Tomographic imaging for reconstructing ionospheric distributions

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Received: October 1, 1997; accepted: July 9, 1999.

RESUMEN

El sistema ionosférico se analiza y se modela usando la técnica tomográfica, formulada en tres segmentos: el sistema de adquisición de datos ionosféricos, el modelo y el algoritmo de reconstrucción. Los límites de resolución del sistema se analizan, seguido de una medición cuantitativa de la capacidad de resolución del sistema completo.

PALABRAS CLAVE: Ionosfera, tomografía, imagen, reconstrucción, algoritmo.

ABSTRACT

The ionospheric system is analysed and modelled using tomographic imaging, formulated in three segments: the ionospheric data acquisition system, the model, and the reconstruction algorithm. The resolution limit of the system is analysed, followed by a quantitative measure of the complete imaging system's resolving capability.

KEY WORDS: Ionosphere, tomography, imaging, reconstruction, algorithm.

INTRODUCTION

Tomographic imaging systems are based on the Radon transform (Radon, 1917) and Fourier optics. A complete imaging system consists of a data acquisition system, a model, and a reconstruction algorithm. These imaging systems have been used in many different disciplines. Medical technology uses Computer-Aided Tomography in CAT scanners (Scudder, 1978; Macovski, 1983; Lewitt, 1983, Censor, 1983), Nuclear Magnetic Resonance (NMR) or Magnetic Resonance Imaging (MRI) (Hinshaw and Lent, 1983; Cho *et al.*, 1982) and ultrasonic imaging (Havlice and Taenzer, 1979; Greenleaf, 1983; Mueller, Kaveh and Wade, 1979).

A new application of tomography has been proposed for the earth's ionosphere (Austen *et al.*, 1988). We analyse the feasibility of tomographic reconstructions, the resolution limitations of ionospheric imaging systems and the sensitivity of the resolution to various parameters. The ionosphere both enhances and degrades the ability to communicate using radio signals. Reflection of signals depend on the electron density distributions at specific positions or along specific directions in the ionosphere. Satellite techniques allow determination of the electron density in the vertical strip.

IONOSPHERIC MEASUREMENTS

Methods of measuring the electron density in the ionosphere include the examination of galactic radio emissions and reflecting signals off the moon to analyse the ionosphere's effect on various signals. The four most widely used techniques for obtaining the electron density in the ionosphere are ionosonde measurements, incoherent scatter radar, Faraday rotation and differential Doppler methods.

The differential Doppler technique examines the effect of the ionosphere on the phase of two harmonic frequency signals. The signal path length of a transmitted wave received at a ground station is proportional to the reciprocal signal wavelength in the ionospheric medium. Let the phase path $\boldsymbol{p}$ be the signal path length received from the transmitter, and let λ_m be the signal wavelength and $\boldsymbol{ds}$ the differential path element, thus

$$p = \int_{\rho} (ds / \lambda_m) . \tag{1}$$

In this way the phase path is a nondimensional quantity. This is a convenient expression to use with geodetic receivers of the Transit satellite navigation system.

The wavelength of the signal in the ionospheric medium can be expressed in terms of the wavelength in free space λ_o and the refractive phase indexes of the ionosphere $\boldsymbol{n}_{phase}$ as

$$\lambda_m = \frac{\lambda_0}{n_{phase}} . \tag{2}$$

The refractive phase index $\boldsymbol{n}_{phase}$ is a function of the plasma frequency, $\boldsymbol{f}_p$, and of the frequency $\boldsymbol{f}$ of the signal which is propagated through the plasma. The index can be approximated by the first two terms of a binomial expansion of the function

$$n_{phase} \simeq 1 - \frac{f_p^2}{2f^2} \ . \quad (3)$$

The plasma frequency in Hz can be approximated in terms of electron density N_e in electrons per cubic meter by

$$f_p^2 \simeq 80.62 N_e \ . \quad (4)$$

The phase path can now be written in terms of electron density along the propagation path using (1) to (4)

