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THE ENHANCED NODAL EQUILIBRIUM OCEAN TIDE AND POLAR MOTION

B. V. SÁNCHEZ*

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RESUMEN

El análisis de los datos recientes del movimiento polar señalan la presencia de un componente cuya periodicidad se corresponde con el movimiento lunar ascendente. Se ha procesado una investigación sobre la respuesta tidal del océano a las fuerzas impelentes durante largos períodos. Los resultados de esta investigación señalan la posibilidad de exitación de un componente ondulatorio o de "cabeceo" con la amplitud y frecuencia coincidentes con los datos registrados. Se ha postulado, así, una función de facilitación del equilibrio, bajo la forma de una expansión en las armonías zonales y los coeficientes de dicha expansión parecen llenar las condiciones de componentes del movimiento polar de la magnitud requerida.

ABSTRACT

Recent data analysis of polar motion indicates the presence of a component with periodicity corresponding to the motion of the lunar ascending node. An investigation of the tidal response of the ocean to long period forcing functions has been conducted. The results of the investigation indicate the possibility of excitation of a wobble component with the amplitude and frecuency indicated by the data. An enhancement function for the equilibrium tide has been postulated in the form of an expansion in zonal harmonics and the coefficients of such an expansion have been estimated so as to obtain polar motion components of the required magnitude.

* *Geodynamics Branch NASA/Goddard Space Flight Center Greenbelt, Maryland 20771.*

INTRODUCTION

Recent data analysis by Markowitz (1979) indicate the existence of a wobble component with periodicity and phase corresponding to the motion of the lunar ascending node.

The objective of this investigation is to ascertain the possibility of the existence of a 18.6 year wobble component due to a modified equilibrium ocean tide. The study of the equilibrium response of the oceans dates back to Darwin (1886). More recently Proudman (1960) reached the conclusion that the tidal constituent with nodal period will follow the equilibrium law while the semiannual and annual constituents will probably follow it.

Wunsch (1967) tested the equilibrium hypothesis by computing periodograms for the fortnightly and monthly tides on islands of the Pacific which he found to deviate significantly from equilibrium. He considers the nodal tide to tend towards equilibrium with a certain degree of admissible uncertainty.

The ocean responds not only to the gravitational potential of the Moon and the Sun, but also to the second degree potential of the Earth rotation. The "pole tide" or ocean response to the Chandler wobble has been analyzed by various investigators. Haubrich and Munk (1959) analyzed mean monthly values of sea level from 11 tide stations and found the average pole tide with period of 14 months to have an amplitude twice that predicted by equilibrium theory.

Hosoyama, *et al.* (1976) found the amplitude ratios of the observed to equilibrium pole tides to increase at high and low latitudes in the northern hemisphere; they also find latitude dependent phase delays and advances with implications concerning the possible excitement of the Chandler wobble. Naito (1977, 1979) computed the secular variations of the amplitude and phase of the observed pole tide for the period 1900-1964 and compared them with those of the equilibrium pole tide, he found the largest amplitudes of the observed pole tide to take place in the coasts of the Baltic and North Seas, he concludes that the observed pole tides seem to have a certain relation with the equilibrium tides, but

to have their own secular variations. Dickman (1979) studied the effects of the pole tide on the Chandler wobble assuming pole tide amplitudes up to 10 time equilibrium both for the case of global enhancement and for regional enhancements in the North and Baltic seas. Dahlen (1976) developed a general theory to determine the influence of the pole tide upon rotation of the Earth.

The concept of an enhancement function which modifies the equilibrium tide can be applied also to the ocean response to the gravitational disturbing forces. Such an enhancement function can be expressed in terms of spherical harmonics with adjustable coefficients. It is then possible to express the tidal contributions to the products of inertia in terms of the enhancement function coefficients which can be estimated so as to obtain the polar motion components indicated by the analysis of the data. The estimated coefficients then can be used to predict the modified behavior of the equilibrium tide.

SOLUTION TO THE LIOUVILLE EQUATIONS

The Liouville equations of motion were first given by Liouville (1858). The following assumptions are now made,

(1) the external moments and relative angular momentum terms vanish.
(2) the moments of inertia are constant and considerably larger than the products of inertia; the equatorial moments of inertia are equal.
(3) the ω_z component of angular velocity is a constant and much larger than ω_x and ω_y.

