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## A CODA-LENGTH MAGNITUDE SCALE FOR SOME MEXICAN STATIONS

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## RESUMEN

Se desarrolla una escala empírica de magnitud de coda para algunas estaciones mexicanas de período corto, usando una regresión de mínimos cuadrados de $\mathrm{m}_{\mathrm{b}}$ versus $\log _{10} \mathrm{~T}$, donde T es la duración de la coda en segundos. Se usaron en total 61 eventos, con un rango de magnitud entre 4.0 y 5.8 y registrados en 12 estaciones o menos. Se obtuvo la siguiente relación promedio

$$
M_{c}=-1.59+2.40 \times \log _{10} T+0.00046 x D
$$

donde $M_{c}$ es la magnitud de coda y $D$ la distancia epicentral en km . Además, para 12 estaciones se calculó una corrección que se adiciona a la magnitud $}$


#### Abstract

An empirical coda-length magnitude scale is developed for some Mexican short period stations using a least squares regression of $m_{b}$ versus $\log _{10} T$, where $T$ is the coda-length in seconds. $A$ total of 61 events in the magnitude range 4.0-5.8 and recorded on up to 12 stations were used, giving the following average relation $$
M_{c}=-1.59+2.40 \log _{10} T+0.00046 x D
$$ where $M_{c}$ is the coda-length magnitude and $D$ the epicentral distance in $k m$. Furthermore, station corrections to be added to magnitudes obtained by the average relation were calculated for all 12 stations.


[^0]
## INTRODUCTION

Magnitude for earthquakes in Mexico are reported locally on different scales. The national seismological service calculates Richter local magnitude $M_{L}$ using a WoodAnderson instrument at the Tacubaya Observatory. However, a large number of events, for which epicenters are calculated, are not recorded on the Wood-Anderson instrument (Mota, personal communication) and $m_{b}$ from PDE is given when available. Red Sismologica Mexicana de Apertura Continental (RESMAC), calculates $\mathrm{M}_{\mathrm{L}}$ using synthetic Wood-Anderson records generated from the digital records. Again the problem is lack of completeness since many events, especially from the southern part of the country, are not recorded by RESMAC. The Institute of Engineering is calculating magnitudes using coda-length, however the various relations used have never been adapted to Mexico. At the moment there are more than 25 permanent short-period seismographs (period $=1 \mathrm{sec}$ ) in operation in Mexico. Most record in analog form, and gain and frequency response are widely different and most often unknown. It thus seems that there is a need for a general accepted empirical codalength magnitude scale.

Many empirical relations between coda-length $T$ and coda-length magnitude $M_{c}$ are found to be of the form (e.g. Lee et al., 1972; Real and Teng, 1973)

$$
\begin{equation*}
M_{c}=A+B x \log _{10} T+C x D \tag{1}
\end{equation*}
$$

where D is the epicentral distance in km and $\mathrm{A}, \mathrm{B}$ and C constants. Sometimes a second order term $\left(\log _{10} \mathrm{~T}\right)^{2}$ is added for a slightly better fit (e.g. Real and Teng, 1973). However the improvement is small and we will use the simpler and generally accepted eq. 1.

To determine A, B and C we followed the method of Lee et al. (1972). They assume that for a single event, variation in coda-length is only a function of distance:

$$
\begin{equation*}
\log _{10} T_{0}=\log _{10} T+C 1 \times D \tag{2}
\end{equation*}
$$

where $\mathrm{T}_{0}$ is the coda-length at the epicenter. By plotting $\log _{10} \mathrm{~T}$ versus D for different stations, the distance dependence factor C 1 can be determined and (1) can be written in the form:

$$
\begin{equation*}
M_{c}=A+B\left(\log _{10} T+C 1 x D\right)=A+B x \log _{10} T_{0} \tag{3}
\end{equation*}
$$

where $\mathrm{C}=\mathrm{C} 1 \mathrm{xB}$. By reducing T to $\mathrm{T}_{0}, \mathrm{~A}$ and B can be determined from the least squares regression between $\log _{10} T_{0}$ and magnitude.

[^1]
## DATA

Due to lack of events where $M_{L}$ have been determined, it was decided to use $m_{b}$, reported by PDE, as calibration magnitudes. Coda-lengths were defined as the time from onset of $P$-waves until the signal disappears into the noise. Coda-lengths were read on 12 stations using 61 events for the period 1977-81.

The earthquakes had depths up to 250 km and were distributed along the Mexican subduction zone (Figure 1).


Fig. 1. Epicenters and stations used. Events with $\mathrm{h}>100 \mathrm{~km}$ are shown with crosses, and for $\mathrm{h} \leqslant 100 \mathrm{~km}$ with diamonds. The dotted rectangle is shown as an insert in the left hand corner.

