A color representation of two-dimensional discontinuous seismic structures

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RESUMEN

Presentamos un procedimiento computacional para representar en colores a una estructura sísmica bidimensional, mostrándose como ejemplo la variación de velocidades sísmicas. La estructura está compuesta por capas verticales lateralmente heterogéneas y separadas por discontinuidades estructurales. En términos más generales, la variable a representar es bivariante, suave y sujeta a discontinuidades finitas (en las interfaces de las capas). Cada capa está dividida por líneas verticales con lo que se forman celdas trapezoidales: los lados no paralelos corresponden a las interfaces que separan las capas entre sí. La parametrización del modelo consiste en asignar valores de la variable analizada a cada vértice de las celdas trapezoidales. Para obtener un valor en un punto interno de una celda, se aplica una interpolación lineal en la intersección de cada lado no paralelo y la línea vertical que pasa por el punto en cuestión; luego, se hace otra interpolación lineal sobre la línea vertical. Este ya reportado procedimiento asigna unívocamente a cada punto un color asociado al valor de la variable que se representa, utilizando una paleta de colores y una discretización de los valores de la variable estructural. La representación en colores resulta muy exacta y no introduce características espurias debido al control que se tiene del esquema numérico de interpolación y suavización.

PALABRAS CLAVE: Modelo sismológico 2-D, parametrización de modelos, representación en color, interpolación, suavización.

ABSTRACT

We present a computational procedure to represent with colors two-dimensional piecewise continuous functions. Seismic wave velocities are used as examples. Vertically and laterally heterogeneous layers are separated by structural discontinuities. Vertical lines divide each layer into trapezoidal cells; the non-parallel sides are the interfaces. A bivariate function is parametrized by assigning a value to each trapezoid vertex. The value at any point inside a trapezoid is obtained by linear interpolations applied along the non parallel sides up to the vertical line passing through the point and, then, along this line. Based in this already reported interpolation scheme, the procedure searches at each layer for points associated with a specific value range of the seismic parameter being represented; the resulting contiguous points define on area which is filled with a unique color from a palette. This procedure gives an accurate representation of a smooth, bivariate function with finite step discontinuities along layer interfaces, and does not introduce spurious features in the interpolation/smoothing numerical process.

KEY WORDS: 2-D seismic model, model parametrization, color representation, interpolation, smoothing.

INTRODUCTION

Color representation of two-dimensional structures is a useful tool for visually detecting incompatibilities among elements of an individual structure as well as among neighboring structures in two or three-dimensional models of an earth region. An isotropic seismic structure is usually described by values of five spatial coordinate functions (to be called "seismic parameters" from here on): density, the two seismic body-waves velocities, and the corresponding pair of attenuation factors. In this paper, we use the longitudinal (V_p) and transverse (V_s) seismic wave velocities in a layered vertical section. The mathematical problem to be solved is the color representation of a layered, bivariate function $v(x,z)$ that may have finite discontinuities at interfaces; the thickness of the layers is not constant. Formally, the solution of this problem is done in the following steps. First, we find the best way to represent in discrete form the function $v(x,z)$ in order to interpolate values at any point. Secondly, we associate the values of the seismic parameter with a color palette taking into account the range of this function as well as the required resolution and an intuitive appeal of the color distribution. Finally, an algorithm is designed to fill the region between consecutive contours with a suitable color, according to the palette. Commercial graphic libraries that draw contours or, more generally, make color representation of bivariate functions, do not commonly take into account the presence of discontinuities. Besides, their schemes of interpolation commonly introduce a smoothing that is difficult to evaluate and control, and consequently

may produce a diffuse interface at the locations of discontinuities.