Neglecting products of small quantities the equation of motion then become

$$-\dot{I}_{xz}\omega_z + A\dot{\omega}_x + I_{yz}\omega_z^2 + (C - A)\,\omega_y\omega_z = 0 \qquad (2.1)$$

$$-\dot{I}_{yz}\omega_z + A\dot{\omega}_y - \dot{I}_{xz}\omega_z^2 + (A - C)\,\omega_x\omega_z = 0$$

Now let the products of inertia be given by

$$I_{xz} = I^r_{xz} + I^T_{xz}$$
$$I_{yz} = I^r_{yz} + I^T_{yz} \qquad (2.2)$$

where the superscript r denotes the contribution due to rotational deformation of the solid Earth and T indicates the contribution due to the ocean tide. The rotational deformation is known to be given by

$$I^r_{xz} = -\frac{1}{3} k_2 \frac{R^5}{G} \omega_x \omega_z \qquad (2.3)$$

$$I^r_{yz} = -\frac{1}{3} k_2 \frac{R^5}{G} \omega_y \omega_z$$

where R denotes the radius of the Earth, G is the gravitational constant and k_2 is the second degree Love number.

Equations (2.1) can then be written as follows,

$$\dot{\omega}_x + n^2 \omega_x = \frac{1}{(A + a_1)} (\dot{f}_x - n f_y) \qquad (2.4)$$

where

$$\dot{\omega}_y + n^2 \omega_y = \frac{1}{(A + a_1)} (\dot{f}_y + n f_x)$$

$$a_1 = \frac{1}{3} k_2 \frac{R^5}{G} \omega_z^2 \qquad (2.5)$$

$$n = \frac{[(C - A) - a_1]}{(A + a_1)} \omega_z$$

$$f_x = \dot{I}^T_{xz} \omega_z - I^T_{yz} \omega_z^2$$

$$f_y = \dot{I}^T_{yz} \omega_z + I^T_{xz} \omega_z^2$$

Let

$$I^T_{xz} = M_{xz}\cos(\zeta t - \phi_{xz})$$

$$I^T_{yz} = M_{yz}\cos(\zeta t - \phi_{yz}) \qquad (2.6)$$

Then

$$\omega_x = W_x\cos(\zeta t - \phi_{\omega x})$$

$$\omega_y = W_y\cos(\zeta t - \phi_{\omega y}) \qquad (2.7)$$

$$W_x = \left\{K_1^2 + K_2^2 + 2K_1K_2\sin(\phi_{xz} - \phi_{yz})\right\}^{1/2}$$

$$W_y = \left\{K_3^2 + K_4^2 + 2K_3K_4\sin(\phi_{yz} - \phi_{xz})\right\}^{1/2}$$

$$\phi_{\omega x} = \arctan\left\{(K_1\sin\phi_{xz} + K_2\cos\phi_{yz})/(K_1\cos\phi_{xz} - K_2\sin\phi_{yz})\right.$$

$$\phi_{\omega y} = \arctan\left\{(K_3\sin\phi_{yz} + K_4\cos\phi_{xz})/(K_3\cos\phi_{yz} - K_4\sin\phi_{xz})\right.$$

$$K_1 = -M_{xz}\omega_z(\zeta^2 + n\omega_z)/[(A + a_1)(n^2 - \zeta^2)]$$

$$K_2 = M_{yz}\omega_z\zeta(\omega_z + n)/[(A + a_1)(n^2 - \zeta^2)]$$

$$K_3 = (M_{yz}/M_{xz})K_1$$

$$K_4 = -(M_{xz}/M_{yz})K_2$$

The polar motion components are given by

$$x = (\omega_x/\omega_z)R$$

$$y = (\omega_y/\omega_z)R \qquad (2.8)$$

EQUILIBRIUM OCEAN TIDE

The subject of an equilibrium ocean tide including the effects of ocean loading and the self attraction of the water has been the object of various investigations: Hendersholt (1972), Dahlen (1976), Agnew and

Farrell (1978) and others. It follows that a global equilibrium tide is given by the following expression:

$$\xi = \frac{(1 + k_n - h_n)}{1 - \alpha_n(1 + k'_n - h'_n)} \frac{U_n}{g} \tag{3.1}$$

$$\alpha_n = \frac{3}{2n + 1} \frac{\rho}{\rho_E}$$

where U_n is the disturbing potential due to the mass of the tide generating body and its motion relative to the Earth; k_n, h_n, k'_n and h'_n are the Love numbers of degree n, ρ denotes the density of water and ρ_E is the mean density of the solid Earth, g is the acceleration of gravity.