$$p = \frac{1}{\lambda_0}\int (1 - 40.3\frac{N_e}{f^2})ds \ . \quad (5)$$

By examining the difference between the phase paths of two harmonic signals, the effect of the ionosphere can be found. Let f_1 and f_2 be the frequencies of two harmonic signals. The ratio of these frequencies will be a constant. As the two frequencies are related by an integer rate, a weighted difference of their respective paths p_1 and p_2 can be defined:

$$\Delta p = p_1 - \left(\frac{f_1}{f_2}\right) p_2 \ . \quad (6)$$

Without the effect of the ionosphere, this weighted difference would be zero. Substituting into the expressions for the phase paths, it can be seen that this difference is directly proportional to a line integral of the electron density along the propagation path:

$$\Delta p = \int_{\rho} \frac{n_{phase1}}{\lambda_{01}} ds - \left(\frac{f_1}{f_2}\right) \int_{\rho} \frac{n_{phase2}}{\lambda_{02}} ds \ . \quad (7)$$

The wavelength in free space is given by the ratio between the speed of light c and the frequency of the signal, $\lambda_{oi} = c / f_i$ for i = 1, 2 . Thus (7) can be further simplified to obtain the relationship of the electron density

$$\Delta p = k \int_{\rho} N_e ds \quad (8)$$

where

$$k = \frac{40.3 f_1}{c}\left(\frac{1}{f_2^2} - \frac{1}{f_1^2}\right) . \quad (9)$$

The weighted difference of the phase paths is a constant, k, times the Total Electron Content (TEC) along the propagation path defined by

$$TEC = \int_{\rho} N_e ds \ . \quad (10)$$

The differential Doppler method of imaging the electron density uses the total electron content and corrects it for angles, thus obtaining the total electron content in a vertical strip in the ionosphere (Rishbeth and Garriott, 1969; Hargreaves, 1979; de Mendonca 1962; Ross, 1960). Considerable work has been done on improving the accuracy of the Faraday rotation and differential Doppler methods (Garriott, 1960; Yeh and Swenson, 1961; de Mendonca, 1962; Burgess, 1962). Since these techniques are based upon the measurement of total electron content (TEC) along a path they can give the electron density in only one dimension. They cannot provide information about the electron density at various altitudes. Incoherent scatter techniques can detect electron concentrations along the signal path; however, this also results in one-dimensional information. The importance of ionospheric tomography lies in imaging a two-dimensional region of the ionosphere.

IONOSPHERIC TOMOGRAPHY

Tomographic imaging systems are based upon the idea of reconstructing a source from data taken in multiple views of the image area. In some tomographic applications, three-dimensional objects are reconstructed from two-dimensional data. Computer-Aided Tomography (CAT) is used in systems that are suitable for x-ray illumination (Scudder, 1978; Lewitt, 1983; Censor, 1983). Applications range from the medical professions for diagnostic purposes, to geophysics for determining structures in the earth, and even to imaging nuclear fuel pin bundles (Macovski, 1983; Sanderson, 1979; Yeh and Swenson, 1961). Diffraction tomography is used where diffraction effects of the illumination wavefield must be considered, such as remote sensing and nondestructive evaluation (Munson *et al*., 1983). Synthetic Aperture Radar (SAR) is used for target location (Elachi *et al*., 1982; Dines and Lytle, 1979; Austen *et al*., 1988) and Nuclear Magnetic Resonance (NMR) imaging is a medical tool (Hinshaw and Lent, 1983; Cho *et al*., 1982). In CAT, the illumination source is a set of x-ray beams, and the detected signal is a projection. The principles of CAT directly apply to the ionosphere.

Computer-Aided Tomography (CAT) is based on the work of J. Radon (1917). He proved that a two-dimensional distribution can be uniquely reconstructed from an infinite number of projections. In x-ray parallel beam tomography, x-rays are emitted from a plane source, attenuated by the object to be imaged and detected by receivers in a plane which lies parallel to the source plane. The received data is an attenuated shadow of the object. Projections are taken from different angles by rotating the object or the source and the receiving planes. Since shadows of an object are identical from opposite views of the object, projection covering 180° is sufficient (Figure 1).

