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TWO-STREAM COMPUTATIONS OF WATER-BODY REFLECTANCES

L. LEMUS*
C. GAY*
S. CAMACHO*
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RESUMEN

Se utiliza el método de dos haces para resolver la ecuación de transferencia de radiación. Se calculan la intensidad hacia adelante y la retro-intensidad, así como la reflectancia de un cuerpo de agua. El método de dos haces se compara con otro y se presentan los resultados de una aplicación práctica.

ABSTRACT

A two-stream approximation is applied to the radiative transfer equation in order to compute the upward and downward intensities and the water-body reflectance. The results of the method are compared with previous ones and a practical application is presented.

* *Instituto de Geofísica, UNAM, México 04510, D. F., MEXICO.*

INTRODUCTION

The interest in remote sensing of water resources has grown considerably in the last few years. This has lead to a more careful analysis of the problem of radiative transfer in water-bodies. Duntley (1963), presented a review of the radiation field in the ocean. Plass and Kattawar (1972), Gordon and Brown (1975) and McCluney (1974) among others, have made calculations of the radiation field in the atmosphere-ocean system using the Monte-Carlo method. This technique simulates the passage of individual photons through a medium that absorbs and scatters them. Estimates of the reflectance and transmittance can be obtained counting those photons that emerge from the medium (Plass and Kattawar, 1969).

In our case, the recent studies of water bodies by remote sensing (Lemus, 1980) have also shown the need of a theoretical framework for the treatment of the radiative processes.

The purpose of this paper is to apply the known two-stream method (Schuster, 1905; Schwarzchild, 1906) to solve the radiative transfer equation and compute the reflectance of water-bodies. The theoretical values are compared with the absolute water-body reflectance computed from the microdensitometry of the multispectral photography of El Bosque dam located in Michoacán, México.

The two-stream method is simple and does not imply a large amount of computer time while maintaining the basic information of the radiation transfer.

METHODOLOGY

The interaction between radiation and a scattering medium is governed by the radiative transfer equation that for a plane parallel medium is given by Chandrasekhar (1960) as

$$\mu \frac{dI_\lambda(\tau, \mu, \varphi)}{d\tau} = I_\lambda(\tau, \mu, \varphi) - J_\lambda(\tau, \mu, \varphi) \tag{1}$$

where μ is the cosine of the zenith angle, $I_\lambda(T, \mu, \varphi)$ is the monochromatic intensity (radiance) defined as the energy dE_λ in the wave-length interval $d\lambda$, that passes through an area $d\sigma$, in the direction (μ, φ), within the solid angle dw in the interval of time dt (Liou, 1980) and τ is the optical depth defined as $\int c_\lambda(z)dz$ where c_λ is the monochromatic extinction coefficient at the wavelength λ. The left hand side of this equation represents the total changes of the intensity within the distance $d\tau$ due to absorption (first term on the right) and emission (second term). When the radiative transfer equation is referred exclusively to the diffuse intensity I_d (intensity that has been scattered more than once) the source function J_λ is given according to Stamnes (1982) by

$$J(\tau, \mu, \varphi) = \frac{w_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} P(\mu,\varphi;\mu',\varphi') I(\tau;\mu',\varphi') d\mu' d\varphi' + Q(\tau,\mu,\varphi), \qquad (2)$$

where w_0 is the single scattering albedo given by the ratio of the scattering coefficient to the extinction coefficient. $P(\mu, \varphi; \mu', \varphi')$ is the phase function and the last term, $Q(\tau, \mu, \varphi)$ represents the contribution to the diffuse intensity due to the scattering of the direct solar intensity.