In order to introduce the concept of a modified equilibrium tide an enhancement function is postulated and its functional form is assumed to be given by an expansion in spherical harmonics of the following form,

$$E(\theta) = r_0^0 P_0^0 + r_1^0 P_1^0 + r_2^0 P_2^0 + r_3^0 P_3^0 \tag{3.2}$$

where the P's are the Legendre polynomials and the r's are undetermined coefficients. The modified equilibrium tide is then given by

$$\xi_M = E(\theta)\xi \tag{3.3}$$

The unmodified tide is recovered when

$$r_0^0 = 1, \quad r_1^0 = r_2^0 = r_3^0 = 0.$$

Now let Ω denote the surface area covered by the oceans and define ξ'_M as follows:

$$\xi'_M \Omega = \iint_C \xi_M \, ds \tag{3.4}$$

where the surface integral is taken over the area of the continents. ξ'_M represents the quantity which must be added to ξ_M in order to satisfy conservation of mass. Note that,

$$\iint_C \xi_M \, ds = \iint_{Sphere} \xi_M \, ds - \iint_{Oceans} \xi_M \, ds$$
$$\iint_{Oceans} \xi_M \, ds = \iint_{Sphere} f(\theta,\psi)\, \xi_M \, ds \tag{3.5}$$

$$f(\theta,\psi) = \sum_n \sum_m P_n^m \begin{pmatrix} a_n^m \cos m\psi \\ b_n^m \sin m\psi \end{pmatrix} = \begin{cases} 0 \text{ over continents} \\ 1 \text{ over oceans} \end{cases} \tag{3.6}$$

Consider the case when the disturbing potential is given by

$$U_2 = R^2 q_2^0 P_2^0 \tag{3.7}$$

q_2^0 being a function of the position and mass of the disturbing body. Making use of Equations (3.1)-(3.7):

$$\xi'_M = (B_2/5a_0^0)\left[a_2^0 r_0^0 + \left(\frac{2}{3}a_1^0 + \frac{3}{7}a_3^0\right) r_1^0 + \left(a_0^0 + \frac{2}{7}a_2^0 + \frac{2}{7}a_4^0 - 1\right) r_2^0 + \left(\frac{3}{7}a_1^0 + \frac{4}{21}a_3^0 + \frac{50}{231}a_5^0\right) r_3^0\right] \tag{3.8}$$

$$B_2 = -\frac{(1 + k_2 - h_2)}{1 - \alpha_2(1 + k'_2 - h'_2)} \frac{R^2}{g} q_2^0$$

The complete expression for the modified equilibrium tide is then

$$\overline{\xi}_M = \xi_M + \xi'_M \tag{3.9}$$

PRODUCTS OF INERTIA

The contribution to the products of inertia due to a modified equilibrium tide are given by

$$I_{xz} = \iint_{\text{Sphere}} f(\theta, \psi)\, xz\, dm$$

$$I_{yz} = \iint_{\text{Sphere}} f(\theta, \psi)\, yz\, dm \tag{4.1}$$

where

$x = R \sin\theta \cos\psi$

$y = R \sin\theta \sin\psi$

$z = R \cos\theta$

$$dm = \rho \overline{\xi}_M(\theta)[R\,d\theta \cdot R \sin\theta\, d\psi]$$

Making use of Equations (3.1-(3.9) yields

$$\begin{Bmatrix} I_{xz} \\ I_{yz} \end{Bmatrix} = -\frac{\pi}{3}\rho R^4 B_2 \left\{ \left[\frac{48}{35}\begin{pmatrix} a_2^1 \\ b_2^1 \end{pmatrix} + \frac{16}{7}\begin{pmatrix} a_4^1 \\ b_4^1 \end{pmatrix} \right] r_0^0 \right.$$

$$+ \left[\frac{48}{75}\begin{pmatrix} a_1^1 \\ b_1^1 \end{pmatrix} + \left(\frac{576}{525} + \frac{1008}{3675}\right)\begin{pmatrix} a_3^1 \\ b_3^1 \end{pmatrix} + \frac{10080}{8085}\begin{pmatrix} a_5^1 \\ b_5^1 \end{pmatrix} \right] r_1^0$$

$$+ \left[\left(\frac{24}{25} + \frac{96}{245}\right)\begin{pmatrix} a_2^1 \\ b_2^1 \end{pmatrix} + \left(\frac{160}{245} + \frac{480}{2695}\right)\begin{pmatrix} a_4^1 \\ b_4^1 \end{pmatrix} + \frac{15120}{15015}\begin{pmatrix} a_6^1 \\ b_6^1 \end{pmatrix} \right] r_2^0$$

