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COMMUNICATION

MEASUREMENT AND ANALYSIS OF GROUND MOVEMENT USING MICROGEODETIC NETWORKS ON ACTIVE FAULTS

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RESUMEN

Se realizaron repetidas mediciones geodéticas sobre una falla de derrumbe en los Andes Peruanos que llamativamente revelan muy poco movimiento, pese a la continuidad de sismicidad de bajo nivel. La instrumentación empleada, rudimentaria y de bajo costo, permitió una seguridad mayor de 5×10^{-6} en tensión. La reducción de datos de la investigación, para demostrar movimiento de puntos, podría ser reemplazada por el análisis de tensiones. El sistema de análisis lineal aporta un método de reducción alternativo, cuya teoría se desarrolla en este artículo, aunque su práctica es más compleja de lo previsible.

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ABSTRACT

Repeated geodetic surveys over a thrust fault in the Peruvian Andes, demonstrate surprisingly little movement in spite of continued low level seismicity. The inexpensive rugged instrumentation used can achieve accuracies of better 5×10^{-6} in strain. Reduction of the survey data to show movements of points could be replaced by strain analysis but probably should provide a direct connection to fault slip. Linear system analysis provides an alternate method of reduction whose theory is developed here but whose practice is more complex than expected.

Since most earthquake prediction theories make implicit or explicit use of a failure criterion such as Mohr-Coulomb failure, assessment of stress states near faults is central to adequate studies of the earthquake mechanism. Unfortunately direct measurements of stress are only effectively done in bore holes and the results are often of doubtful value. (Ranalli, 1975). Sufficient measurements to clarify local perturbations are rarely done. This problem of local variability can be reduced by measuring strain changes over larger areas, assuming a stress strain relationship and deducing corresponding average stress changes. Large area strain can, in principle, be measured with arrays of strain or tilt meters or, more practically, by repeated surveys of carefully monumented geodetic networks. We report here on this last option. We have made substantial observations on and some progress in analysis of a fault in the Peruvian Andes.

The use of microgeodetic networks (strongly braced and of an aperture not exceeding ~ 2 km) is appealing because the necessary equipment for such measurements is relatively inexpensive, does not have to be left in place, is quite reliable, and if used carefully can achieve geodynamically useful accuracies when operated by reasonably competent technicians.

OBSERVATION TECHNIQUES

We measure distances with an HP 3800 infrared electronic distance measurement unit (EDM) and angles with a Wild T2 theodolite. During the 1978 field season we acquired experience with AGA 14 and AGA 12A instruments but have not completed data analysis for this field work.

We do not use laser based instruments or sophisticated electronics. These more sophisticated instruments are usually heavier, more sensitive to transportation shocks, less reliable and consume more power than our infrared EDM units. This and the fact that some laser units do not work well at low ambient pressures, make such units unattractive for work at high altitudes or in remote areas far from technical support. Our initial experimental area was at elevations of over 4000 meters near the Peruvian city of Huancayo. The high altitude has the benefit that EDM measurements become less sensitive to meteorological effects. A 1°C change of temperature at this site affects the index of refraction of air by $.6 \times 10^{-6}$.

A second investigation area has been developed near Huaraz in Peru. This site is interesting, for the fault we study has been associate with about 40,000 casualties in 1970. (Ericksen *et al.*, 1970). This network is also at a high elevation (3000 + m) so similarly good results should be expected. Huaraz may be better, for the network has a larger number of degrees of freedom in adjustment of a similar number of points. The net has more redundancies.

Preliminary site studies are complete for three networks in Mexico. They are at relatively low elevations (500-1000 m) and so will be monumented with concrete pillars with forced centering plates. In the state of Chiapas the Comisión Federal de Electricidad is constructing an earth fill dam in the Sumidero Canyon and is carrying out evaluation of a potential concrete arch dam in the Itzantun gorge. The reservoirs that result from these structures will substantially exceed 200 m in depth and can be expected to result in easily observable ground deflection. Strain observations combined with the known load will provide an unusual opportunity to measure parameters of a constitutive equation on a geological scale.

The Motocintla area near Guatemala is an accessible junction of faults known to be active (Schwartz *et al.*, 1977) at least in neighbouring Guatemala. Lines of sight are possible from mountain peaks in spite of heavy jungle on the slopes and we expect to monument this area in early 1979.