MATHEMATICAL MODEL

To develop a mathematical model of the system, denote

Fig. 1. Standard parallel beam tomography system.

Fig. 2. Definition of rotated coordinate system.

the source magnitude as $\boldsymbol{I_o}$, the object attenuation profile as ***g(x,y)*** and the received magnitude as ***I(x)***. The problem is to find ***g(x,y)***, given ***I(x)***, with $\boldsymbol{I_o}$ known (Macovski, 1983). ***I(x)*** is related to a line integral of the object attenuation profile along the path of the x-ray beam as

$$I(x) = I_0 e^{-\int g(x,y)dy} \ . \tag{11}$$

Define a new function

$$p(x) = 1n\left(\frac{I_0}{I(x)}\right) = \int g(x,y)dy \ . \tag{12}$$

For any point $\boldsymbol{x_o}$, $\boldsymbol{p(x_o)}$ is a line integral of the attenuation produced by the object along a ray path and is the shadow of the object at $\boldsymbol{x_{o.}}$ Thus ***p(x)*** is the complete shadow and is called a projection. The data acquisition system obtains samples which are often called projection data.

To obtain multiple projections, the object is rotated and the line of integration is taken at different angles. For a projection at an angle θ, define a new rotated coordinate system with axes, X-Y, as shown in Figure 2. The two coordinates are related as follows:

$$X = x\cos\theta + y\sin\theta \quad x = X\cos\theta + Y\sin\theta \tag{13}$$

$$Y = -x\sin\theta + y\cos\theta \quad y = X\sin\theta + Y\cos\theta \ . \tag{14}$$

The projection will then be a line integral along Y, namely

$$p_\theta(X) = \int g(X\cos\theta - Y\sin\theta, X\sin\theta + Y\cos\theta)dY \tag{15}$$

By varying θ, expressions for all the projections can be obtained.

The Central Slice Theorem is the basis for the reconstruction of images from projection data. Consider a projection of an object taken at an angle θ. The Central Slice Theorem states that the spectrum of this projection is a slice of the spectrum of the object, passing through the origin at an angle θ. To illustrate this relationship, consider a projection taken at an angle θ with a rotated coordinate system defined as before. A similar rotated coordinate system can be defined in a spatial frequency domain. Let F_x and F_y be the frequencies in the rotated system, and let f_x and f_y be the frequencies in the unrotated system. By the Central Slice Theorem the frequencies are related as:

$$F_x = f_x\cos\theta + f_y\sin\theta \quad f_x = F_x\cos\theta - F_y\sin\theta \tag{16}$$

$$F_y = -f_x\sin\theta + f_y\cos\theta \quad f_y = F_x\sin\theta - F_y\cos\theta \tag{17}$$

The object ***g(x,y)*** can be written equivalently in the rotated coordinate system as ***G (X,Y)***. Using the rotated coordinate system, the integration along the ray paths becomes an integration along Y, or

$$p_\theta(X) = \int G_\theta(X,Y)dY \ . \tag{18}$$

The spectrum of the projection can be expressed in terms of the object spectrum as

$$P_\theta(f) = \int p_\theta(X)e^{-j2\pi fX}dX \ . \tag{19}$$

The projection function can be expressed in terms of the original source distribution as

$$P_\theta(f) = \int\int G_\theta(X,Y)e^{-j2\pi fX}e^{-j2\pi 0Y}dYdX \ . \tag{20}$$

The projection spectrum is then the object spectrum evaluated at F_x = f and F_y = 0, or

$$P_\theta(f) = G_\theta(f, 0) \ . \qquad (21)$$

This represents one slice of the object spectrum at the same rotated angle. This slice can also be written in terms of the original coordinate system as

$$P_\theta(f) = G(f\cos\theta, f\sin\theta) \ . \qquad (22)$$

This relationship is the Central Slice Theorem and is illustrated in Figure 3. Thus can be reconstructed using an algorithm based upon the Central Slice Theorem. Ideally, an infinite number of projections would be taken so that the slices would cover the entire spectrum of the object. Then a two-dimensional inverse Fourier Transform would give an exact reconstruction of the object.

