

# The solar dust ring

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## RESUMEN

Una gran cantidad de partículas de polvo se concentra alrededor del Sol cuando nuestra estrella está en fase de actividad mínima. La concentración está localizada a 4 radios solares. La dinámica de estas partículas está modulada por las fuerzas: gravitacional, electromagnética y por los arrastres, así como por la presión de radiación. Todas estas fuerzas están incluidas en la ecuación de momento. Al mismo tiempo, el polvo está perdiendo masa por sublimación y tanto la presión de radiación como la sublimación controlan el tiempo de vida del polvo y modifican los parámetros orbitales. Se considera que el campo magnético sólo tiene una componente; en este caso, esa componente es la radial. Se discuten algunos aspectos físicos de la dinámica del polvo.

**PALABRAS CLAVE:** Sol, plasmas de polvo.

## ABSTRACT

A theoretical model is used to explain the dynamical behavior of dust particles (grains) near the Sun. The equation of motion includes gravitation, electromagnetic forces and radiation pressure as well as drags. All dust particles which are near the Sun lose mass by sublimation, and both radiation pressure and sublimation control the dust grain life time and modify the orbital parameters. In the model, a physical consideration is made for the solar magnetic field: it is assumed that the field has only a radial component. The set of equations which describes all forces is presented in detail. Some physical aspects of the dynamical behavior of the dust are discussed.

**KEY WORDS:** Sun, dusty plasmas.

## 1.- INTRODUCTION

An annular feature containing dust particles was detected near the Sun (Mukai *et al.*, 1974, Mukai 1983, Lamy, 1974). The light scattered by these particles forms the F corona which is the innermost region of the zodiacal cloud and is part of the zodiacal light. The feature was described as a ring located at  $4 R_{\odot}$  (solar radii).

The dynamics of dust particles around the Sun involves several important physical interactions with the heliosphere, the solar magnetosphere and the solar wind (Mann *et al.*, 1997). These interactions modify the grain sizes and the orbital parameters. In particular, the grain size is reduced by sublimation and the orbital parameters are modulated by the radiation pressure force and sublimation pseudo-force. The equation of motion includes gravitational, Lorentz, radiation pressure, drag and sublimation forces which represent a complete model in order to analyze the dynamical behavior of the dust. The chemical composition of the dust particles plays an important role because not all materials can survive near the Sun. Perhaps the best candidates are silicates and carbons.

## 2.- THE MODEL

The equation of motion of a dust particle which is near the Sun is

$$\frac{d}{dt}(m\vec{v}) = \vec{F}_L - \vec{F}_g + \vec{F}_{P-R} + \vec{F}_{drag} \quad (1)$$

where  $\vec{F}_L$  is the Lorentz force,  $\vec{F}_g$  is the gravitational force,  $\vec{F}_{P-R}$  is the radiation pressure and  $\vec{F}_{drag}$  are the drag forces.

In equation (1), the left side includes the sublimation term and

$$m = \frac{4}{3}\pi\rho a^3$$

where  $m$  is the mass,  $\rho$  is the density and  $a$  is the grain radius. Thus,

$$\frac{4}{3}\pi a^3 \rho \frac{d\vec{v}}{dt} + 4\pi\rho a^2 \frac{da}{dt} \frac{d\vec{r}}{dt} = \vec{F}_L - \vec{F}_g + \vec{F}_R + \vec{F}_{drag} .$$

From the energy balance equation on grains

$$\pi a^2 (1 - A_B) \frac{L_\oplus}{4\pi r^2} f(r) = 4\pi a^2 \epsilon_s \sigma T_s^4 + 4\pi a^2 \frac{L_M}{N_A} \frac{dz}{dt} - K(T) \nabla T|_s \quad (2)$$

where

$L_M$  = latent heat of sublimation/mole

$N_A$  = Avogadro's number

$\frac{dz}{dt}$  = flux of sublimated molecules ( $cm^{-2}s^{-1}$ )

$L_\oplus$  = solar luminosity (ergs/s)

In equation (2),  $f(r)$  takes into account the deviation from the inverse law of radiation as the particle gets closer to the Sun (the solid angle effect when  $R=O(R_\oplus)$ , and is given by:

$$f(r) = 2(1 - \cos \alpha) \frac{r^2}{R_\oplus^2} = 2 \frac{r^2}{R_\oplus^2} \left[ 1 - \left( 1 - \frac{R_\oplus^2}{r^2} \right)^{1/2} \right]. \quad (3)$$

Here  $\alpha$  is the solid angle subtended at the Sun and  $f(r) \rightarrow 1$  when  $r \gg R_\oplus$ .