For Mexican events with $\mathrm{m}_{\mathrm{b}}>5.8$ saturation of the body-wave magnitude scale has been observed (Singh, personal communication) and for events with $\mathrm{m}_{\mathrm{b}}<4.0$ an examination of the PDE bulletins showed that magnitudes have often been calculated with one or two stations. Thus to use the most reliable data, only events in the magnitude range 4.0 to 5.8 were selected. The eight first stations shown in Table 1 recorded 25 or more of the 61 events. Thus the first group of 8 stations were used to determine A, B and C while for the last 4 stations, only station corrections were calculated.

The factor C1 was first determined. Since coda-lengths at different stations had variations of up to $100 \%$ for the same event (Figure 2), we chose to determine C1, for each station, by plotting $\log _{10} \mathrm{~T}$ versus D for different events with the same magnitude. A total of 11 events with $\mathrm{m}_{\mathrm{b}}=5.1$ were used and distances ranged from about 100 to 1000 km . Figure 2 shows two examples and Table 2 summarizes the results for the least squares fit. The scatter in the data is large and since errors in PDE locations generally are less than 100 km , this scatter must be due to uncertainties in the coda-lengths (Figure 2).

Table 1
Results from the least squares fit of $m_{b}$ versus $\log _{10} T . N$ is the number of events, A and B the constants given in eq. 1 and $\mathrm{A}_{2.4}$ the values A for $\mathrm{B}=2.4$. The RMS error is in $\mathrm{m}_{\mathrm{b}}$ and COR is the correlation coefficient. Station corrections are to be added to calculated magnitudes when an average value of $\mathrm{A}_{2.4}$ is used instead of the individual values given below.

|  | B allowed to vary |  |  |  | B fixed at 2.4 |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Station | N | A | B | COR | RMS | A $2.4^{c}$ | RMS | Station correction |
| IIM | 57 | -1.61 | 2.46 | 0.78 | 0.28 | -1.46 | 0.32 | 0.13 |
| IIC | 54 | -1.89 | 2.42 | 0.86 | 0.22 | -1.83 | 0.25 | -0.24 |
| IIP | 52 | -0.56 | 2.04 | 0.84 | 0.24 | -1.53 | 0.29 | -0.06 |
| IIT | 50 | -2.17 | 2.58 | 0.78 | 0.28 | -1.68 | 0.31 | -0.09 |
| III | 44 | -0.24 | 1.91 | 0.50 | 0.35 | -1.61 | 0.39 | -0.02 |
| OZC | 25 | -2.23 | 2.72 | 0.78 | 0.25 | -1.40 | 0.31 | 0.19 |
| CSN | 51 | -1.71 | 2.53 | 0.79 | 0.27 | -1.38 | 0.29 | 0.22 |
| VHO | 50 | -0.73 | 2.02 | 0.87 | 0.21 | -1.79 | 0.24 | -0.20 |
| TPN | 14 |  |  |  |  | -1.52 |  | 0.07 |
| CH1 | 5 |  |  |  |  | -1.36 |  | 0.23 |
| CH5 | 4 |  |  |  |  | -1.31 |  | 0.28 |
| CH6 | 5 |  |  |  |  | -1.21 |  | 0.38 |



Fig. 2. The coda-length $T$ for two different stations is shown as a function of epicentral distance to 11 different events with magnitude 5.1. Note the difference in coda-length for the two stations.

Stations III and OZC did not show any clear distance dependence on coda-length. Excluding these two stations, the average value of Cl was 0.00019 . This value can be compared to what has been observed elsewhere. Chaplin et al. (1980), found C1 $=0.00035$ for New England and Lee et al. (1972), C1 $=0.00150$ for California. In an independent study, Canas (personal communication) obtained $\mathrm{Cl}=0.00018$ for Central Mexico using station OXM (Figure 1), and a similar event distribution as in our study. These values could imply a slightly higher $\mathbf{Q}$ for Mexico than for New England and substantially higher as compared to California. In Mexico, Q measured along the Pacific Coast (Rodriguez et al., 1982) and in Central Mexico (Canas, personal communication) seems to indicate higher values than in Califomia but somewhat lower than in Eastern United States (Herrmann, 1980). Thus considering uncertainties in regional variation of Q and varying instrument response, our value of $\mathbf{C 1}=0.00019$ seems reasonable, especially considering the coincidence of the independently determined Cl values for Mexico.