Stable monumentation is crucial to these experiments. Until now we have used small brass plugs cemented with epoxy into outcrops of competent rock. The instruments are aligned over these marks on tripods equipped with optical plummets. In appears that this process is a significant source of error (Chrzanowski, 1977) and we will begin to use concrete pillars with brass fittings for attaching survey instruments in our newer networks. We do not know how long these pillars will require to achieve thermal stability after the concrete sets.

NETWORK DESIGN

Although it is fairly easy to show (Chrzanowski, 1977) that the network in Figure 1 is an ideal network for study of a fault, it is in practice difficult to achieve such an ideal. Figure 2 shows the Huancayo network and some of the reasons it deviates from the ideal. We consider that a minimum network requires at least five well spaced points on each side of the fault. The geometric shape of the subnet on each side of the fault should allow calculation of reasonably accurate strain changes on either side of the fault without reference to data on the other side, and there should be sufficient ties across the fault to detect slip on the fault. The ideal network (Figure 1) has 71 degrees of freedom for an adjustment with minimum constraints. We seek to detect unambiguously a relative displacement of one cm or more between any two points. This requires that the standard deviation of a measured line length be less than 5 mm and the standard deviation of a mesured angle be less than 1.5". Such accuracies can be achieved with 1" theodolites and average short range EDM units.

Particularly in Huancayo, where we study a reverse thrust fault vertical displacements should be considered. The mountainous terrain makes spirit levelling difficult and we have only recently achieved sufficient accuracy with trigonometric levelling by using special targets and telescopic height of instrument measurement. These techniques were first used in 1978 and we have no relative changes data yet.

ANALYSIS OF OBSERVATIONS

It is useful to consider three quantities in the study of deforming geodetic networks. An observation vector $\vec{\Delta}$ consisting of a list of changes in angles and distances (both vertical and horizontal), a relative displacement vector $\vec{u}$ which is a list of relative movements of various points in the network and a source vector $\vec{F}$ which can be considered the source of $\vec{\Delta}$ and $\vec{u}$. The source vector is the body force equivalents (Nyland, 1973) vector for the dislocation distribution postulated to give rise to $\vec{\Delta}$.

Since $\vec{\Delta}$ is small compared with the aperture of the network it is easy by routine analysis (Appendix) to find a matrix $\underset{\sim}{G}$ such that

$$\vec{\Delta} = \underset{\sim}{G}\vec{u} \tag{1}$$

If we interpret $\vec{F}$ as the body force equivalents, the reciprocity theorem (Sokolnikoff, 1956) and the solution to Boussinesq's problem (Landau and Lifshits, 1970) yields (Appendix) another linear relation between $\vec{F}$ and $\vec{u}$.

$$\vec{u} = \underset{\sim}{B}\vec{F} \tag{2}$$

In this case perfect elasticity has to be assumed and the location of the forces vectors $\vec{F}$ must be known a priori. The matrix which relates $\vec{\Delta}$ to $\vec{F}$ is usually rectangular. Only a finite number of observations can be made to constrain a model with an infinite number of geophysical degrees of freedom. However linear inversion techniques are still possible. We can write

$$\underset{\sim}{G}\underset{\sim}{B} = \underset{\sim}{U}\,\underset{\sim}{\Lambda}\left[\underset{\sim}{V}^{T}_{11}, \underset{\sim}{V}^{T}_{21}\right] \tag{3}$$

where U is a square matrix of the order m of the observation vector Δ, $\underset{\sim}{V}$ is a square matrix of the order n of the unknown vector $\vec{F}$ and

$$\underset{\sim}{V} = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

where V_{11} and V_{22} are square submatrices of order m and n-m, $\underset{\sim}{V}_{12}$ and $\underset{\sim}{V}_{21}$ are rectangular submatrices such that $\underset{\sim}{V}$ is a square matrix and

$$\underset{\sim}{U}^{T}\underset{\sim}{U} = \left[V^{T}_{11}, V^{T}_{21}\right]\begin{bmatrix} V_{11} \\ V_{21} \end{bmatrix} \doteq \underset{\sim}{1}$$