In order to obtain  $\frac{dz}{dt}$  we use the Clausius-Clapeyron equation,

$$\frac{dP_s}{P_s} = \frac{L_M}{k_B N_A} \frac{dT_s}{T_s^2} \quad (3a)$$

where

$$L_M = aT_s + b. \quad (3b)$$

Generally  $a$  and  $b$  are constant for a given material. From equations (3a) and (3b),

$$n_s = n_0 \left( \frac{T_0}{T_s} \right)^{\left( 1 - \frac{a}{k_B N_A} \right)} \exp \left[ \frac{b}{k_B N_A} \left( \frac{1}{T_0} - \frac{1}{T_s} \right) \right] \quad (3c)$$

where  $n_0$  and  $T_0$  are material parameters. Also

$$\frac{dz}{dt} = n_s v_s = \left( \frac{2k_B T_s}{\pi \mu} \right)^{1/2} n_s, \quad (3d)$$

thus

$$\frac{dz}{dt} = v_s n_s = \left( \frac{2k_B T_s}{\pi \mu} \right)^{1/2} n_0 \left( \frac{T_0}{T_s} \right)^{\left( 1 - \frac{a}{k_B N_A} \right)} \exp \left[ \frac{b}{k_B N_A} \left( \frac{1}{T_0} - \frac{1}{T_s} \right) \right]. \quad (4)$$

The term  $\mu$  in the equation (4) is the mass of the sublimating molecule.

Since

$$4\pi a^2 \frac{dz}{dt} \mu = -\frac{dM}{dt} = -\frac{d}{dt} \left( \frac{4}{3} \pi a^3 \rho \right) = -4\pi a^2 \frac{da}{dt} \rho$$

where

$$\frac{da}{dt} = -\frac{z\mu}{\rho}. \quad (5)$$

When grains are near the Sun they lose mass because of interaction between the solar wind and the grain surface. The energetic particles coming from the Sun hit and take the surface dust off.

From

$$\frac{d}{dt} (m\bar{v}) = \bar{F}_{ext}$$

we obtain,

$$m \frac{d\bar{v}}{dt} + \frac{dm}{dt} \bar{v} = \bar{F}_{ext}$$

$$\frac{d\bar{v}}{dt} = \frac{1}{m} \bar{F}_{ext} - \frac{1}{m} \frac{dm}{dt} \bar{v}. \quad (5a)$$

Since  $\frac{dm}{dt} < 0$ ,  $-\frac{1}{m} \frac{dm}{dt} \bar{v} > 0$  and acts as a pseudoforce (call it  $F_{sublimation}$ ) to accelerate the particle.

Note that this is similar to the Poynting–Robertson force which is a drag force.

$\bar{F}_{pr} \approx -Cv_t \hat{i}_t$  where  $C > 0$  and  $\bar{F}_{sublimation} \approx Dv_t \hat{i}_t$  where  $D > 0$ .

$$\therefore |\bar{F}_{pr}| \gg |\bar{F}_{sublimation}| \quad (\text{i.e. for larger } r).$$

Grains will spiral toward the Sun in orbits that become ever more circular. When  $|\bar{F}_{sublimation}| \gg |\bar{F}_{pr}|$  i.e. for smaller  $r$ , when sublimation dominates, the grains will spiral out in orbits that also become more elliptical. So they could oscillate back and forth. Of course it is assumed that the particles are not completely kicked out of the solar-system as the radiation pressure term dominates over solar gravity.

The gravitational interaction between the Sun and the dust particles is expressed by the next equation,

$$\bar{F}_g = -\frac{GM_\oplus m}{r^2} \hat{i}_r \quad (6)$$

where  $G$  is the gravitational constant and  $M_\oplus$  is the solar mass.

