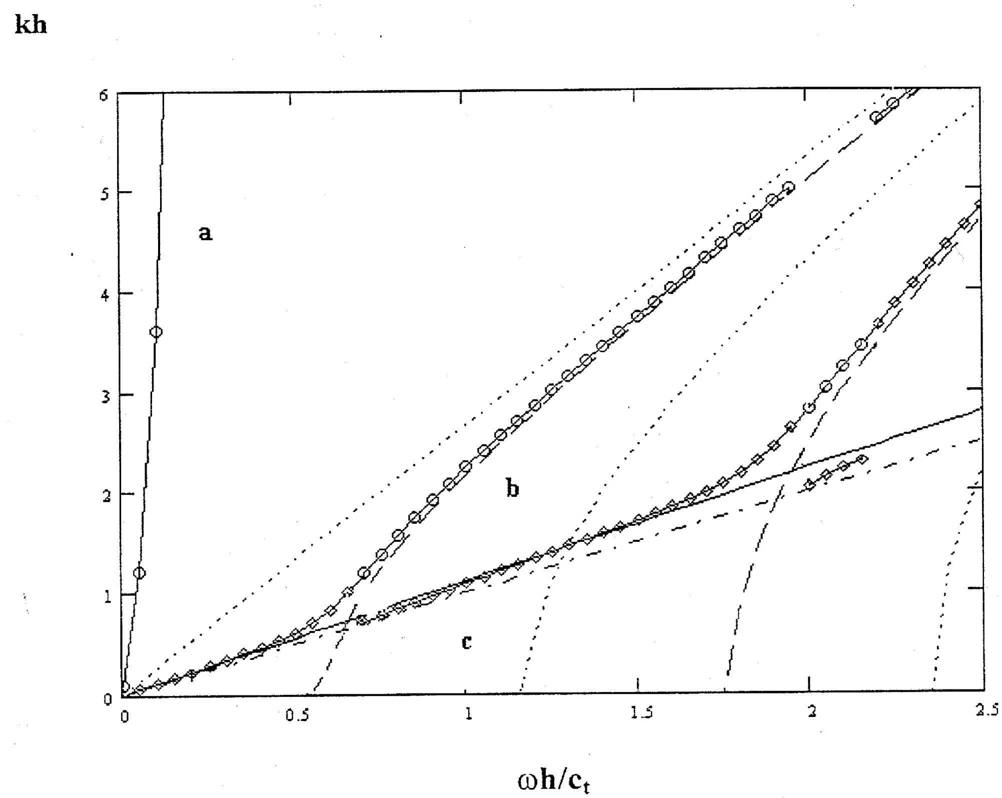

Fig. 2. Dispersive characteristics of a gravity wave (curve *a*) and a coupled Rayleigh wave (curves *b*,*c*) obtained as numerical solutions of Eq. (7). Dashed curves *e* and *g* correspond to $\psi \equiv -i\varphi = (2n+1)\pi/2$, where *e* is for n=0, *g* is for n=1. They describe the *P*-waves for a rigid bottom. Dotted curves *d*, *f* and *h* correspond to $\psi \equiv -i\varphi = n\pi$, where *d* is n=0, *f* is n=1, *h* is n=2, and $i=(-1)^{1/2}$. Line *i* corresponds to the dispersive law $\omega = kc_t$, where c_t is the velocity of shear waves in the halfspace. Line *j* corresponds to the dispersive law $\omega = kC_R$, where C_R is the velocity of Rayleigh waves on the halfspace. Points I and II are the intersections of independent dispersive branches of free Rayleigh waves on the bottom with the *P*-wave in the liquid layer. Numerical solutions show that these branches repel each other due to interactions. The distribution of wave fields for some point on the dispersive branches is shown in Figures 3 to 6.

waves in the liquid layer takes place. At the points of synchronism their dispersion branches (domains *I* and *II*) are repellent and the waves become strongly coupled. The normal modes of the liquid layer cause cutoff frequencies, and also a cutoff on wavenumbers due to coupling with the Rayleigh wave. At small wavenumbers these waves become "leaking" waves.

The propagating waves in the system are superpositions of independent wave motions with different dispersion laws. The appropriate parameter for the classification of wave modes is φ in (7). For the slow branch we have $\varphi^2>0$, and the argument of tanh in (7) is a real function of frequency. These waves are surface modes (see Figure 3).