Fig. 3. Central Slice Theorem.

RECONSTRUCTION USING THE BACKPROJECTION ALGORITHM

The most common algorithm used for reconstructing images from projection data is the Backprojection Algorithm. This algorithm has the adventage of requiring only a one-dimensional inverse Fourier Transform. The Backprojection Algorithm derives its name from the step in which each projection is smeared back across the image area, an operation referred to as «Backprojection». An enhancement filter is applied to the projections prior to backprojection either by multiplication in the frequency or convolution spatial domain. This filter , H(f)=|f| , and the backprojection operation are derived from polar transformation of two-dimensional inverse Fourier Transform of $G(f_x, f_y)$, i.e.,

$$g(x,y) = \int\int G(f_x, f_y) e^{j2\pi(xf_x + yf_y)} df_x df_y \ . \qquad (23)$$

Define the polar coordinate system

$$f_x = R\cos\theta \quad R = sgn(f_y(f_x^2 + f_y^2)^{\frac{1}{2}}) \qquad (24)$$

$$f_y = R\sin\theta \quad \theta = \left(tg^{-1}\left(\frac{f_y}{f_x}\right)\right) \mathrm{mod}\,\pi \qquad (25)$$

$$df_x df_y = |R| dR d\theta \qquad (26)$$

(R, θ) in the spatial frequency domain in terms of the coordinates (f_x, f_y).

Then, the inverse Fourier Transform can be written as

$$g(x,y) = \int_0^\pi \int_{-\infty}^\infty G(R\cos\theta, R\sin\theta) e^{j2\pi(xR\cos\theta + yR\sin\theta)} |R| dRd \ . \qquad (27)$$

By using (22), the inverse Fourier Transform can be written in terms of the spectrum of the projection instead of the object spectrum as

$$g(x,y) = \int_0^\pi \int_{-\infty}^\infty P_\theta(R) e^{j2\pi R(x\cos\theta + y\sin\theta)} |R| dR d\theta \ . \qquad (28)$$

The inner integral of this equation, f_θ (r), is the Inverse Fourier Transform of the product of the projection spectrum, P_θ(r), and the enhancement filter, |R|, evaluated at $x\cos\theta + y\sin\theta$, or

$$f_\theta(r) = F.T.^{-1}(P_\theta(R)|R|) \ . \qquad (29)$$

This integral is referred to as the backprojection integral.

In the Filtered Backprojection Algorithm, |R| can be thought of as a compensation filter for the distribution of the density of backprojected rays. To illustrate, consider the reconstruction of a point source. Figure 4 shows an x-ray tomography system with rays backprojected through the image area. The density of rays increases towards the center of the image. |R| simply compensates for this effect.

Mathematically, the smearing effect can be seen by examining the reconstruction from only one projection at an angle of $\theta=\theta_0$. The object would be reconstructed from this projection directly since the integral in equation (28) reduces to a direct equality, namely

$$g(x,y) = f_{\theta_0}(x\cos\theta + y\sin\theta) = f_{\theta_0}(X) \ . \qquad (30)$$

For any point in the object, its value is the value of the filtered projection at the corresponding location in the projection. The corresponding location for a point in the object is shown in Figure 5.

Fig. 4. Density of backprojected rays.

Fig. 5. Projection location corresponding to point in the object.

Figure 6 shows that, for each projection, the object will be given the same value, f(Xo), for all point along the line $X=X_o$ since they correspond to the same point on the projection. This value can be thought of as being smeared back along this line.

Backprojecting several projections amounts to smearing each projection across the image area and summing the contributions from all of the projections.