where $\underset{\sim}{1}$ is the unit matrix of order m and $\underset{\sim}{\Lambda}$ is diagonal of order m. Combining (1), (2) and (3) the singular value decomposition (Golab & Rinasch, 1970) we get

$$\vec{\Delta} = \underset{\sim}{G}\underset{\sim}{B}\vec{F}$$

$$= \underset{\sim\sim}{U\Lambda} \left[\underset{\sim}{V}_{11}^{T}, \underset{\sim}{V}_{21}^{T} \right] \vec{F}$$

$$\Lambda^{-1} U^{T} \Delta = \frac{\vec{\alpha}}{\Lambda} = \left[V_{11}^{T}, V_{21}^{T} \right] \vec{F} = \vec{\beta}$$

Which means that certain weighted averages of F (indicated here by β) are equal to weighted averages of Δ divided by the square roots of the eigen values of $\underset{\sim}{B}^T \underset{\sim}{G}^T \underset{\sim\sim}{GB}$. It is possible to assess the accuracy of the elements of β. The most accurate can be considered to be diagnostic of geodynamic conditions. The weights for averaging which produces these values depend on the network geometry ($\underset{\sim}{G}$) and the geodynamic model geometry ($\underset{\sim}{B}$). Both are at least in theory known. The necessary formulas for calculating $\underset{\sim}{G}$ and $\underset{\sim}{B}$ are given in the Appendix.

DETERMINATION OF DISPLACEMENTS

We have not yet developed sufficient experience with such a linear inverse theory of geodynamics to be able to show more insight than the standard method of analysis (Lazzarini *et al.*, 1977 for example). Our analysis presently consists of seven steps, some of which can be challenged. They are however traditional in the study of deformation of large structures.

1. Identification of least moved points.
2. Identification of minimum constraints to allow network adjustment.
3. Separate adjustment of two sets of observations at different times, using minimum constraints.

4 & 5. Calculation of difference vectors for different adjustments and acceptance of the analysis which gives minimun displacement.

6. Calculation of standard deviation of displacements σ_d.

7. Displacements less than two σ_d are ignored.

Use of a free adjustment (inner constraints) and development of new approaches in the statistical analysis of the displacements are being now tested of the University of New Brunswick.

RESULTS

We have three sets of measurements on the Huancayo network. The HP3800 performed well in excess of manufacturer's specifications; results from angle measurements (measured in 4 sets) were not as good as expected.

Routine analysis of the field data indicates the following standard deviations of observations:

	Distance		*Angle*
1975	4 mm	(Independent of distance)	2.6"
1976	1.8 mm	"	2.2"
1977	2.8 mm	"	3.9"

Figures 3 and 4 show the vector displacements under two different hypotheses about constraints. In one case we fix a point and an azimuth. In the other we use free adjustment. The free adjustment in Figure 4 is due to Dr. Walter Welsch of West Germany. Who joined our project last year. The results seem more consistent with what we know of the geophysics. The fault is a reverse thrust. Nevertheless a useful interpretation can be found for figure 3 as well. Figure 5 shows a more heavily braced net which we have installed in Huaraz.

Tentatively we suggest for Huancayo, based on a minimum constraint adjustment, that in 1975-1976 the upper block slipped slightly west and perhaps north (note points 2 and 6). In 1976-1977 the upper block slipped slightly east (note point 2) in such a way that no net movement is apparent for the episode 1977-1975. These arguments ignore any movement which is not of sufficient magnitude to exeed a

95% confidence ellipse for the point in question. With these results, and the error analysis done at the Univeristy of New Brunswick, a test of linear inverse theory becomes feasible.

Although we have demostrated that for restricted configuration of sources some characteristic changes of the sources can be determined to better than 10% using the Huaytapallana net, the physical significance of the characteristics (β in the above notation) is not clear. It is further not clear how the postulated source geometry should be constructed. The results and the problems are sufficiently complex that they deserve a separate paper (Nyland 1979, in preparation).

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APPENDIX

Consider two stations i and k observable from another point j. With full surveying equipment five observations are possible.

1. Slope distances from j to i and from j to k we call S_{ji} and S_{jk}.
2. Elevation differences for the same pair we call h_{ji} and h_{jk}.
3. The horizontal angle i to k we call Θ_{ijk}. From these data we get the horizontal vectors from j to i and from i to k which we call $\vec{r}_{ji}$ and $\vec{r}_{jk}$.