Assuming a non-emitting and every opaque medium vertically stratified in paralel planes, azimuthal symmetry, and considering the discrete ordinates approximation of the radiative transfer equation for the diffuse intensity (Chandrasekhar, 1960), Eq.(1) takes the form

$$\mu_i \frac{dI_d(\tau,\mu_i)}{d\tau} = I_d(\tau,\mu_i) - \frac{w_0}{2} \sum_{-N}^{N} a_j P(\mu_i,\mu_j) I_d(\tau,\mu_j) - Q(\tau,\mu_i), \qquad (3)$$

where μ_i (quadrature points) are the cosines of the zenith angle in prescribed directions given by a quadrature formula; $I_d(\tau, \mu_i)$ is the diffuse intensity corresponding to μ_i and a_j are the quadrature weights that satisfy the following normalization condition

$$\Sigma a_j = 1 .$$

We seek the solution of Eq.(3) assuming isotropic scattering (a good approximation for optically thick media (Liou, 1980)) and the two streams approximation. In this case $P(\mu, \mu_j) = 1$ and $N = 1$ and Eq.(3) takes the form

$$\mu_i \frac{dI_d(\tau,\mu_i)}{d\tau} = I_d(\tau,\mu_i) - \frac{w_0}{2} \sum_{-1}^{1} I_d(\tau,\mu_j) - Q(\tau,\mu_i) \quad , \qquad (4)$$

where we have substituted $a_j = 1$

and $$Q(\tau,\mu_i) = \frac{w_0}{4\pi} I_{INC}\, e^{-\tau/\mu_0} \quad , \qquad (5)$$

with $$I_{INC} = \pi F_0\, \delta(\varphi - \varphi_0)\delta(\mu - \mu_0) \quad , \qquad (6)$$

where μ_0 is the cosine of the solar zenith angle and F_0 is the incident flux on the air-water interface ($\tau = 0$). In the Schuster-Schwarzschild, approximation of the radiative field $\mu_1 = 1/2$.

Defining $I_d^+ \equiv I_\lambda(\tau, \mu_i)$ for $0 \leqslant \mu \leqslant 1$ and $I_d^- \equiv I_\lambda^-(\tau, \mu_i)$ for $-1 \leqslant \mu \leqslant 0$ we can rewrite Eq.(4) as

$$\frac{1}{2}\frac{dI_d^+}{d\tau} = I_d^+ - \frac{w_0}{2}(I_d^+ + I_d^-) - \frac{w_0}{4} F_0 e^{-\tau/\mu_0} \quad , \tag{7}$$

for $0 \leqslant \mu \leqslant 1$ (upper hemisphere)

and

$$-\frac{1}{2}\frac{dI_d^-}{d\tau} = I_d^- - \frac{w_0}{2}(I_d^+ + I_d^-) - \frac{w_0}{4} F_0 e^{-\tau/\mu_0} \quad , \tag{8}$$

for $-1 \leqslant \mu \leqslant 0$ (lower hemisphere) $I_d(\tau, \mu_i) = I_d$ is the backward diffuse intensity and $I_d(\tau, \mu_i) = I_d$ is the forward diffuse intensity.

Equations (7) and (8) admit solutions of the form

$$I_d^+ = J_+ e^{-\tau/\mu_0} + A e^{\tau\Gamma} + B e^{-\tau\Gamma} \quad , \tag{9}$$

$$I_d^- = J_- e^{-\tau/\mu_0} + C e^{\tau\Gamma} + D e^{-\tau\Gamma} \quad , \tag{10}$$

where
$$J_\pm = \frac{R_\pm}{1/\mu_0^2 - \Gamma^2} \quad ,$$

and
$$\Gamma^2 = 4(1 - w_0), \quad , \quad R_\pm = -w_0 F_0 (1 \pm 1/2\pi_0) \quad , \tag{11}$$

and A, B, C and D are constants determined by the boundary conditions. From Eqs.(7) and (8) it is clear, that only two constants are independent, if A and B are considered independent, C and D are given as $C = k_- A$, $D = k_+ B$.