The computational procedure presented here overcomes these problems. We represent seismic models by means of heterogeneous layers, a natural choice, since these are actually present in the crust and the upper mantle. Boundaries between layers mark in principle (but not necessarily) a finite discontinuity at least in one of the seismic parameters. Lateral and vertical variations inside each layer can be also discretized by using cells with various geometrical shapes and interpolation properties. The most commonly used cells in seismology are either homogeneous rectangle shaped with a constant value of a seismic parameter, or inhomogeneous triangle shaped where the gradient is constant (see, among others, Gebrande, 1976; Aric *et al.*, 1980; Traslosheros *et al.*, 1990), or trapezoids (to be fully described in the following section) with parameter values assigned to each vertex. Triangular and trapezoidal cells allow smooth variations within them; therefore, for numerically describing a bivariate function, they require a smaller number of parameter values than when we use homogeneous cells. Trapezoidal cells naturally appear in two typical situations. In one of them, we get cross sections from a distribution of one-dimensional layered structures that represent a three-dimensional variation in petrophysical parameters; this method is actually applied in regional lithosphere/asthenosphere mapping (Schwab *et al.*, 1996). In a second example, laterally varying layers and trapezoidal cells are required by computer programs (Zelt y Smith, 1992) that use arrival-time and seismic amplitude data for two-dimensional structure estimation.

After selecting trapezoidal cells to describe the lateral and vertical variations of a seismic parameter inside a layer, we look for a simple interpolation method that allows only monotonic variations inside the cell. The methods used in Zelt and Smith (1992) and Schwab *et al.* (1996) are equivalent and have the required property. With this interpolation scheme, the procedure systematically iterates over the variables of the problem: layers, colors, horizontal distance and depth. Regions of the same color (associated with a specific parameter range) are identified and filled. This procedure produces color visualization of layered, laterally heterogeneous media by applying a simple, minimum smoothing interpolation that has proven useful in our routine work, particularly in detecting model inconsistencies.

METHOD AND ALGORITHM

A discrete representation of a heterogeneous, isotropic, two-dimensional V_p (used as an example of a seismic parameter from here on) model with layers and trapezoidal cells is found in Zelt and Smith (1992). This example is an application to seismic ray tracing and interpretation of arrival time data. The equations shown in this section are taken from Zelt and Smith (1992) and are given only for easy reference. This interpolation method generalizes a one-dimensional linear interpolation, and thus has the property of allowing a monotonic variation inside each cell. It is possible to interpolate uniquely for the velocity at one specific point by making three linear interpolations: first, along each of the two non-parallel sides (a "upper" and "lower" boundaries) of the trapezoid; then, over the vertical line passing trough the point (Figure 1). Let us represent a layered model in a cartesian coordinates for which the Z values increase with depth and the X axis corresponds to the lateral (distance) coordinate. The four vertices of one trapezoid are the points P_i, $i = 1,2,3,4$; velocities are only assigned at each of the trapezoid vertices. The four sides of the trapezoid are given by the following straight lines

$$x = x_1, \quad x = x_2, \quad z = s_1 x + b_1, \quad z = s_2 x + b_2 \quad (1)$$

whose coefficients are readily calculated. The fourth side in equation (1) $(\overline{P_3 P_4}$, in Figure 1) is deeper than the previous one $(\overline{P_1 P_2})$; vertex coordinates are implicitly defined by $P_1(x_1, z_1, v_1)$, $P_2(x_2, z_2, v_2)$, $P_3(x_1, z_3, v_3)$, $P_4(x_2, z_4, v_4)$, where $x_2 > x_1$, while v_i, $i = 1, 2, 3, 4$, represent the velocities.

The algebra associated with the interpolation procedure is straightforward. After applying the three interpolations that are described above, the result is

$$v(x,z) = \frac{(c_1 x + c_2 x^2 + c_3 z + c_4 xz + c_5)}{(c_6 x + c_7)} \quad (2)$$

where the coefficients can be calculated from

$$
\begin{aligned}
c_1 &= s_2(x_2 v_1 - x_1 v_2) + b_2(v_2 - v_1) - s_1(x_2 v_3 - x_1 v_4) - b_1(v_4 - v_3), \\
c_2 &= s_2(v_2 - v_1) - s_1(v_4 - v_3), \\
c_3 &= x_1 v_2 - x_2 v_1 + x_2 v_3 - x_1 v_4, \\
c_4 &= v_1 - v_2 + v_4 - v_3, \\
c_5 &= b_2(x_2 v_1 - x_1 v_2) - b_1(x_2 v_3 - x_1 v_4), \\
c_6 &= (s_2 - s_1)(x_2 - x_1), \\
c_7 &= (b_2 - b_1)(x_2 - x_1).
\end{aligned}
\quad (3)
$$

Equation (2) represents a rational function (*i.e.*, second degree polynomial) in x and z, simplified by the assumption that the interpolation is linear in z for a constant value of x. Expressions (2) and (3) reproduce the given numerical values of coordinates and velocities at each trapezoid vertex.