In order to calculate the Lorentz force  $\bar{F}_L$  we need the grain charge  $Q(t)$  which is given by

$$\frac{dQ}{dt} = \sum I = I_e + I_i + I_p + I_s + I_t \quad (7)$$

with

$e \equiv$  electron collection

$i \equiv$  ion collection

$p \equiv$  photoemission

$s \equiv$  secondary electron emission

$t \equiv$  thermoionic emission

All these contributions are the currents in equation (7).

Assuming that  $\phi_s > 0$  where  $\phi_s$  is the surface potential, all currents on grains are:

(a) Electron current

$$J_e = -en_e \left( \frac{k_B T_e}{2\pi m_e} \right)^{1/2} \left( 1 + \frac{e\phi_s}{k_B T_e} \right) \quad (8)$$

where  $e$  is the electron charge,  $n_e$  is the electron density,  $k_B$  is the Boltzmann constant,  $T_e$  is the electron temperature,  $m_e$  is the electron mass and  $\phi$  is the electrostatic potential.

(b) Ion current

$$J_i = -en_i \left( \frac{k_B T_i}{2\pi m_i} \right) \exp\left( -\frac{e\phi_s}{k_B T_i} \right). \quad (9)$$

All parameters correspond to ions.

(c) Thermoionic current produced by hot grains.

$$J_t = f(a) \frac{4\pi m_e k_B^2 T_s^2}{h^3} \exp\left\{ -\frac{w + e\phi_s - \frac{5e^2}{8a}}{k_B T_s} \right\} \approx f(a) \frac{4\pi m_e k_B^2 T_s^2}{h^3} \exp\left\{ -\frac{w + e\phi_s}{k_B T_s} \right\} \quad (10)$$

where  $w$  is the work-function of the material and  $f(a) \approx 1$  when  $a \geq 0.1 \mu\text{m}$  and  $h$  is Planck's constant.

(d) Secondary emission current. This current is produced when the background plasma is sufficiently hot.

$$J_s = \frac{2\pi e}{m_e^2} \exp\left( -\frac{e\phi_s}{k_B T_s} \right) \left( 1 + \frac{e\phi_s}{k_B T_s} \right) \int_{e\phi_s}^{\infty} E \delta(E) f_e(E - e\phi_s) dE \quad (11)$$

with  $k_B T_s \approx 3eV$  (Chow, 1996) and:

$$f_e(E) = n_e \left( \frac{m_e}{2\pi k_B T_e} \right)^{3/2} \exp\left( -\frac{E}{k_B T_e} \right) \quad (11a)$$

is the Maxwellian distribution function and:

$$\delta(E) = 7.4 \delta_m \left( \frac{E}{E_m} \right) \exp\left[ -2 \left( \frac{E}{E_m} \right)^{1/2} \right]. \quad (11b)$$

Equation (11b) is the Sternglass formula for secondary electron emission and is valid for  $a \geq 0.1 \mu\text{m}$  with  $\delta_m$  and  $E_m$  being constants for a given material; when,  $a \leq 0.1 \mu\text{m}$ ,  $\delta(E)$  becomes larger than that given by (11b).

(e) Photoemission current: It is created when UV radiation is present.

$$J_p = \frac{e}{4} Q_{abs} Y F(r) \quad (12)$$

where  $Y \approx 1$  for conductors and  $\approx 0.1$  for dielectrics.  $Q_{abs}$  is the absorption coefficient and  $F(r)$  is a function of  $r$  (equation 12<sup>a</sup>).

$Q_{abs} = 1$  when  $x \left( = \frac{2\pi}{\lambda} \right) > 1$  and  $Q_{abs} = x$  when  $x \leq 1$ . In this case  $\bar{\lambda}$  is the mean wave-length in the solar spectrum  $\approx 0.6 \mu\text{m}$ .  $\therefore x \approx 10a$  ( $\mu\text{m}$ ) i.e.  $Q_{abs} \approx 1$  when  $x \geq 0.1 \mu\text{m}$ .