We assume that the incident diffuse intensity at the air-water interface is zero and that there is no contribution to the diffuse intensity from the bottom of the strata. The two boundary conditions for the diffuse field take the form

$$I_d^-(\tau = 0) = 0 \quad ,$$

$$I_d^+(\tau = \tau^*) = 0 \quad . \tag{12}$$

These correspond to the case in which there are no sources of diffuse intensity coming from outside the water medium.

The solutions for each hemisphere, assuming no bottom reflectance (Gay, 1979) are given by

$$I_d^- = J_- e^{-\tau'/\mu_0} + k_- [(k_+ J_+ e^{\tau(\Gamma-1/\mu_0)} - J_-)/k_- - k_+ e^{2\tau\Gamma}] e^{\tau\Gamma} +$$

$$k_+ [J_- - k_+ J_+ e^{\tau(\Gamma-1/\mu_0)}/k_- - k_+ e^{2\tau\Gamma} + J_+ e^{\tau(\Gamma-1/\mu_0)}] e^{-\tau'\Gamma} \quad , \qquad (13)$$

for $-1 \leqslant \mu \leqslant 0$ and

$$I_d^+ = J_+ e^{-\tau'/\mu_0} + [k_+ J_+ e^{\tau(\Gamma-1/\mu_0)} - J_-)/k_- - k_+ e^{2\tau'\Gamma}] e^{\tau'\Gamma} +$$

$$[(J_- - k_+ J_+ e^{\tau(\Gamma-1/\mu_0)}/k_- - k_+ e^{2\tau\Gamma}) e^{2\tau\Gamma} + J_+ e^{\tau(\Gamma-1/\mu_0)}] e^{-\tau'\Gamma} \quad , \qquad (14)$$

for $-0 \leqslant \mu \leqslant 1$.

Due to total internal reflection at $\tau = 0$ only a fraction of the upward intensity $I_d^+(\tau = 0)$ goes through the water-air interface. In terms of the upward intensity and $T_{w\text{-}a}$ the water-body reflectance ρ_w is defined as

$$\rho_w(\tau' = 0) = \frac{T_{w\text{-}a}}{n^2} \frac{I_d^+(\tau' = 0)}{I_0(0)} \qquad (15)$$

where $I_0(0) = \pi F_0$

For the computer analysis, the optical depth τ was considered as $\tau = c_\lambda z$. This form implies that the medium is homogeneous. This approximation is valid for a well mixed medium.

The results of the calculations for I_d^+ and I_d^- for different values of μ_0 and w_0 are plotted in Figures 1 and 2. It is clear that upward and downward intensities depend strongly on μ_0 and w_0 . The marked dependence on wavelength is also shown in these figures.

In order to compare with Monte Carlo results, the water-body spectral reflectance was also computed for different albedos and values of μ_0 (Figures 3 and 5), the results shown in Figure 3 correspond to the assumption of isotropic scattering. The results obtained by the Monte Carlo method (McCluney, 1984; Plass, 1972) are