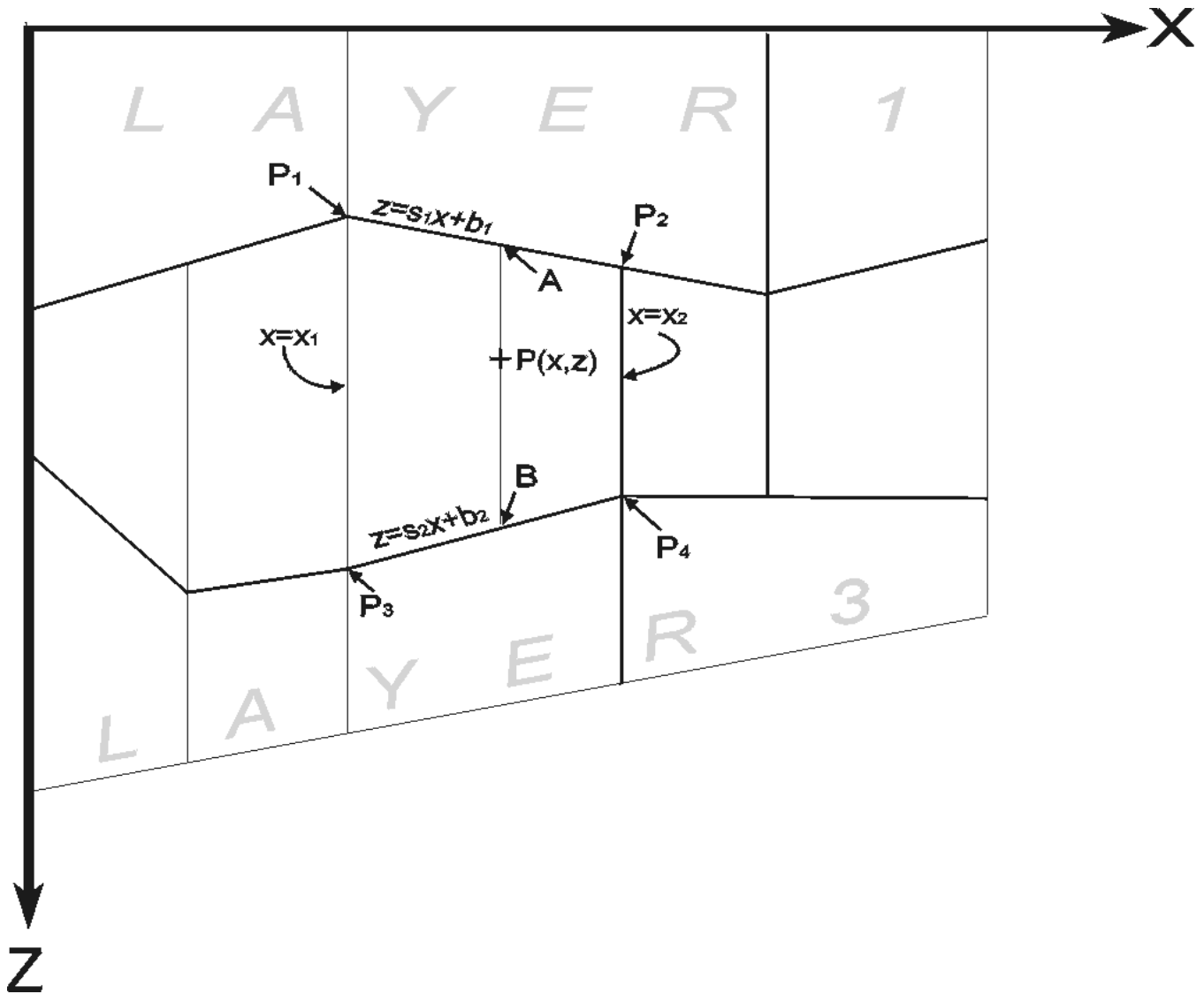The interpolated values $v(x,z)$ have a monotonic varia-