Table 2
Constants Cl , as defined by eq. 2, determined by least squares regression of $\log _{10} \mathrm{~T}$ versus epicentral distance. RMS errors are in $\log _{10} T$.

| Station | C1 | Correlation coefficient | RMS |
| :--- | ---: | :---: | :--- |
| IIM | 0.00020 | 0.64 | 0.063 |
| IIC | 0.00017 | 0.71 | 0.044 |
| IIP | 0.00023 | 0.75 | 0.052 |
| IIT | 0.00012 | 0.41 | 0.059 |
| III | 0.00004 | 0.16 | 0.067 |
| OZC | -0.00004 | -0.32 | 0.024 |
| CSN | 0.00023 | 0.77 | 0.044 |
| VHO | 0.00016 | 0.64 | 0.041 |

Using $\mathrm{Cl}=0.00019$, all coda-lengths were reduced to $\mathrm{T}_{0}$ ant A and B determined for each station. Table 1 gives the parameters obtained by the least squares regression and Figure 3 shows some examples. Data from events deeper than 100 km were plotted with a different symbol, however deeper events do not seem to have systematically different coda-lengths. Except for station III, which has a large RMS error, it is seen that B varies between 2.0 and 2.7. This variation could be due to scatter in the data (similar results have been observed for the California network (Real and Teng, 1973)). For practical purposes, we averaged the factors B (not including station III) obtaining the value 2.40. With this fixed value, the constants A


Fig. 3. Magnitudes $m_{b}$ as a function of coda-length T. Note the small difference in the least squares fit and the fit with a fixed slope of 2.40.
(now called $\mathrm{A}_{2.4}$ ) were redetermined for all stations (Table 1). By fixing $\mathrm{B}=2.40$, the fit to the data is only slightly worse as seen by the small increases in the RMS errors (Table 1 and Figure 3). It thus seems reasonable to use an average value of $B$ for all stations. To get an average coda-length magnitude relation for Mexico, the first 8 values of $A_{2 a}$ in Table 1 were averaged and (1) becomes:

$$
\begin{equation*}
M_{c}=-1.59+2.40 \times \log _{10} T+B x \log _{10} T_{0} \tag{4}
\end{equation*}
$$

Stations corrections to be added to magnitudes found by (4) are calculated as $\mathbf{A}_{2.4}+1.59$ and given in Table 1.

## DISCUSSION

Using the Wood-Anderson instrument at stations PBJ (Fig. 1) (Presa Benito Juárez, Oaxaca), González (1980) found coda-length magnitude scales for 3 groups of aftershocks $\left(3 \leqslant M_{L} \leqslant 5\right)$ to the 1978 Oaxaca earthquake

$$
\begin{gather*}
\mathrm{M}_{\mathrm{c}}=-0.13+1.92 \log _{10} \mathrm{~T}  \tag{5a}\\
\mathrm{M}_{\mathrm{c}}=-1.64+2.43 \log _{10} \mathrm{~T}  \tag{5b}\\
\mathrm{M}_{\mathrm{c}}=-0.86+1.87 \log _{10} \mathrm{~T}  \tag{5c}\\
\mathrm{M}_{\mathrm{c}}=-1.49+2.32 \log _{10} \mathrm{~T}+0.00042 * \mathrm{D} \tag{6}
\end{gather*}
$$

where the 3 relations represent different time periods and gain and filter settings. Equation 5c represent the longest time period ( 15 days) and filters and gains were set in the same position as before the main shock. Gonzalez (1980) therefore considered (5c) to be representative of earthquakes in the area. However (5c) gives lower magnitudes than our general relation (Fig. 4), while (5b) (lower gain, see González (1980)) is almost identical to our relation. Since (5) is a regional relationship, one should however be careful to compare with our results.

A study more comparable to ours is the earlier mentioned by Canas, where magnitudes $\mathrm{m}_{\mathrm{b}}(\mathrm{Lg})$ (similar to $\mathrm{m}_{\mathrm{b}}$ ) in the range 2.4 to 5.5 were used for calibrating the $\mathrm{M}_{\mathrm{c}}$ scale for station OXM. The following relation was obtained (Canas, personal communication).


Fig. 4. Comparison of various $M_{c}$-Log10 $T$ relations available in the literature.

This relation is almost identical to the one obtained in our study and thus supports our results. It is however difficult to make a comparison between relation (4), (5), (6) and results reported from other countries due to difference in system response, different definition of coda-length, the use of different types of magnitudes for calibration and regional differences in coda-lengths. Figure 4 shows some examples of reported relations using short-period systems and calibration events of magnitudes larger than 4.0. Usually most coda-length magnitude scales give $\mathbf{M}_{c}=3.0$ $\pm 0.3$ for a coda-length of 100 sec (Bakun and Lindh, 1977) while for a coda-length of 1000 sec much more variation is found, probably due to the few calibration events with magnitudes above 5.0. However, our relation seems to be in reasonable agreement with other studies, and it can probably be used for the stations given in Table 1 to obtain $\mathrm{m}_{\mathrm{b}}$-compatible magnitudes for Mexican events with coda-lengths between 100 and 1000 sec , and possibly for coda-lengths as low as 50 sec considering the study by Canas.

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