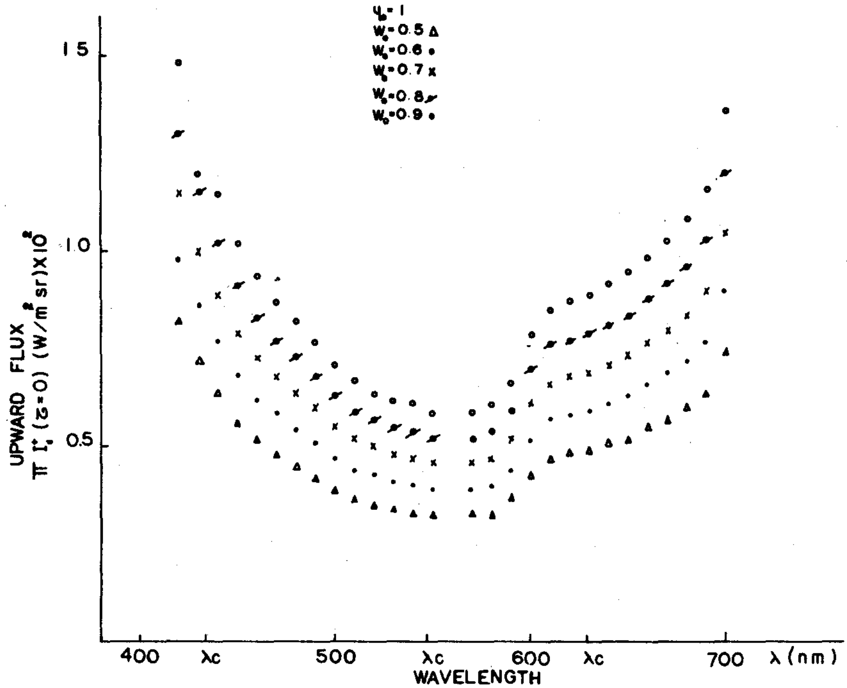


Fig. 1. Upward flux as a function of wavelength for $\mu_0 = 1$.

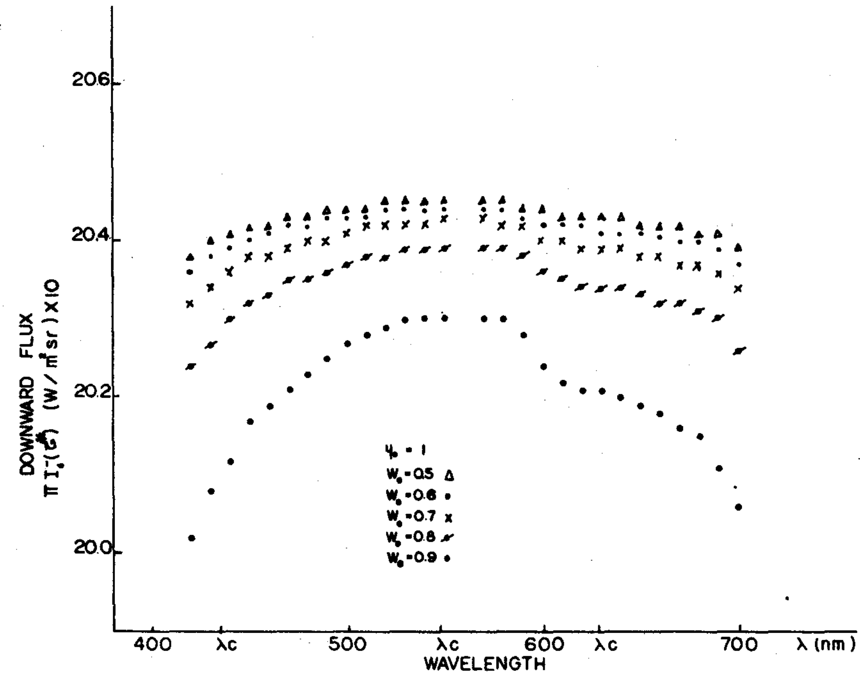


Fig. 2. Downward flux as a function of wavelength for $\mu_0 = 1$.

shown in Figures 6 to 8. It is clear from Figures 6 to 8 that the curves corresponding to the medium and turbid ocean show the same behavior as those obtained by the two-stream method (Figures 1 to 5).

In spite of the fact that the water-bodies are different in each case the comparison shows that the two-stream method produces similar results to the Monte Carlo ones.