Fig. 1. Example of the velocity model parametrization. A three-layer model is divided into 12 trapezoidal blocks. Symbols used in the text are given for one trapezoid. The interpolation procedure consists of two linear interpolations along $\overline{P_1 P_2}$ and $\overline{P_3 P_4}$ that produces values at A and B; subsequently, another linear interpolation along the segment $\overline{AB}$ produces the value at $P(x,z)$. Adapted from Zelt and Smith (1992).

tion inside the corresponding trapezoid; thus, the interpolated seismic parameters cannot have an absolute maximum or minimum there. However, the four velocities assigned to the trapezoid vertices might determine an interpolated saddle point; this numerical nuisance is avoided by requiring the same signs for the velocity gradients along each of the vertical sides of the trapezoid. The interpolation method makes unavoidable the use of more than one "parametric layer" to describe inhomogeneous low and high velocity ("physical") layers.

The next step consists in selecting a color palette associated with the total range of values of the interpolated parameter. We believe that it is preferable to select a palette that calls on the reader's trained intuition, in which the colors are easily associated with physical properties, specifically petrophysical properties of the earth structure. For instance, it seems desirable that the asthenosphere, taken as a low velocity layer and associated with partial fusion and high temperatures, be represented by "warm" colors like red, yellow or combinations; whereas the lithosphere, associated with lower temperatures and solid material be represented by "cold" colors such as blue and green. The selected palette must also separate "physical" layers of the earth by taking into account, in range and resolution, values of the seismic parameters associated with these layers. To appeal to intuition, the palette does not necessarily need to be defined in a linear scale over the velocity range, even though it is conve-

nient to try to preserve this property. The most important criterion is that any physical layer can be distinctly visualized in the figure; however, this criterion is difficult to apply when numerical values of a seismic velocity are equal in different layers (i.e., S-wave velocities in the lower crust and the asthenosphere).

The computational procedure finds maximum and minimum velocity values at each layer and then associates colors according to the range of velocities. In two other loops, horizontal distance and depth are scanned looking for contiguous points that have the velocity to be represented by this color. A well-defined polygon that encloses these adjacent points is then filled with the color. Interpolated velocities are calculated by using equation (2) for which the coefficient values given by equations (3) have been stored (as arrays) in a previous program section. A layer might have more than one polygon with the same color.

The procedure has been implemented for both a PC (FORTRAN Microsoft and Lahey compilers) and a SUN Workstation (F77). The program actually used to generate Figures 2 and 3 is written in FORTRAN 77; a PostScript file is the standard—but not the only possible form—of output. Each palette color is calculated as a linear combination of primary colors. The file is sent to a laser/color printer or can be depicted in a console. However, different output units are not necessarily equivalent regarding colors and shading patterns. The program uses the following CALCOMP type plotting routines: PLOTS (initialization), PLOT (moves the "pen" in straight line between two points with the "pen" up or down), NUMBER (draws numbers at a specified position), SYMBOL (draws special symbols including characters, at a specified position), AXIS (draws a scaled linear axis), COMPLX (selects a character set), STDASH (defines the characteristics of a dashed line) and PLOTD (generates straight, dashed line between two points). Routines that handle colors in two-dimensional figures are: PALETTE (forms the palette according to an index set), COLOR (defines a specific color to use), FILL (fills a polygon with a specified color and shading pattern) and STFILL (selects a shading pattern used by FILL). These routines belong, with the exception of a modified version of AXIS, to the graphic library PLOT88 (Young and Van Woert, 1992). Similar routines or functions are available in compilers (TURBO C++, FORTRAN MICROSOFT, for example) or other graphic libraries.