After a period of time stations i and k move with respect to j by vectors $\vec{d}_{ji}$ and $\vec{d}_{jk}$ which have horizontal component vectors $\vec{u}_{ji}$ and $\vec{u}_{jk}$ and vertical scalar components e_{ji} and e_{jk}. Let $\underset{\sim}{\epsilon} = \vec{\nabla u}$ be the gradient of the displacement field in the area. Then it is easy to see that

$$(\vec{u}_{ji} + \vec{k}e_{ji}) = (\vec{r}_{ji} + \vec{k}h_{ji}).\underset{\sim}{\epsilon}$$

$$(\vec{u}_{jk} + \overrightarrow{ke}_{jk}) = (\vec{r}_{jk} + \vec{k}h_{jk}).\underset{\sim}{\epsilon}$$

where $\vec{k}$ is a unit vertical vector and $\underset{\sim}{\epsilon}$ is assumed constant over the region contained by ijk. We suggest however that calculation of $\underset{\sim}{\epsilon}$ or the strain dyadic, interferes with a direct look at geodynamic causes of the changes in the network. Instead, in order to obtain the matrix $\underset{\sim}{G}$ (eq 1) begin by calculating the first order change in the cross product of the horizontal relative position vectors.

$$(\vec{r}_{ji} + \vec{u}_{ji}) \times (\vec{r}_{jk} + \vec{u}_{jk}) - \vec{r}_{ji} \times \vec{r}_{jk} \simeq \vec{r}_{ji} \times \vec{u}_{jk} - \vec{r}_{jk} \times \vec{u}_{ji}$$

$$\simeq \left\{ |\vec{r}_{ji} + \vec{u}_{ji}|\ |\vec{r}_{jk} + \vec{u}_{jk}| \cos(\Theta_{ijk} + \delta_{ijk}) - |\vec{r}_{ji}|\ |\vec{r}_{jk}| \cos\Theta_{ijk} \right\} \vec{k}$$

$$\simeq |r_{ji}|\ |r_{jk}| \left\{ \cos\Theta_{ijk} \left\{ \frac{\vec{r}_{ji} \cdot \vec{u}_{ji}}{|\vec{r}_{ji}|^2} + \frac{\vec{r}_{ji} \cdot \vec{u}_{jk}}{|\vec{r}_{jk}|^2} \right\} - \delta_{ijk} \sin\Theta_{ijk} \right\} \vec{k}.$$

Expressing the relation in Cartesian coordinates

$$r_{ji}r_{jk}\cos\Theta_{ijk}\left(\frac{r^{\beta}_{ji}u^{\beta}_{ji}}{r^{2}_{ji}}+\frac{r^{\beta}_{jk}u^{\beta}_{jk}}{r^{2}_{jk}}\right)-\epsilon^{3\alpha\beta}(r^{\alpha}_{ji}u^{\beta}_{jk}-r^{\alpha}_{jk}u^{\beta}_{ji})$$

$$=r_{ji}r_{jk}\delta_{ijk}\sin\Theta_{ijk}.$$

This is merely a compact derivation and slight extension of a well known (Frank, 1966) result. Here δ_{ijk} is the increase in Θ_{ijk} as time increases, ϵ^{LMN} is the completely antisymetric third order tensor, summation over greek indices is from 1 to 2 and there is no summation over lower case Roman subscripts. The value of $r_{ij} = (r^{\alpha}_{ij}r^{\alpha}_{ij})^{1/2}$. In order to express the elements of the row of $\underset{\sim}{G}$ corresponding to ijk we introduce δ_{iI} which is zero unless I = i when it equals 1, and permit summation over upper case Roman subscripts which can have values over the indices identifying all network points. With this

$$r_{ji}r_{jk}\cos\Theta_{ijk}\left(\frac{r^{\beta}_{ji}}{r_{ji}^{2}}\delta_{jJ}\delta_{iI}+\frac{r^{\beta}_{jk}}{r_{jk}^{2}}\delta_{jJ}\delta_{kI}\right)-$$

$$\epsilon^{3\alpha\beta}\left(r^{\alpha}_{ji}\delta_{jJ}\delta_{kI}-r^{\alpha}_{jk}\delta_{jJ}\delta_{iI}\right)u^{\beta}_{JI}=\delta_{ijk}r_{ji}r_{jk}\sin\Theta_{ijk}$$