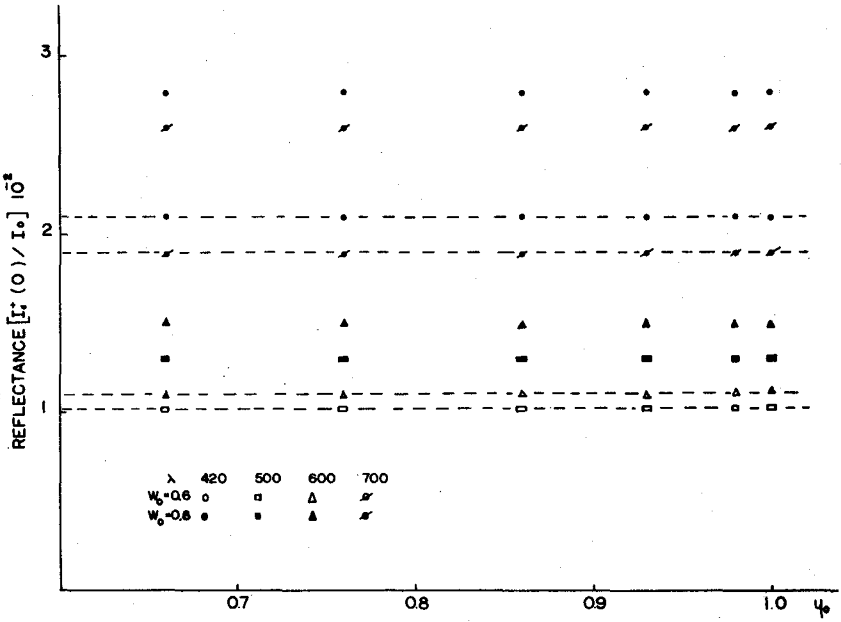


Fig. 3. Dependence of the two-stream calculated reflectance on μ_0 for $\lambda = 420, 500, 600, 700$ nm.

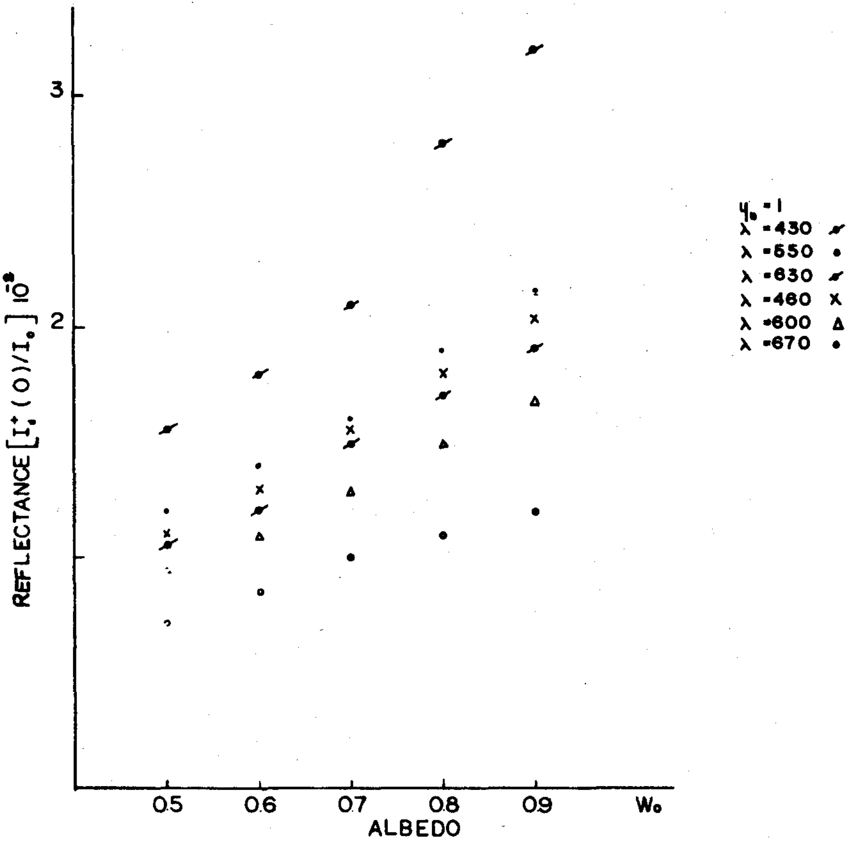


Fig. 4. Dependence of the two-stream calculated reflectance on w_0 for λ= 430, 460, 550, 600, 700 nm.