EXAMPLES

Figure 2 shows an application of the procedure; it represents a structure used as an initial model for the interpretation of a seismic refraction profile between Ensenada, Baja California, Mexico, and Blythe, Arizona, USA (Frez *et al.*,

1995). In this figure, we use 100 ppi as a search step; the PostScript file has a size of about 3 Mb that is reduced to one fifth after being compressed. This figure helps to depict three distinct structural provinces; from left to right, the Peninsular Ranges, the Salton Trough, and the Chocolate Ranges. The tectonically active Salton Trough is visualized with thin crust, thick sediments and upwelling of hot upper mantle material; the other two regions are tectonically stable. Also, lower mean crustal V_p values beneath the Chocolate Ranges than beneath the Peninsular Ranges can be discerned.

Figure 3 illustrates a seismic shear velocity (V_s) cross-section adapted from Figure 8 of Zeng *et al.* (1989). End-point coordinates of the section are (78.00° N, 53.00° E) and (78.00° N, 52.10° W). Some modifications have been made for simplicity, consistency with adjacent structures, and available geophysical information. The maximum depth of the low velocity layer has been set arbitrarily at 275 km. An intersection between the profile and an active oceanic ridge is clearly appreciated, with the consequent lithospheric thinning. Palettes are not in linear scales; instead, we focus our attention in some part of the color mapping to detect inconsistencies.

The analysis of both figures helps us to detect structural inconsistencies that need to be taken care of. For example, Figure 3 shows the presence of a few points with anomalously large velocities at the bottom of the low-velocity layer; one is between distances of 7° and 8°, the other is at about 12°. An attentive reader might find a few other incompatibilities in both figures.

DISCUSSION

Computer programs that implement the procedure described above are routine tools in our research. For non-PostScript printers, a suitable palette can be found by simple trial and error, combining primary colors, possibly with the additional use of shading patterns. Some printers have a more limited set of colors, typically eight counting black and white; however, an ample color palette can generally be simulated for them by combining colors and shading patterns, which are also typically eight, counting solid black. We have done this in other versions of the program, as for HP DeskJet printers that run on a PC, using both Microsoft and Lahey FORTRAN compilers.