For waveguide P-modes we have $\varphi^2<0$ and the argument of tanh in (7) is imaginary. If we put $\varphi=i\psi$, the hyperbolic expression tanh $\varphi/\varphi=\tan\psi/\psi$ becomes a trigonometrical function. For these modes the distribution of pressure and velocity oscillates with depth (Figure 4 to 6).

As the waveguide modes are located near $\psi_n=(2n+1)\pi/2$, $n = 0,1,2...$, it is convenient to classify the wave branches by specifying the value of ψ_n. In Figure 2 the corresponding curves are dashed. The dotted lines show the case of absence of body waves, with values $\psi_n=\pi n$. At such points we may expect no perturbation of the Rayleigh waves due to the water layer. Case n=0 corresponds to the slow gravity surface wave.

The dependencies of the wave values on depth for various frequencies can be seen more clearly in Figures 3 to 6 as a function of the pressure p, the velocity $\boldsymbol{v}$ and the flux of energy P.

The Rayleigh wave is dispersed because of the influence of the water layer. In the vicinity of the interaction of Rayleigh and waveguide P-modes the dispersion properties become essential. Where waveguide modes are absent, the dispersion of waves is rather insignificant. Thus the wave field of this wave is rather complicated.

Let us consider the distribution of amplitudes, velocities and average fluxes of energy in the vertical coordinate z (Figures 3 to 6). For average fluxes of energy we may use the expression

$$P_k=-(1/2)Re(p_{kj}\,\partial u_j/\partial t)$$

and for the liquid layer

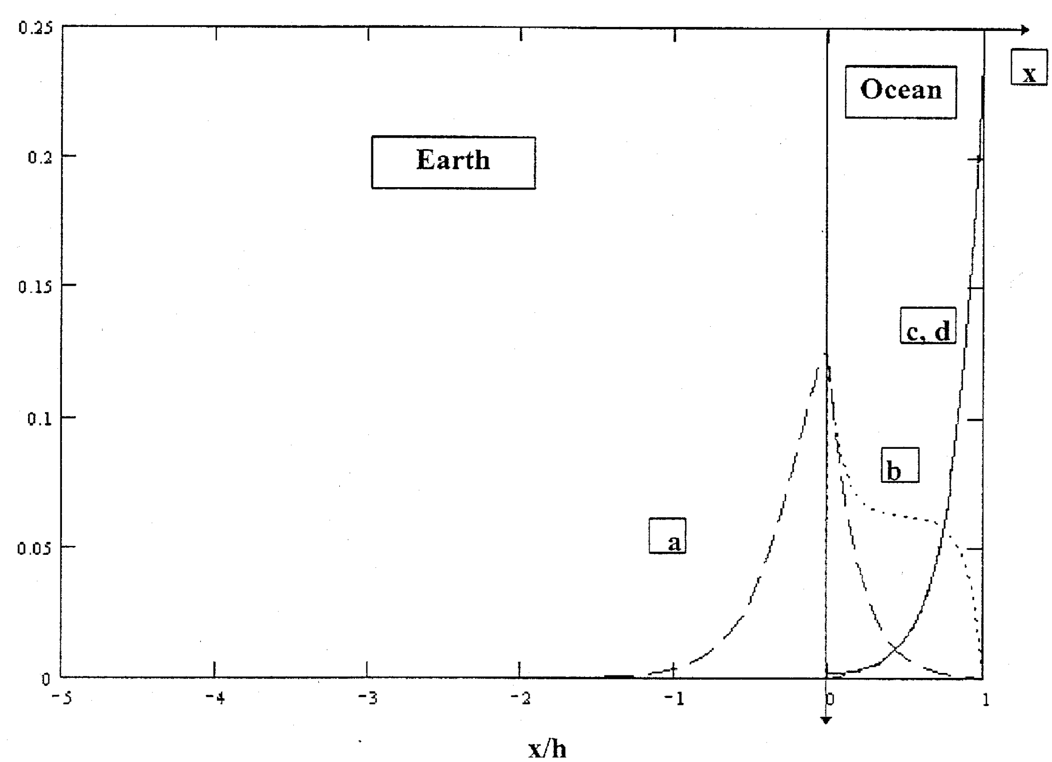

Fig. 3. Distribution of pressure, velocity and energy flux for the slow branch of gravity and Rayleigh waves in Ocean and Earth in dimensionless units (a=normal pressure; b=tangential flux of energy; $c=v_z$ and $d=v_x$. This case is $\varphi\rightarrow 0$ in Eq. (7).