Consider a source for u^{β}_{JI} which has fixed geometry. If the earth is elastic we can in principle solve for a Green's function $\underset{\sim}{B}$ of the model such that

$$u^{\beta}_{JI} = B^{\beta\gamma}_{JL} F^{\gamma}_{L} - B^{\beta\gamma}_{IL} F^{\gamma}_{L} = (B^{\beta\gamma}_{JL} - B^{\beta\gamma}_{IL}) F^{\gamma}_{L}$$

$B^{\beta\gamma}_{JL}$ is the displacement at J in direction β due to a force F^{γ}_{L} ($\gamma = 1,3$) at location L. Observe that γ indexes the components of F_{L} . If l_{ji} is the change in slope distance from j to i

$$l_{ji} = |\vec{r}_{jk} + \vec{k} h_{ji} + \vec{u}_{ji}| - |\vec{r}_{ji} + \vec{k} h_{ji}|$$

$$= |\vec{v}_{ji} + \vec{u}_{ji}| - |\vec{v}_{ji}| = \vec{u}_{ji} \cdot \frac{\vec{v}_{ji}}{v_{ji}}$$

$$= v^{\epsilon}_{ji} \frac{(B^{\epsilon\gamma}_{JL} - B^{\epsilon\gamma}_{iL})}{v_{ji}} F^{\gamma}_{L}$$

where $v_{ji} = ((\vec{r}_{ji} + \vec{k} h_{ji}) \; (\vec{r}_{ji} + \vec{k} h_{ji}))^{1/2}$

and v^{ϵ}_{ji} is the ϵ component of $\vec{r}_{ji} + \vec{k} h_{ji}$ ($\epsilon = 1,3$)

If e_{ji} is the change in h_{ji}

$$e_{ji} = (B^{3\gamma}_{jL} - B^{3\gamma}_{iL}) F^{\gamma}_{L}$$

Each of these equations constitutes a linear relation between the source strengths F^{γ}_{L} and the observations. δ_{ijk}, l_{ji}, l_{jk}, e_{ji}, e_{jk}. If the correct equation is written for each observation, the result is, in principle, amenable to analysis by singular value decomposition.

In order to derive one set of exact formulas for $B^{\alpha\beta}_{JL}$, the displacement in direction α at J due to a unit force in direction β at L, consider also $S^{\beta\delta}_{LJ}$ the displacement in direction β at point L due to a unit force on the boundary surface in direction δ at J. If the material is described by Hooke's law it is easy to show by the reciprocity theorem (Sokolnikoff, 1956, p. 391) that

$$B^{\delta\beta}_{JL} = S^{\beta\delta}_{LJ}$$

If we further assume the physical model of the geophysical reality is a half-space, the necessary formulas for $S^{\beta\delta}_{LJ}$ can be found in Landau &

Lifshitz (1970, p 29). Let $r = |\vec{v}_{JL}|$, $x = v^{1'}_{JL}$, $y^{1} = v^{2}_{JL}$, $z = v^{3}_{JL}$, E = Young's modulus of elasticity, σ = Poisson's ratio.

$$\underset{\sim}{S}_{LJ} = \frac{1+\sigma}{2\Pi E}$$

$$\frac{2(1-\sigma)r+z}{r(r+z)}+\frac{x^2(2r(\sigma r+z)+z^2}{r^3(r+z)^2}, \quad \frac{xy(2r(\sigma r+z)+z^2}{r^3(r+z)^2}, \quad \frac{xz}{r^3}-\frac{(1-2\sigma)x}{r(r+z)}$$

$$\frac{xy(2r(\sigma r+z)+z^2)}{r^3(r+z)^2}, \quad \frac{2(1-\sigma)r+z}{r(r+z)}+\frac{x^2(2r(\sigma r+z)+z^2)}{r^3(r+z)^2}, \quad \frac{yz}{r^3}-\frac{(1-2\sigma)y}{r(r+z)}$$

$$X\left(\frac{(1-2\sigma)}{r(r+z)}+\frac{z}{r^3}\right), \left(\frac{(1-2\sigma)}{r(r+z)}+\frac{z}{r^3}\right)y, \quad \frac{2(1-\sigma)}{r}+\frac{z^2}{r^3}$$