Occasionally, seismic functions at depths between 0 and 300 km have a large variation in their numerical values. For instance, the presence of an ocean produces this situation with the quality (attenuation) factor for transverse waves, Q_s. Thus, a palette associated with a linear scale of Q_s might mask important details in this parameter distribution. A non-linear scale will alleviate this problem either (1) by the use
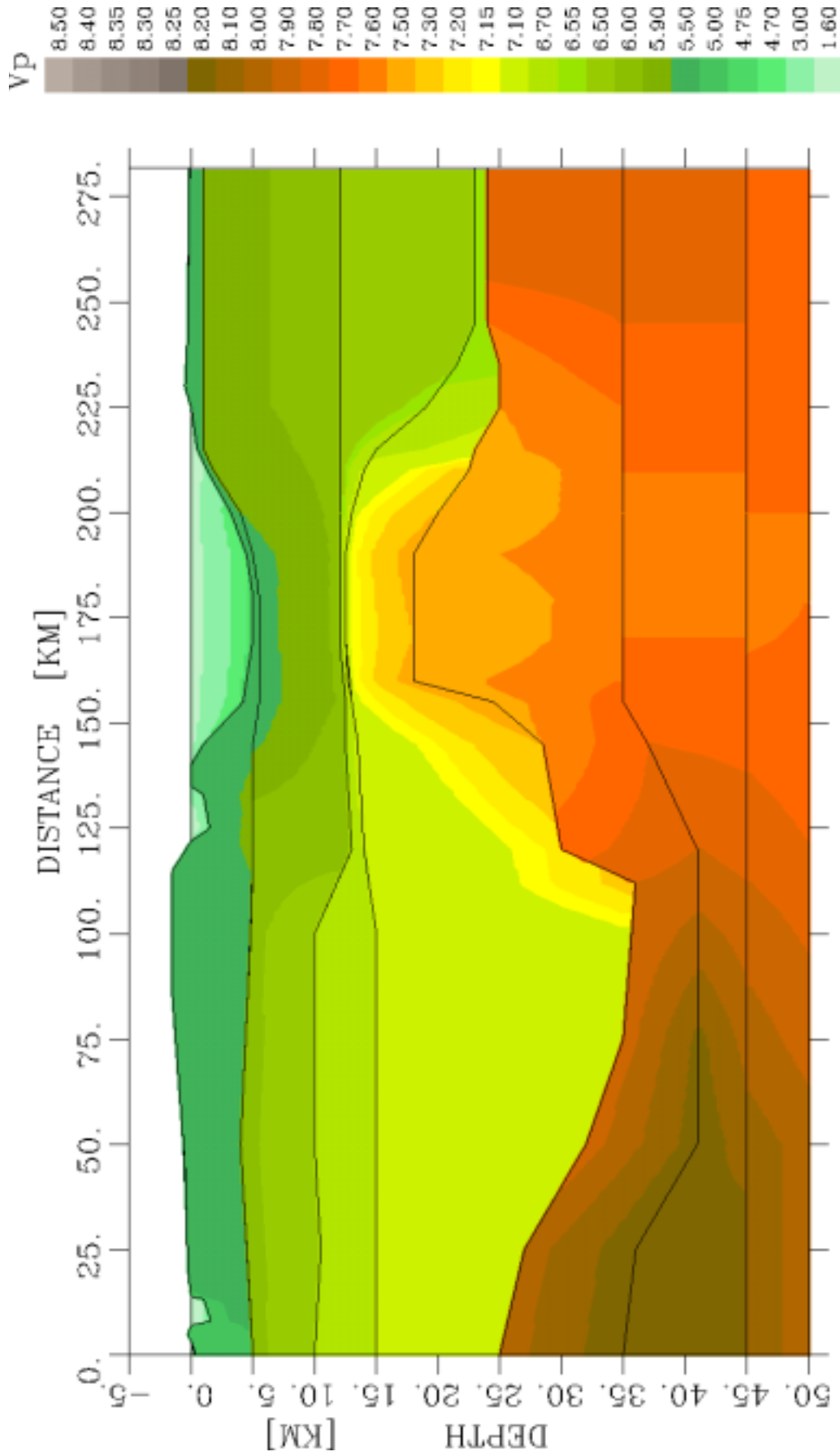
Fig. 2. Initial model for the interpretation of a refraction profile between Ensenada, Baja California, Mexico, and Blythe, Arizona, USA. Longitudinal wave velocity (V_P, en km/s) is depicted. Black continuous lines mark layer boundaries. See text for details.
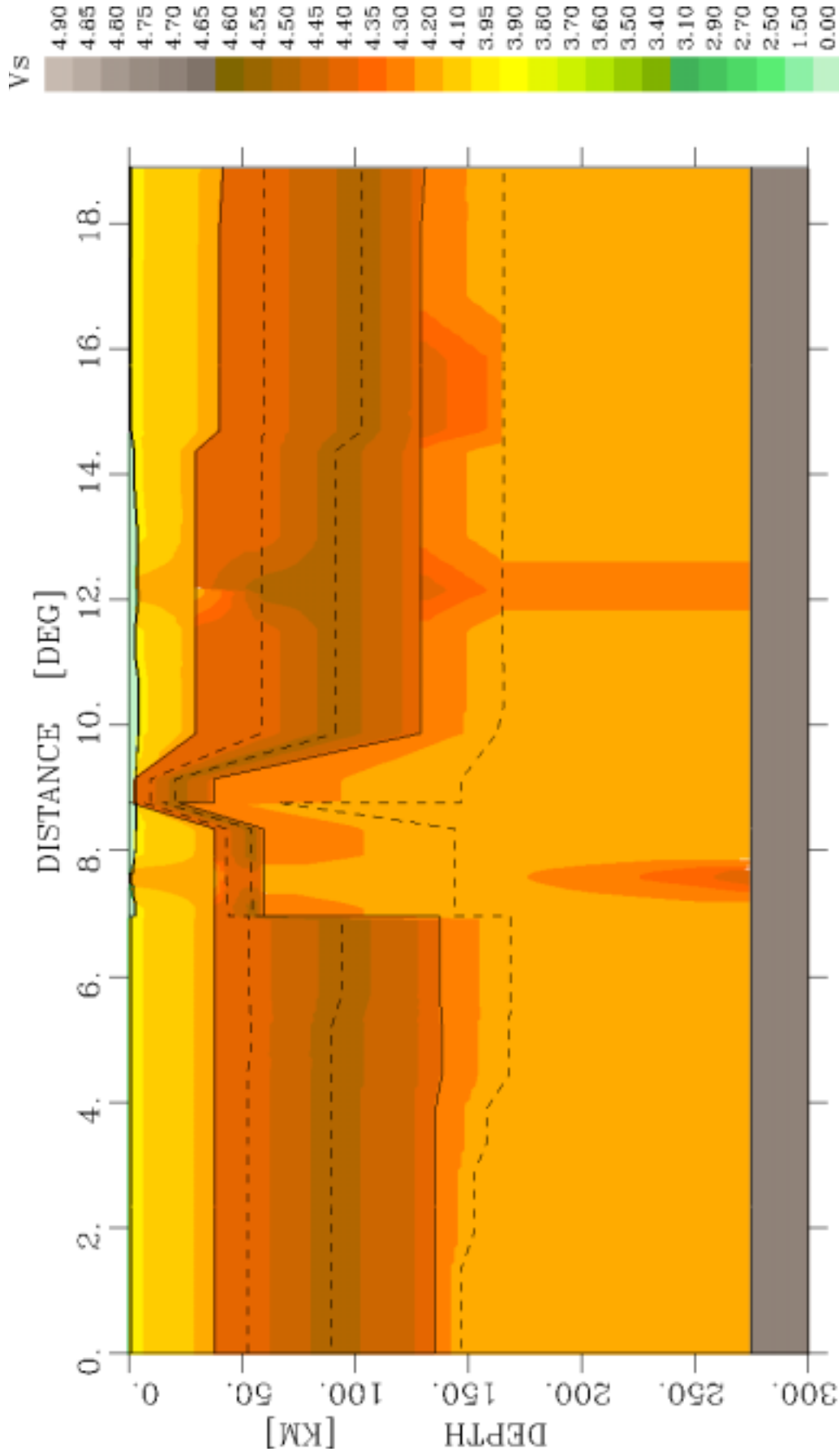
Fig. 3. Cross-section for transverse wave velocity (V_s, en km/s) connecting 44 one-dimensional models between two points in the Arctic region and crossing an active oceanic ridge. Layer boundaries are marked by solid lines; discontinuous lines denote boundaries between at least a "sub-layer", i.e, layers that exist inside the crust, lithosphere and/or asthenosphere, taken as principal "layers". See text for other details.

of the transformed variable $1/Q_s$ or (2) by magnifying the scale of Q_s in sub-ranges where it is necessary to emphasize color resolution.

Portability of the program is affected by the use of routines from a graphic library. However, they can be replaced by equivalent procedures from other libraries or compilers. PostScript output files are currently easy to print; alternatively, we have the options of printing, either to output files or directly in HP DeskJet or DesignJet printers; or, to translate PostScript files into these printers' languages.

Another aspect that deserves discussion is the use of trapezoidal vs. triangular cells. Both are used in seismology for ray tracing and travel-time calculations, mainly because of the existence of analytical interpolation formulas. For triangular cells, they can be found in Traslosheros *et al.* (1990), for example. In general, trapezoidal cells allow a much stronger function variation than do triangular cells. Ultimately, however, the use of one or the other of these cells usually depends on the more complex and important computational tools used in a research project.

Apart from these considerations, we believe that trapezoidal cells are more appropriated when dealing with either (1) a three-dimensional structure that is parametrized by a set of one-dimensional models or (2) with a two-dimensional layered structure. On the other hand, a two-dimensional structure without layers might be more easily parametrized by triangular cells. In the latter case, the simple idea of finding and filling polygonal areas with the same color can also be applied.

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