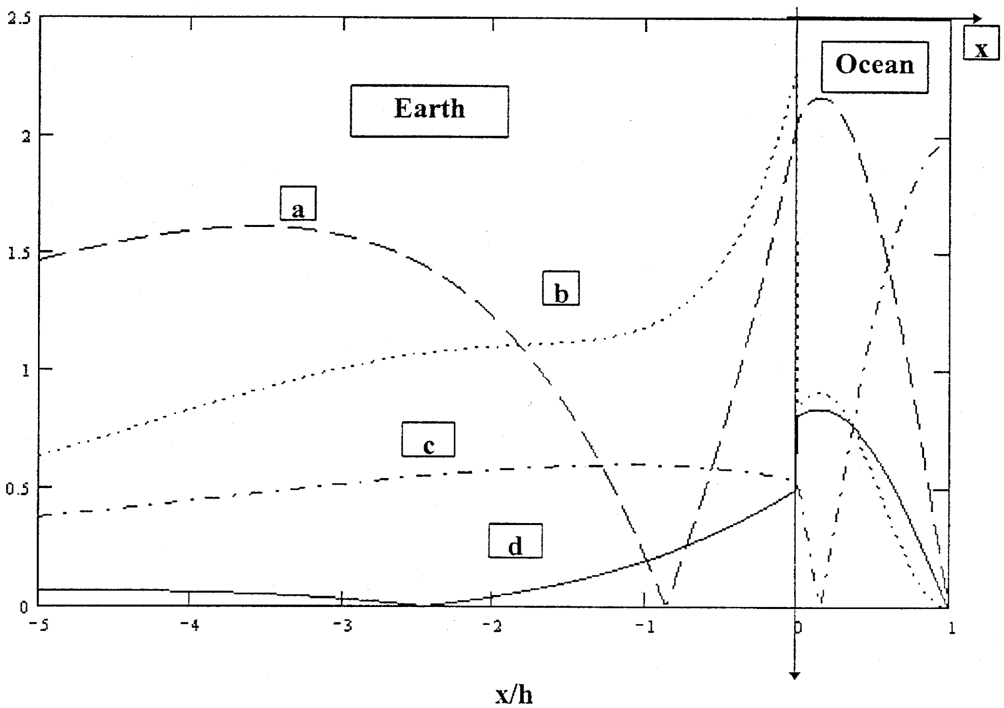

Fig. 4. Distribution of pressure, velocity and energy flux for fast branch of gravity-acoustic and Rayleigh waves in Ocean and Earth in dimensionless units (*a*=normal pressure; *b*=vertical flux of energy; $c=v_z$ and $d=v_x$, for $\psi_n=-i\phi_n=\pi/2$ near the first intersection of branches of P-waves and Rayleigh waves (domain I) in Fig. 2.