IONOSPHERIC RECONSTRUCTION ALGORITHM

An ionospheric reconstruction algorithm can be developed based on the Filtered Backprojection Algorithm of x-ray tomography. The data received in the Doppler system lends itself to a tomographic interpretation, allowing a two-dimensional reconstruction of any slice of the ionosphere (Austen *et al*., 1988). This can be seen by an examination of the data acquisition system. The physical system used for data acquisition in the differential Doppler method consists

Fig. 6. Line of point in the object that corresponds to a point in the projection.

of a satellite orbiting the earth at an altitude of about 1000 km above the earth's surface, and a set of ground stations lying in the straight line on the earth's surface (Figure 7). At each satellite position, two coherent signals of harmonic frequencies are transmitted and are received by ground stations. The data received by each ground station can be used to calculate the total electron content along the propagation path between the satellite position and the ground station. The measured Total Electron Content (TEC) along this path can be represented by an integral of electron density along this path. Define N(s) to be the number of electrons per unit volume and ρ to be the propagation distance, then

$$TEC = \int_{\rho} N(s)ds \qquad (31)$$

Comparing (30) and (12) shows that each TEC value corresponds to a point $p(x_o)$ on a projection. The angle of the projection on which this TEC value would lie is determined by the angle of the propagation path. This angle is defined with respect to x-axis (Figure 8). This coordinate system is determined by choosing a central receiver as a point of reference and fixing the origin at the centre of the earth, with this central receiver lying on the y-axis. By interpreting the TEC data as a sample on a projection, a tomographic reconstruction approach can be used.

There are a few assumptions made on the nature of both the ionosphere and the data acquisition system in this analysis. First, it is assumed that the electron density does not fluctuate greatly during the data acquisition. In reality, the ionospheric system is time varying, and the differential Doppler data are not taken simultaneously. Second, the ground stations are assumed to lie on a straight line on the earth's surface. If ground stations are not located on a straight line, it is assumed that a geometric correction will provide the desired corrections with negligible error. With these

Fig. 7. Data acquisition system for ionospheric tomography.

Fig. 9. Formation of tomographic projections.

Fig. 8. Acquisition of one piece of data.

assumptions, it is possible to reconstruct a slice of the ionosphere.

It has been shown that an individual piece of data in the ionospheric system can be interpreted as a sample in a tomographic projection. In traditional x-ray tomography, the data are taken as projections from which reconstructions are made. However, ionospheric data are taken individually. Therefore, projections must be formed artificially by a process called reindexing.

The ionosphere data are stored with the TEC value, the receiver position and the satellite position. From the latter two, the angle of the propagation path can be determined. The reindexing process first sorts the data according to their angle of propagation. The data is then reindexed into projections by grouping data with similar angles. Figure 9 shows a simplified grouping scenario in which data of various angles are reindexed into two projections. Since each TEC value is equivalent to a point on projection, this grouping is equivalent to gathering data points or rays with the same projection angle and defining them to be a projection.

However, as shown in Figure 10, the rays in the ionosphere are not easily reindexed into projections, the angles of propagation do not naturally fall into separate projections, thus it is necessary to allow a certain amount of error during this reindexing to avoid producing projections containing only one sample. In addition, the geometry of this system is also unusual since the receivers lie on a concave curve as opposed to a straight line as in x-ray tomography. However, the actual locations of the ground stations along the propagation paths are not crucial since the assumption that the receivers lie on straight line amounts to increasing the length of the propagation path, and this extended portion of the path make zero contribution to the integral.

Each ground station in the data acquisition system receives signals from many closely spaced satellite positions. The receiver and satellite positions are not chosen for specific projection angles, and the actual angles of the propagation paths usually cover a certain almost continuously range of values.As a result, in such ranges, the separating factor may not be the actual angle. Instead, the separating factor may be the receiver location because, for any one projection, each receiver can contribute only one data point. Classifying two data values corresponding to one ground station to a single projection is equivalent to assigning two values to a single sample on a projection, which is impossible, therefore, when a data point is classified into a projection that already contains a value for that receiver, a new projection must be started. A simplified example is shown in Figure 11. The projection angles of rays A1, B2, and B3 are very close; however, B2 and B3 cannot belong to the same projection because both are data received by receiver B. Therefore, they must be separated into two projections.