In order to compute the singular value decomposition of A note that if

$$\underset{\sim}{A} = \underset{\sim}{U}\Lambda V^T$$

the $\quad A^T A = V \Lambda U^T U \Lambda V^T = V \Lambda^2 V^T$

and $\quad A A^T = U \Lambda V^T V \Lambda U^T = U \Lambda^2 U^T$

so that U is composed of the eigenvectors of $\underset{\sim}{A} A^T$

V is composed of the first n eigenvectors of $A^T A$

Λ is a diagonal matrix consisting of the square roots of

eigenvalues of $\underset{\sim}{A}\,\underset{\sim}{A}^T$

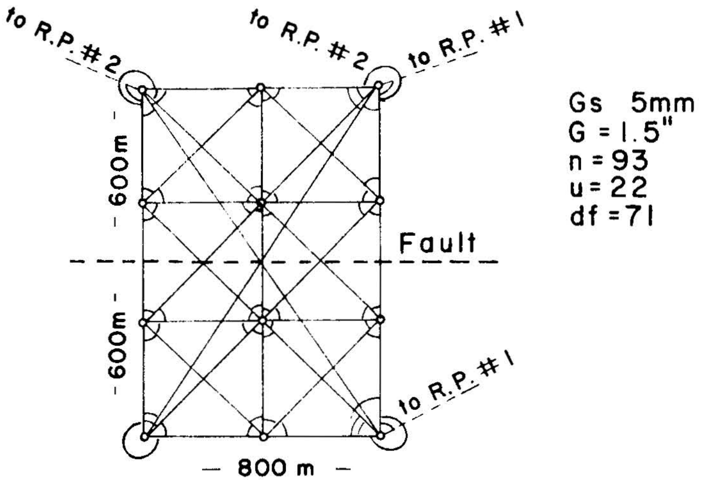

FIG. 1. "IDEAL" MICRO-GEODETIC NETWORK

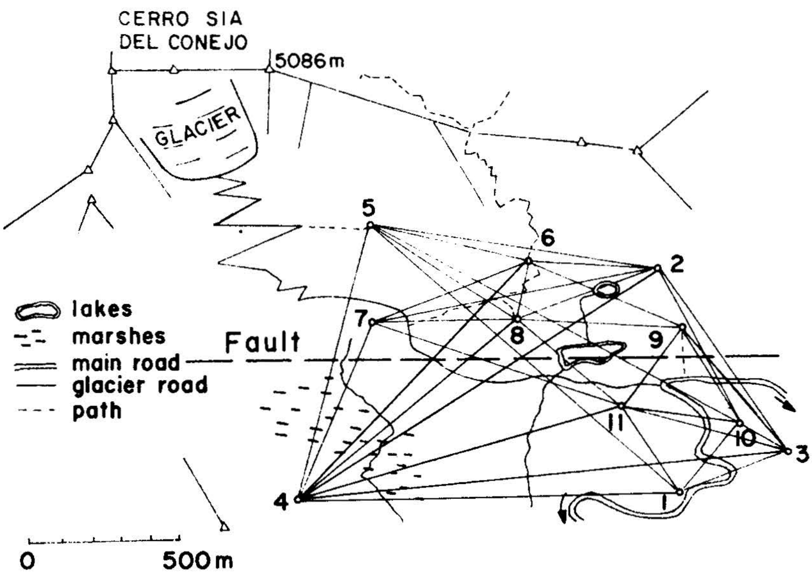

FIG. 2. HUAYTAPALLANA NETWORK

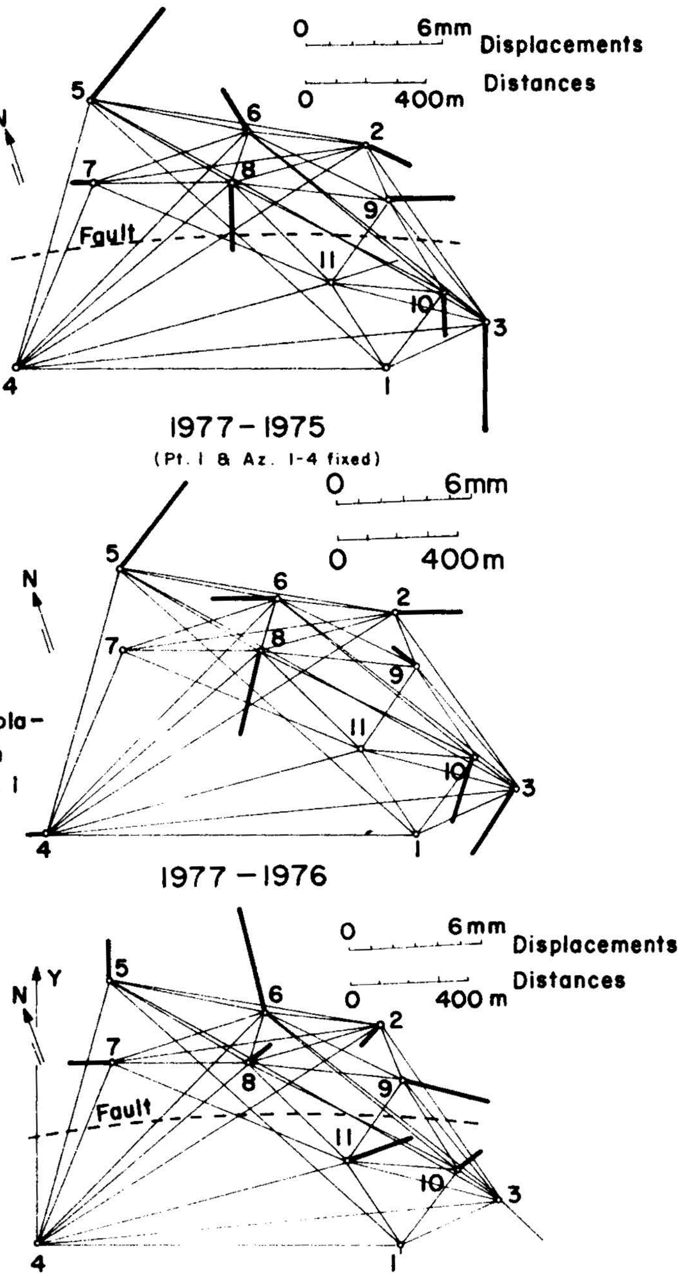

FIG. 3. Vectors of displacement (adjustment with minimum constraints Pt. 1 and Az. 1-4 fixed)