APPLICATION OF THE METHOD

To have a realistic model, the computed values must agree with experimental data. In this case, to compare the two-stream computed results (Figure 5) with experimental data, multispectral photography was taken at 'El Bosque' dam, located at Zitácuaro, Michoacán, México, with Tri X-pan film and 25, 57, 47B Kodak filters (Lemus, 1983). The multispectral photography was taken at a height of 20 m and a maximum view angle of 30°. The maximum solar zenith angle was of 10°. From the microdensitometry of the films, the absolute water-reflectance in three bands was obtained. The results are plotted in Figure 5. The experimental values are plotted with dark lines and the two-streams results are indicated with points. From

Figure 5, it is clear that the set of theoretical curves obtained for different values of the single scattering albedo permits us to choose the best fit for the experimental values of the absolute water-body reflectance.

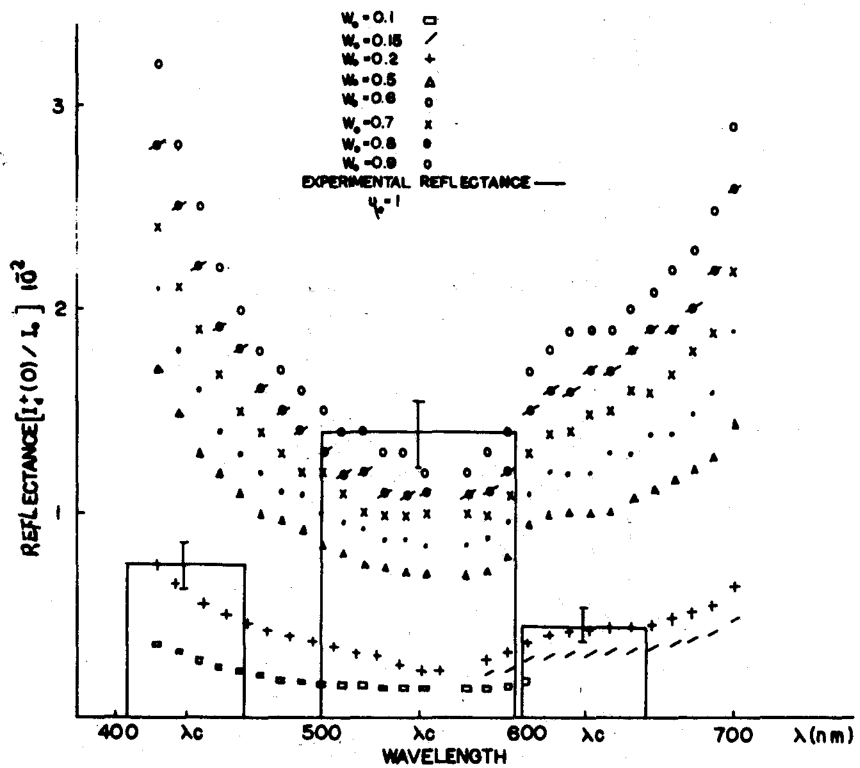


Fig. 5. Comparison of the two-stream computed reflectance for different w_0 with the experimental data (continuous line).