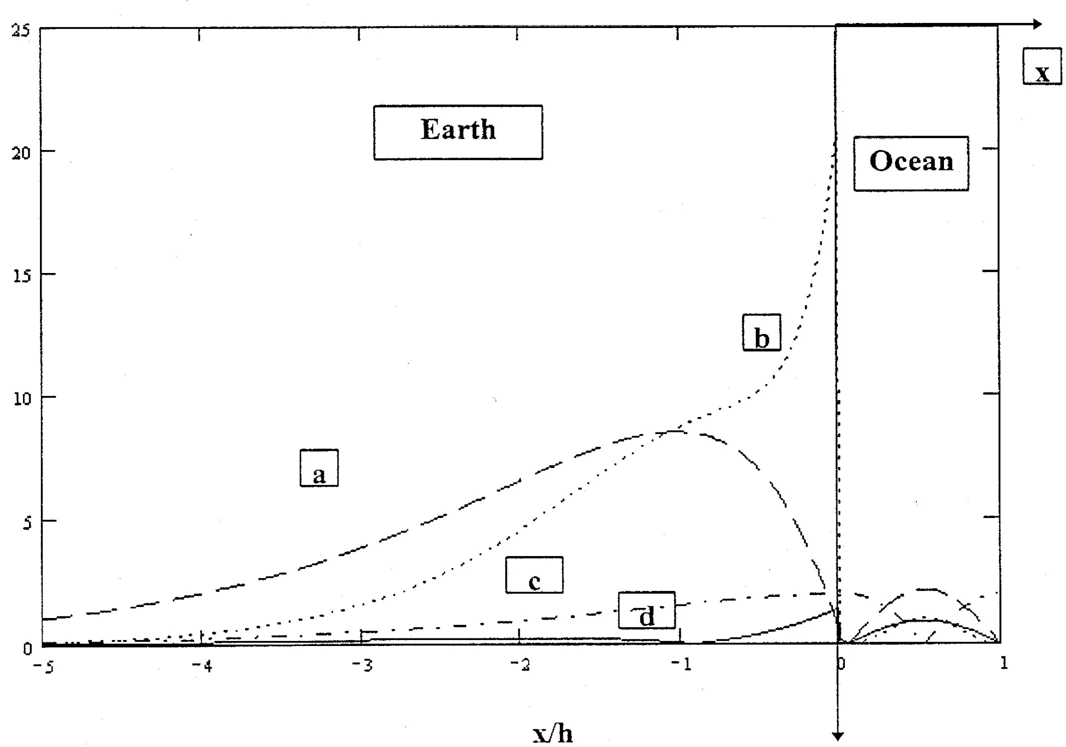

Fig. 5. Distribution of pressure, velocity and energy flux for fast branch of gravity-acoustic and Rayleigh waves in Ocean and Earth (*a*=normal pressure; *b*=vertical flux of energy; $c=v_z$ and $d=v_x$. Case $\psi_n=-i\phi_n=\pi$ near the intersection of curve *f* with Rayleigh waves (see Fig. 2).

$$P_k=(1/2)Re(pv_k)\ .$$

On Figures 3 to 6 we show the wave field for the slow and fast wave branches for some points of the dispersion curves. In Figure 3 the slow branch has a surface wave character and practically does not couple with the Rayleigh wave on the ocean floor. For this branch the curve of $v_{x,z}$ practically coincide. The flow of energy is concentrated in the layer. The picture becomes more complex in the vicinity of an intersection of branches and the formation of coupled P-wave and gravity-Rayleigh wave (Figure 4 to 5). In Figures 4 to 6 the waves become internal: the pressure ***p*** and the flow of energy ***P*** is concentrated in the vicinity of the water - substrate interface. Also the distribution of velocity components ***v*** in the layer is interesting.

From Figure 4 to 6 the velocity ***v*** on the free surface has only a normal component ($v_z \neq 0$). However, with depth there begins to prevail the longitudinal component due the change of orientation of the velocity vector. The pressure in the liquid increases with depth. However, near the bottom pressure reaches a maximum and the decreases. This is due to the Rayleigh wave on the ocean floor.

When $\psi_n=\psi_1=\pi$ the main wave is the Rayleigh wave, and from Figure 5, $p_{zz}=0$ at the boundary $\xi=0$. In this case the influence of the liquid layer is small because the region lies outside of the dispersed branches of P-waves in a liquid. Case $\psi_n=\frac{3}{2}\pi$ is shown in Figure 6. For this branch the distribution of all values has an oscillating character. As the modes of high order are usually strongly absorbed, they have little influence on seismic propagation. For the 1969 Kurile earthquake recorded in Honolulu (Weaver *et al.*, 1970) only the fast branch of the seismic excitation propagated along the interface Ocean-Earth crust was recorded. For such strong seismic excitation, a preliminary nonlinear analysis should be required.

DISCUSSION

The propagation of waves in semi-confined and layered media is a classical problem of hydrodynamics and the theory of elasticity (see Lamb 1930, Ewing 1957). However, the physics even of rather simple wave motions in isotropic layered systems can be rather complex. Because of the presence of several characteristic scales, including wavelengths and thickness of layers, the waves in such systems have es-