$$F(r) = 5.4 \times 10^{16} \left( \frac{R_{\oplus}}{r} \right)^{1/2} \quad (12a)$$

$$\bar{F}_L = \frac{Q(t)}{c} (\bar{v}_{rel} \times \bar{B}) \quad (13)$$

Several physical considerations have been done on the magnetic field,

- (1) The solar magnetic field is approximately dipolar up to the source surface ( $r \approx 2.5 R_{\oplus}$ ) and becomes radial beyond it.
- (2) We assume rigid co-rotation up to the source surface and radial expansion beyond it (i.e.  $\bar{v}_s = \left( \frac{\bar{\Omega} \times \bar{r}}{c} \right)$  inside  $r = r_s = 2.5 R_{\oplus}$  and  $\bar{v}_s = v_{sw} \hat{r}$  outside  $r = 2.5 R_{\oplus}$ )
- (3) The dipole field changes polarity every 11 years, and does so rather rapidly (i.e. within about 1 year) near solar maximum.

#### RADIATION PRESSURE

$$\bar{F}_r = H(a, r) \hat{i}_r - H(a, r) \frac{\bar{v}}{c} \quad (14)$$

where

$$H(a, r) = Q_{pr} \pi a^2 f(r) \frac{L_{\oplus}}{4\pi r^2} \frac{1}{c}. \quad (14a)$$

The first term on the right-hand side in equation (14) represents the radiation pressure term. It is due to the initial interception by the particle of the incident momentum in the beam. The second term is a mass loading drag which is due to the effective rate of mass loss from the moving particle as it continuously reradiates the incident energy.

$$\begin{aligned} \bar{F}_r &= -H(a, r) \left[ \left(1 - \frac{v_r}{c}\right) \hat{i}_r - \frac{v_t}{c} \hat{i}_t \right] \approx Q_{pr} \frac{\pi a^2 f(r) L_{\oplus}}{4\pi r^2 c} \left[ \hat{i}_r - \frac{v_t}{c} \hat{i}_t \right] \\ &= \beta GM_{\oplus} m f(r) \frac{1}{r^2} \left[ \hat{i}_r - \frac{v_t}{c} \hat{i}_t \right] \end{aligned} \quad (14b)$$

where

$$\beta = \frac{Q_{pr} \pi a^2 L_{\oplus}}{4\pi c} \frac{1}{GM_{\oplus} m} = \frac{3Q_{pr} L_{\oplus}}{16\pi c GM_{\oplus} \rho} \frac{1}{a}. \quad (14c)$$

From equation (5a):

$$\begin{aligned} \frac{d\bar{v}}{dt} &= -\frac{1}{m} \frac{dm}{dt} \bar{v} + \frac{1}{m} \bar{F}_g + \frac{1}{m} \bar{F}_r + \frac{1}{m} \bar{F}_L \\ &\approx -\frac{(1-\beta)}{r^2} GM_{\oplus} \hat{i}_r - \left[ \frac{GM_{\oplus}}{r^2} \frac{\beta}{c} + \frac{1}{m} \frac{dm}{dt} \right] v_t \hat{i}_t + \frac{\bar{F}_L}{m}. \end{aligned} \quad (14d)$$

The mathematical term  $-\frac{GM_{\oplus}}{r^2} \frac{\beta}{c} v_t \hat{i}_t$  is the Poynting-Robertson drag. Since  $\frac{dm}{dt} < 0$ ,  $-\frac{1}{m} \frac{dm}{dt} v_t \hat{i}_t$  = sublimation “force”. This force acts as a “push” on the grain’s surface.

#### DRAG FORCES

$$\bar{F}_d = \bar{F}_{coll} + F_{Coulomb}. \quad (15)$$

The first term on the right hand side is due to direct collection and the second one is the scattering of ions by grains:

$\bar{F}_{coll} = n_i m_i v_i v_s \pi b_c^2$  where:

$$v_s = \left( \frac{8k_B T_i}{\pi m_i} + v_i^2 \right)^{1/2} \quad (15b)$$

and

$$b_c^2 = a^2 \left( 1 - \frac{2e\phi}{m_i v_s^2} \right) \quad (15b)$$

$$\bar{F}_{Coulomb} = n_i m_i v_i v_s \cdot 4\pi b_{\pi/2}^2 \Gamma \quad (15c)$$

where

$$b_{\pi/2} = \frac{e\phi_s}{m_i v_s^2}$$

and

$$\Gamma = \ln \left[ \frac{\lambda_{De}^2 + b_{\pi/2}^2}{b_c^2 + b_{\pi/2}^2} \right].$$

### 3.- DISCUSSION

The dust density increases at 4 solar radii as a result of stabilizing the semimajor axis of dust particle orbits due to a slow, continual decrease in the particle size from evaporation and sputtering (Mukai *et al.*, 1979, 1981). As the ratio of radiation pressure to gravitational force continually increases as particle size decreases, a pseudo-force is generated which balances the Poynting-Robertson drag (Rusk *et al.*, 1988). This is called the sublimation effect.