There is also the question of how close the projection angles should be to be classified together into one projection. If the classification is made with relatively flexible rules, then there will be an error in the projection angle. However, less computation will be required since there will be fewer projections. On the other hand, if the classification is made with stringent rules, then most often each projection will consist of only one data point, but the projection angle will

Fig. 10. Collection of propagation paths in set of ionospheric data.

Fig. 11. Classification problem in reindexing data.

be accurate. This significantly increases the computation involved since, after the filtering process, the same number of points will be backprojected across the image for each projection despite the original number of samples in the projection.

Another difference from the standard x-ray tomography system is the non uniform sample spacing. If this uneven spacing is ignored, the reconstruction will be concentrated in one area and spread out in another. For example, the reconstruction of a point source would result in an asymmetric image. Interpolation can be used; however, the sample spacing is difficult to choose since the spacing between receivers on the projection can vary up to a factor of ten. Choosing a small sample spacing about the smallest spacing between receivers would require a very large array and would increase processing time. Choosing a large sample, spacing would group several receivers that are closely spaced into one sample.

After reindexing, each projection is then Fourier Transformed, filtered by the enhancement filter |f| and inverse Fourier Transformed (Figure 13). The filtered projection is then backprojected across the image area. In the original Filtered Backprojection Algorithm the enhancement filter was referred to as |R| where R was the radial component of the spatial frequency of the object spectrum. From the Central

Fig. 12. Uneven spacing of receivers in a projection.

Slice Theorem it was shown that the spectrum of a projection is simply a rotated slice of the object spectrum. Therefore, the spatial frequency of the projection spectrum is exactly the radial component of spatial frequency of the object spectrum at the corresponding angle. Thus, the enhancement filter is called |f| in this algorithm.

RESOLUTION ANALYSIS

The ionospheric imaging system consists of the data acquisition system and the reconstruction algorithm. It is desirable to measure the resolving capability of this imaging system given the controllable parameters, such as the number of receivers and the amount of the earth' surface they cover. Two different approaches to resolution analysis are presented. First the system is considered from a holographic point of view in which an illuminating source is used to measure the system's limitations. The analysis results in a resolution index giving an upper limit to resolving capability of the system. The second approach uses simulations of point source reconstructions to obtain a quantitative evaluation for resolution of this imaging system. In the first approach, only the physical setup of the receivers is analysed to determine an upper limit for the resolution of the system. This is a general imaging system's approach and is not concerned with the actual differential Doppler data adquisition system, since this ionospheric imaging system can be interpreted as a tomographic imaging system. The resolving capability of imaging, in general, is determined by the bandwidth of the signal and the size of the detecting aperture. For any such system, an upper limit to resolving capability can be found by considering the limits imposed by the system. The maximum bandwidth of any signal detected by the system is determined by the sample spacing. In the ionospheric system, the maximum sample spacing is the angular aperture, $\Delta\theta$, divided by the number of receivers, N, multiplied by a factor R which contains the range distance information. This assumes the ideal case of evenly distributed receivers. The

Fig. 13. Direct Fourier Method for tomographic reconstruction.

maximum angular bandwidth can be defined as follows:

$$sample\ spacing = \frac{\Delta\theta * R}{N} \tag{32}$$

$$angular\ frequency\ bandwidth = \frac{N}{\Delta\theta}\ . \tag{33}$$

This bandwidth (BW) is a maximum upper limit since not all signals will occupy the entire bandwidth. The other limit imposed by the system is the size of the detecting aperture. This is viewed relative to an infinite aperture in terms of the energy of detected signals. Although the receivers in the ionospheric system are not used to form a receiving aperture, this can be thought of as a measure of the range of propagation paths. The finite aperture restricts the angular range of the propagation paths that, in turn, restricts the range of projection angles. In the infinite case, projections are available from all angles.