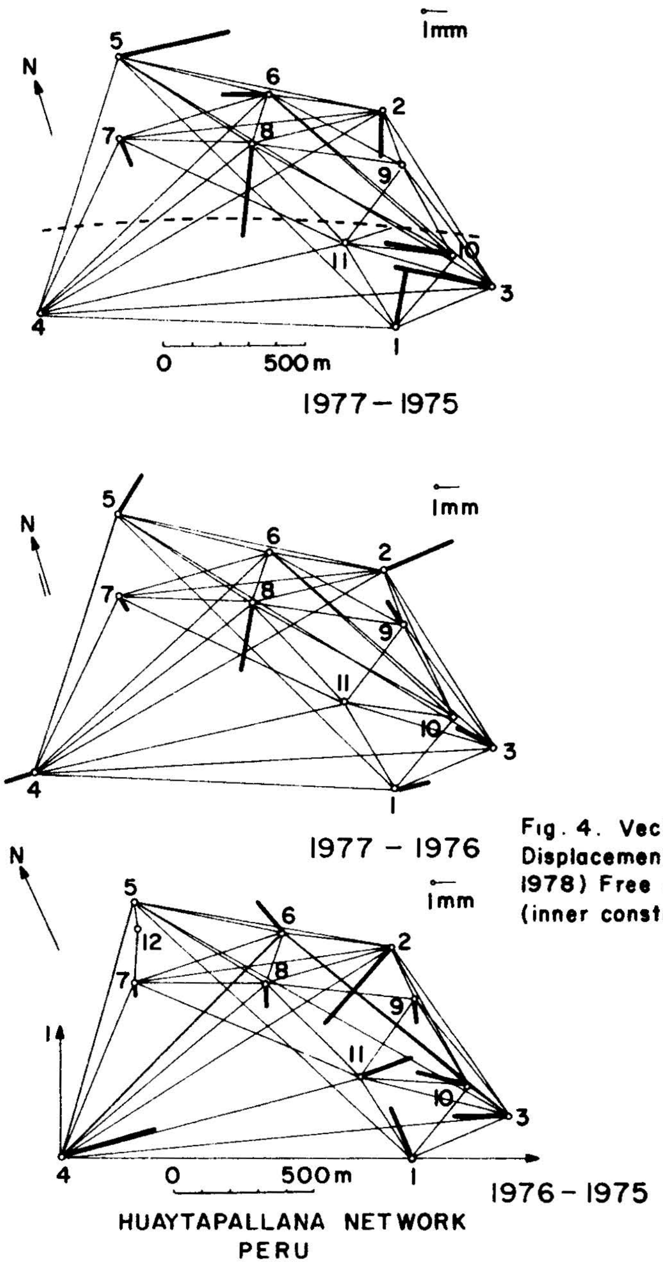

Fig. 4. Vectors of Displacements (Welsch 1978) Free Adjustmen (inner constraints)

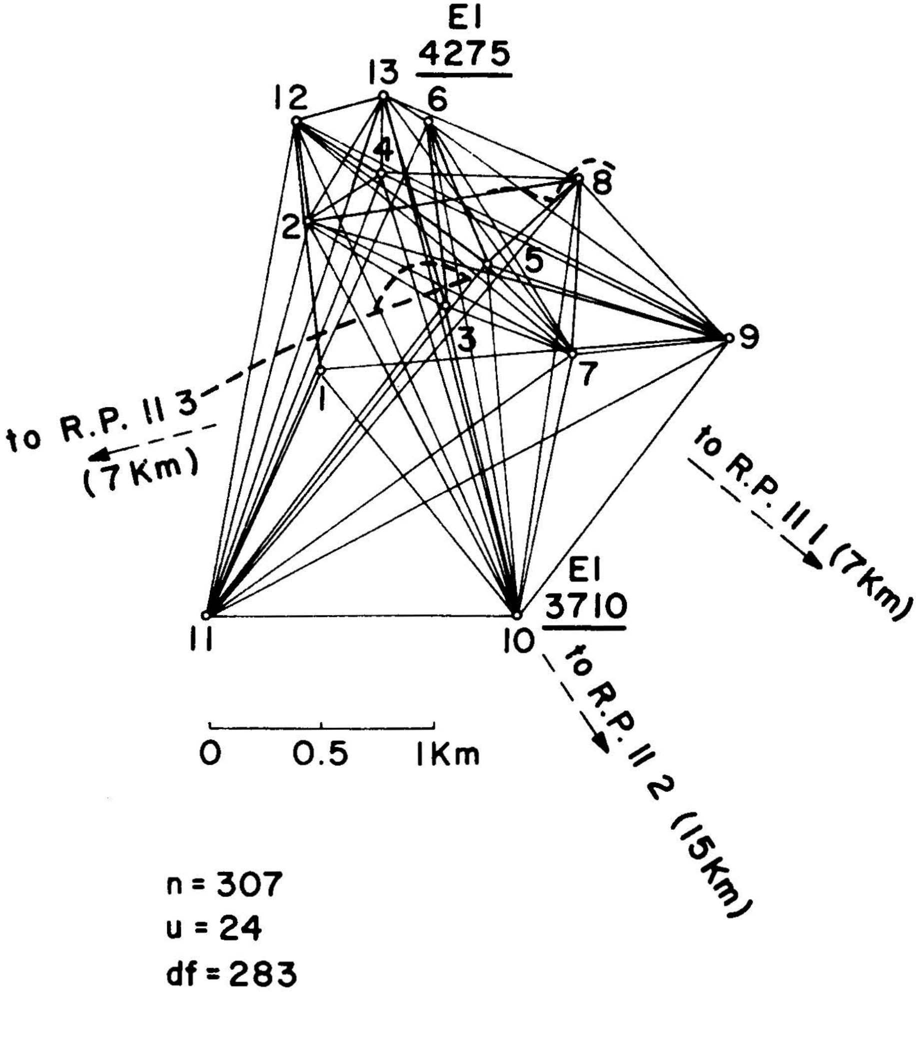

FIG. 5. PITEC NETWORK (HUARAZ)

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