The solar magnetic field also interacts with dust particles through the Lorentz force. Solar magnetic field is almost a dipole up to the solar source surface and is approximately radial beyond. The field radiates outward from the Sun at the velocity of the solar wind, then the Lorentz force on a dust particle includes both terms. The first one is the velocity of the dust grain with respect to the inertial coordinate system, and the second is the solar wind velocity.

On the other hand the Lorentz force is due to the particle’s velocity because the solar wind velocity only has a radial component. The resulting Lorentz force is normal to the particle’s orbital plane and it can affect the longitude of the ascending node and the inclination. The ascending node and the inclination are the orbital parameters which determine the orientation of the dust particle orbital plane, and the semimajor axis of a particle at the edge of the evaporation zone remains nearly constant over more than one hundred orbits (Lamy, 1974). This is due to a balancing between the Poynting-Robertson drag and the sublimation effect. The consequence is an enhancement in density at the outer edge of this zone. The evaporation zone distance depends on the thermohydrodynamical properties of the dust particles which are orbiting the Sun. This means that there are several regions of enhanced dust density (MacQueen, 1968).

Several forces are affecting the dynamical behavior of the dust near the Sun. However sublimation affects the grain size and increases the acceleration due to the magnetic field, because the charge on the dust particle is reduced linearly with particle radius while the mass varies as the cube of the particle radius.

We used a numerical representation of the field (Hoeksema, 1986) where the multipole components of the magnetic field can be generated by the coefficients  $g_m$  and  $h_m$  which are generated for each Carrington rotation. We took the coefficients from 1974 to 1996.

We have assumed that the charge depends on several currents which come from ions, electrons, photoionization, thermoionization and secondary electron emission.

Regarding the chemical properties we used different materials in order to know the sublimation rate.

In a model by Krivov *et al.* (1998), the authors included gravitational, electromagnetic and radiation pressure forces and sublimation. They found the following results:

- (a) Carbon grains sublimate between 3-4 solar radii.
- (b) Carbon grains possess high  $\beta$  ratios and must be larger than 2.4  $\mu\text{m}$  to reach the near-solar region.
- (c) For  $a > 2.4 \mu\text{m}$ , Lorentz force is relatively weak, comes basically from the dipole zonal component of the field and leads to low-amplitude oscillations of orbital inclinations and a precession of the lines of nodes.
- (d) Results for silicate grains are the same.
- (e) Silicate grains disappear at a heliocentric distance of  $\cong 2$  solar radii.
- (f) After several oscillations, carbon grains are eventually ejected as  $\beta$  meteorites when they approach a critical radius of 2.4  $\mu\text{m}$ , which corresponds to the radiation pressure to solar gravity ratio  $\beta$  equal to unity.
- (g) The orientation of the orbital planes of the particles is dictated by the Lorentz force.

In the present model drags are included in the equation of motion and we expect to find the following results:

- (a) As gravitational and electromagnetic, forces, radiation pressure, drags forces and sublimation are included in a complete model, we expect to obtain representative results about the dynamics of dust particles around the Sun.
- (b) The mass leads to a secular acceleration in a way similar to the Poynting-Robertson force and causes a secular retardation.

- (c) As Poynting-Robertson force causes the orbit to shrink and become more circular, the mass loss term causes the orbit to expand and become more elliptical.
- (d) The mass loss causes the “effective gravitational constant” to decrease. This causes the orbit to expand and become more elliptical.
- (e) Poynting-Robertson effect will dominate at larger distances where evaporation is negligible, and the mass loss effect will dominate closer to the Sun.
- (f) A size limit of sublimation for several materials when grains are near the most representative distances of the dust’s concentration.
- (g) The distances where dust grains are sublimated.
- (h) The critical radius for  $\beta$  meteorites.

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