A measure of resolution can incorporate both limiting factors (Lee, 1988). Define a resolution index as

$$Index = \frac{BW * D}{R} \tag{34}$$

where D is a certainty factor, R is a constant defined as in (31). D is the normalized energy of the detected signal based upon the size and relative offset of the detecting aperture. Although signals are not propagated in the ionospheric system, this certainty factor can be thought of as an indicator of the relative size of the aperture. In the ionospheric system, if the aperture lies between $\theta_{\min}$ and $\theta_{\max}$, then the certainty factor can be written as

$$D = \frac{\sin\theta_{\max} - \sin\theta_{\min}}{2}\ . \tag{35}$$

This certainty factor ranges between zero and one. For the limiting case of an infinite aperture, the certainty factor is one since it is calculated by taking a ratio of the energy of the detected signal and the energy of the signal detected by an infinite aperture. For this index, the larger the value, the better the resolution. As the aperture increases in size, the resolution improves and the certainty factor also increases, giving a larger index. In the ionospheric system, this index can be determined in both the x and y directions for any particular aperture and set of receivers. The index is different in the x- and y-directions because the detecting aperture has a different size and relative offset when viewed from the two directions. In Figure 14, the detecting aperture ranges from θ_1 to θ_2 in the x-direction and from θ_1' to θ_2' in the y-direction. The indices can then be written as:

$$Index_x = \frac{N(\sin\theta_2 - \sin\theta_1)}{2R\Delta\theta} \tag{36}$$

$$Index_y = \frac{N(\sin\theta'_2 - \sin\theta'_1)}{2R\Delta\theta}\ . \tag{37}$$

Fig. 14. Receiving aperture viewed from both directions.

Or, in terms of the aperture size $\Delta\theta$ and a central angle ϕ the indices can be rewritten as:

$$Index_x = \frac{N\left[\sin(\phi + \frac{\Delta\theta}{2}) - \sin(\phi - \frac{\Delta\theta}{2})\right]}{2R\Delta\theta} \qquad (38)$$

$$Index_y = \frac{N\left[\cos(\phi + \frac{\Delta\theta}{2}) - \cos(\phi - \frac{\Delta\theta}{2})\right]}{2R\Delta\theta} \ . \qquad (39)$$

For any fixed aperture and set of receivers, the sum of the squares of the indices in the x- and y-directions is related by a constant. Thus, there is a tradeoff between improving resolution in the x- and y-directions, as expected

$$Index_x^2 + Index_y^2 = \left(\frac{N}{2R\Delta\theta}\right)^2 \left[2 - 2\cos(\Delta\theta)\right] \ . \qquad (40)$$

This resolution index is useful for understanding the limitations imposed by the system. A second approach to resolution analysis will use this traditional definition to analyse the complete imagine system.

The minimum distance between two resolvable points is widely accepted as the definition of resolution. However, several interpretations of resolvable exist. In one definition, two points are resolvable if their first zero crossings are separate. Alternatively, two points can be said to be resolvable if their 3dB contours are separate. There are many problems associated with using such a definition for resolution.

CONCLUSIONS

The ionosphere imaging system analysed consists of three stages. The first, is the physical data acquisition system where the differential Doppler method is used to measure the phase path lengths and to calculate the total electron content. The last is the tomographic reconstruction algorithm that is a special case of the more general x-ray tomography Filtered Backprojection Algorithm. The second stage provides the link between the others taking the ionospheric data through the reindexing process. Although this idea seems straightforward at first glance, there are many subtleties that present difficulties in the imaging process. This entire imaging system built on the foundations of x-ray tomography and the idea of backprojection form the base to develop an algorithm to reconstruct ionospheric images. The resolution of this system is analysed. A resolution index for tomographic imaging systems give a quantitative measure of the effect of the controllable parameters, namely, the number of receivers and the region covered by them. This approach provides an upper limit to the resolving capability of the system.

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