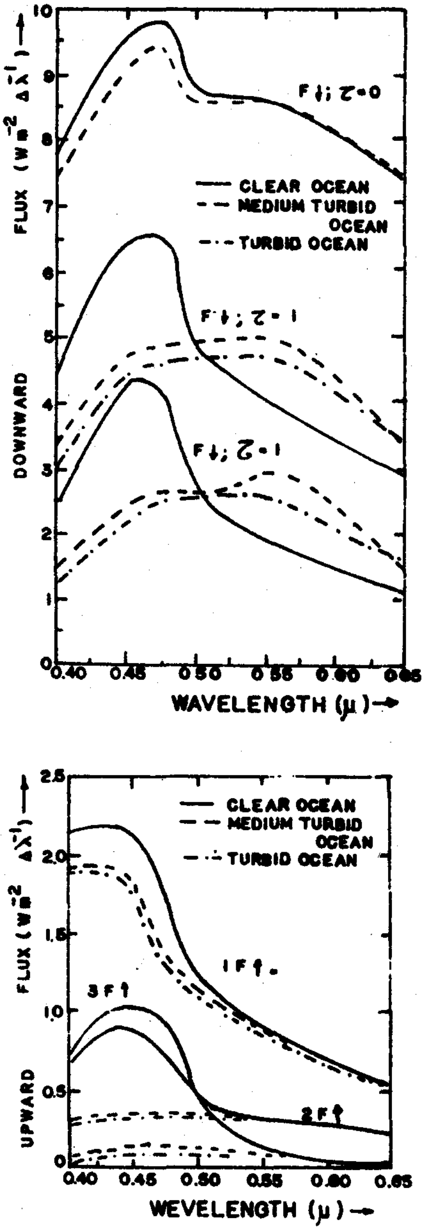


Fig. 6. Downward and upward flux as a function of wavelength. 1F at the top of atmosphere, 2F just above and 3F just below ocean surface, Plass (1972).

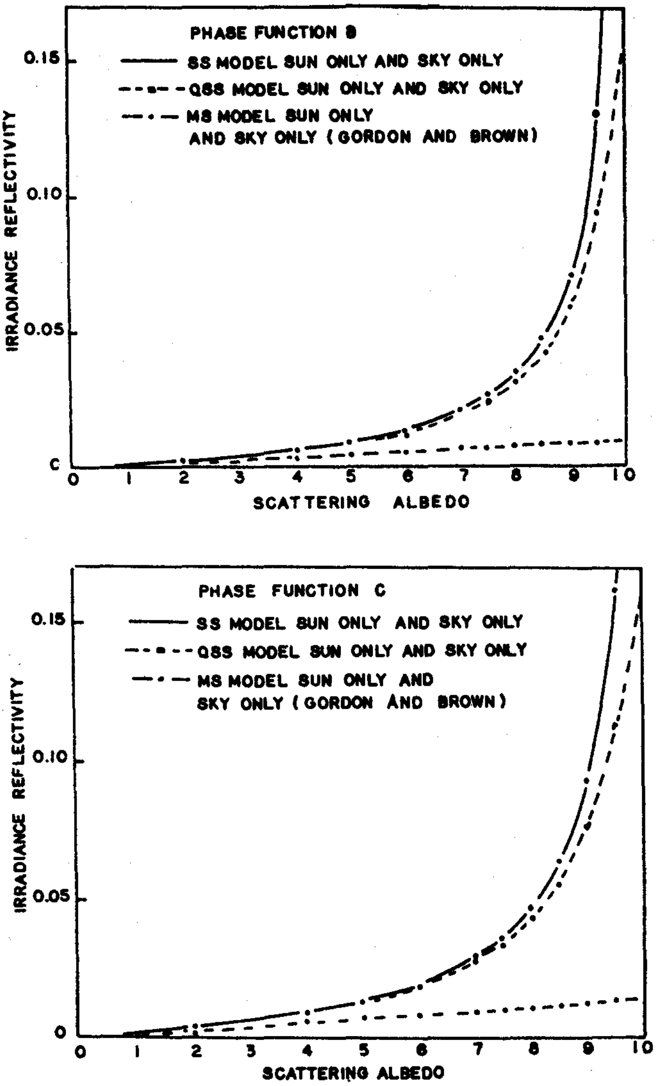
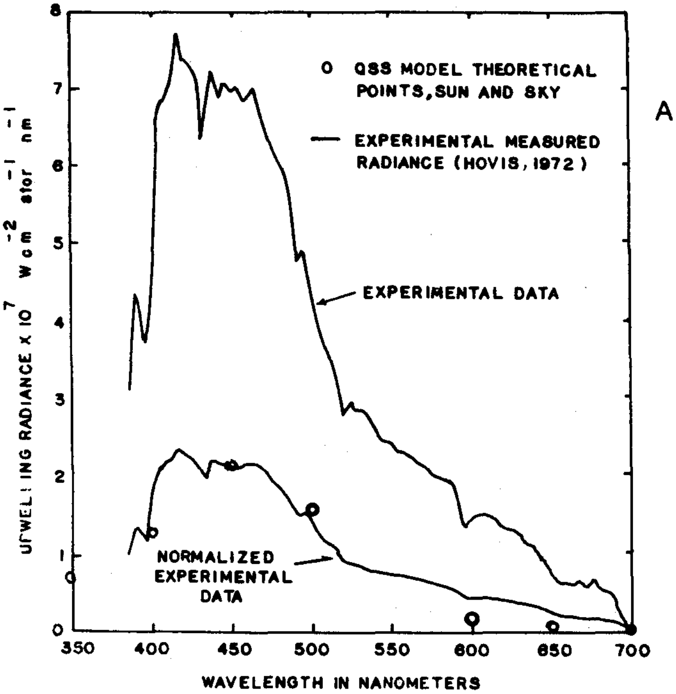