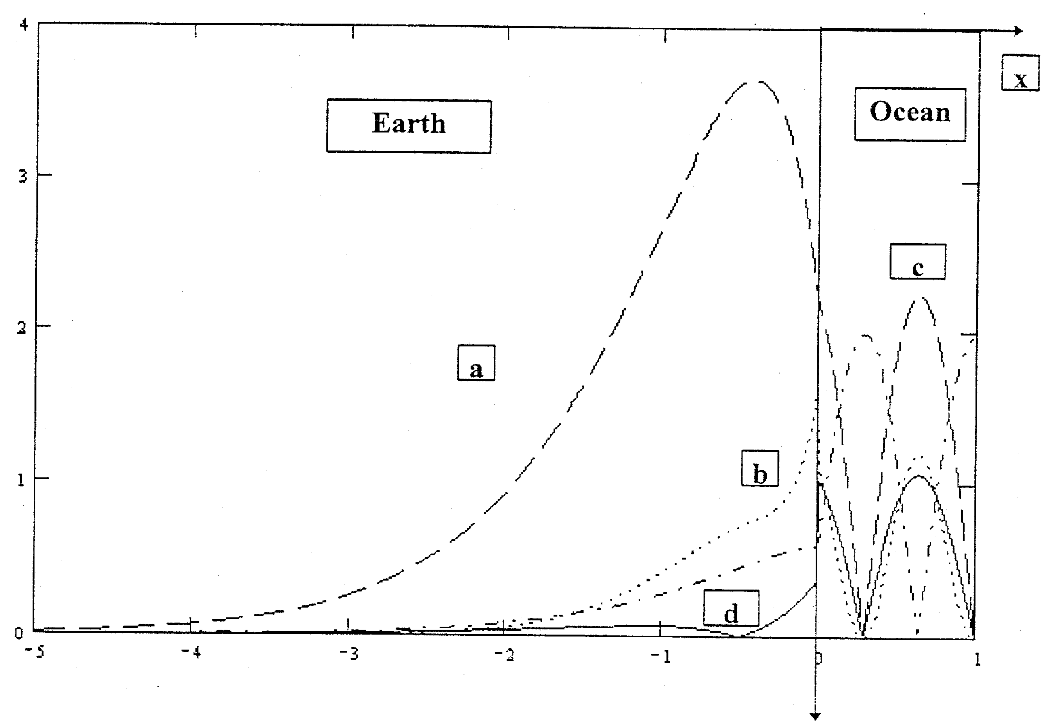

Fig. 6. Distribution of pressure, velocity and energy flux for fast branch of gravity-acoustic and Rayleigh waves in ocean and Earth in dimensionless units (*a*=normal pressure; *b*=vertical flux of energy; $c=v_z$ and $d=v_x$. Case $\psi_n=-i\phi_n=3\pi/2$ near the second intersection of branches of *P* waves and Rayleigh waves (domain II in Fig. 2).

sentially dispersive properties. Similar problem was studied in Ewing, 1957 as applied to microseismic theory. We will concentrate our attention to nonlinear waves propagation.

The dispersive equations are rather bulky. For simplification one of several wave form are selected and factors such as compressibility are neglected. However, this may obscure essential relationships between waves of various kinds. Often is assumed that the fluid is incompressible for gravity waves on its surface. This approach is common for waves propagating in combined Ocean-Earth models when the velocity is small.

In a liquid layer, gravity and sound waves may approach the velocity of Rayleigh waves on the bottom. For a hard bottom the phase velocity of sound in the liquid may tend to infinity, especially near points of frequency cutoff. However, in a more realistic elastic halfspace the velocity of these waves is restricted to the velocity of Rayleigh waves because of radiation into the halfspace. This results in frequency and wave number cutoffs.

We solve the problem in a more general form by writing down the complete equations of hydrodynamics for the water layer and the equations of elasticity for the bottom. This accounts for all the main physical factors without neglecting the constraints due to scale. This is a more exact theory, which includes the various waves in the connected boundary system.

If we assume a compressible liquid and hard bottom, we may introduce gravity waves on the surface of the liquid, or internal gravity waves in the liquid. Such waves may cause elastic displacements of the bottom and the waves on the bottom may cause a response in the water layer. Thus an incompressible liquid is a zero-order approximation for the coupling constant $\rho gh/\mu \ll 1$. This approximation is good when $\mu \to \infty$. Otherwise if $\rho gh/\mu \gg 1$(in the case of deep water, $h \to \infty$) then the resulting wave processes become spatially shared. But this approach is no longer valid for a water layer of finite depth h. It does apply to internal waves in a liquid.

In general it is not possible to consider P waves in the water layer and in the elastic halfspace as independent, and it is necessary to solve the complete system of the connected equations.

Lomnitz *et al*. (1999) have found that coupling between P waves and Rayleigh waves can occur when the two modes share the same phase velocity at an interface. They show that a monochromatic mode of large amplitude may generate stationary patterns and may be responsible for earthquake damage.

4. CONCLUSION

The propagation of waves in a two-layer Ocean-Earth model has been described. We provide a physical picture of the structure of coupled waves. It is shown that the Rayleigh wave has increased dispersion due to the influence of the water layer.

A general theory should include such factors as inhomogeneous density distribution in both media and attenuation (Whitham, 1974). However, these factors yield only a small contribution to the phase velocity.

A major question is the analysis of propagating waves in the presence of nonlinearity due to large earthquakes (Weaver *et al*., 1970; Yuen *et al*., 1982). Such perturbations can produce solitary waves or solitons. The propagating of solitons can be investigated in details only after clarification of the main mechanisms of nonlinearity.

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