Fig. 7. Irradiance reflectivity v.s. single scattering albedo for three optical models of the sea using phase function B and C, McCluney (1974).



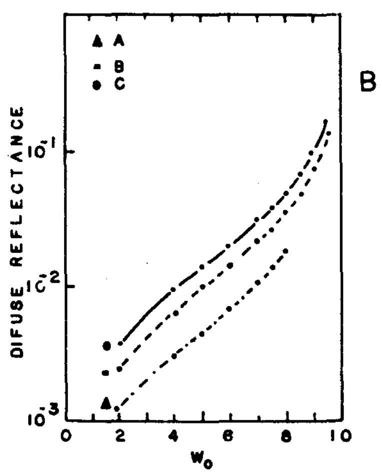


Fig. 8. a) Upwelling radiance spectrum for the Sargasso Sea with measurements made by Hovis, McCluney (1974).

b) Dependence of diffuse reflectance on w_0, Gordon (1975).

DISCUSSION

In spite of the fact that the two-stream approximation represents a simplified model of the radiation field, in the case of water bodies, the results of this work show that it is possible to compute the upward and downward intensities and the water-body reflectance; it is also possible to estimate the optical properties of the medium, as the single scattering albedo.

From Figure 5, we observe that the single scattering albedo of the water-body for the blue-spectral band is $w_0(\Delta\lambda) = 0.2$, for the green-spectral band $w_0(\Delta\lambda) = 0.9$ and for the red-spectral band $w_0(\Delta\lambda) = 0.2$. From the curve plotted in Figure 2, we find that 30% of the incident light remains after the first meter. This fact agrees with the results of Reid and Good (1976), who showed that for pure water the light intensity is reduced approximately 53% in the first meter. For Silver Lake in Wisconsin, they found that 31% of the incident light remains after the first meter. From the behavior of the curves in Figure 5, it can be assumed that there are microscopic particles present in the water-body that reflect in the blue, but absorb less in the green and reflect also, but considerably less in the red-spectral band. This also agrees with Silver Lake's results. From this, it might be inferred that there are green, green-yellow-red algae and other suspended particles. From the ground-truth sampling and the laboratory analysis (Lemus, 1983), the presence of Chlorophyts, Phyrrophyts and Detritus was determined. Their effect on the water reflectance spectrum, as expected from the Chlorophyll (a) extract spectra obtained by Thorne (1977), appears in the theoretical and experimental results plotted in Figure 5.

The results of this work indicate that we can estimate some optical properties of a water-body using a two-stream method in a remote sensing study. However, to have exact values of the single scattering albedo $w_0(\lambda)$, and for the determination of the scattering particles of the water-body, ground truth samplings and laboratory analysis are necessary.

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