$$+ \left[\frac{72}{175}\begin{pmatrix} a_1^1 \\ b_1^1 \end{pmatrix} + \left(\frac{864}{1225} + \frac{192}{1575}\right)\begin{pmatrix} a_3^1 \\ b_3^1 \end{pmatrix} + \left(\frac{13440}{24255} + \frac{13200}{99099}\right)\begin{pmatrix} a_5^1 \\ b_5^1 \end{pmatrix} \right.$$

$$\left. \left. + \frac{147840}{165165}\begin{pmatrix} a_7^1 \\ b_7^1 \end{pmatrix} \right] r_3^0 - \frac{12}{5}\begin{pmatrix} a_2^1 \\ b_2^1 \end{pmatrix}\frac{\xi'_M}{B_2} \right\} \qquad (4.2)$$

The integration of the product of three and four spherical harmonics has been performed by means of the 3-j symbols of Wigner (Rotenberg *et al.*, 1959; Winch and James, 1973).

NUMERICAL RESULTS

The expressions for the products of inertia given by Equations (4.2) can be put in the form of Equations (2.6) since

$$q_2^0 = -(G_D/R^2) \sum_i M_i \cos(\alpha_i) \qquad (5.1)$$

where G_D is Doodson's constant and M_i, α_i are the amplitudes and arguments for the various tidal constituents. The principal terms of the low frequency tides are given by Cartwright and Edden (1973). For the

purpose of this investigation the nodal term is the one of interest, then

$$M = -0.06556$$

$$\alpha = \zeta t - 259.18328^0 \qquad (5.2)$$

$$\zeta = 2\pi/18.613 \text{ years}$$

epoch: 1899 December 31, 12h, Om, Os
ephemeris time.

The solution of the problem consists in estimating the values of r_0^0, r_1^0, r_2^0 and r_3^0 appearing in Equation (4.2) which will yield polar motion components satisfying the results from data anlysis.

Markowitz (1979) analyzed 79 years of ILS (International Latitude Service) data and 17 years of IPMS (International Polar Motion Service) data, he obtained the following results:

ILS

$$x = (28 \pm 13) \cos(\zeta t - 1903.7 \pm 1.2y)$$

$$y = (22 \pm 13) \cos(\zeta t - 1904.4 \pm 1.5y)$$

IPMS

$$x = (22 \pm 13) \cos(\zeta t - 1905.7 \pm 1.5y)$$

$$y = (25 \pm 13) \cos(\zeta t - 1906.7 \pm 1.4y)$$

The results are expressed in a geodetic coordinate system in cm units. The polar motion components corresponding to the case of a non-modified equilibrium tide are obtained by letting $r_0^0 = 1, r_1^0 = r_2^0 = r_3^0 = 0$, this yields

$$x = 0.31 \cos(\zeta t - 1906.14)$$

$$y = 3.02 \cos(\zeta t - 1913.38)$$

In order to estimate the values of the coefficients that will fit the polar motion data a general purpose adaptive iterator for monlinear problems (Campbell *et. al.*, 1964) has been used. The results are given below:

ILS

$$r_0^0 = -5.2260094$$

$$r_1^0 = 2.7151830$$

$$r_2^0 = 3.1766850$$

$$r_3^0 = -0.92669194$$

$$x = 27.48 \cos(\zeta t - 1905.37)$$

$$y = 22.84 \cos(\zeta t - 1904.97)$$

IPMS

$$r_0^0 = -2.0466899$$

$$r_1^0 = 1.9925059$$

$$r_2^0 = 2.0108450$$

$$r_3^0 = 0.0091169942$$

$$x = 21.27 \cos(\zeta t - 1905.44)$$

$$y = 25.85 \cos(\zeta t - 1905.05)$$

In both cases the coefficients were estimated so as to obtain polar motion components within 1 cm of the mean values given by Markowitz analysis. No constraints were imposed on the values obtained for the phase angles.

The corresponding enhancement functions and tide heights follow from Equations (3.2) and (3.9), they are shown in Figures (5.1)-(5.5) below, also shown is the unmodified equilibrium tide.

CONCLUSIONS

The results indicate that a modified equilibrium tide could provide the excitation required to generate the polar motion component detected by Markowitz. However since no ocean dynamics have been incorporated into the formulation of the enhancement function the results have to remain speculative. The concept of enhancement is not a new one although its application has been limited to the pole tide, in such cases different investigators have considered the possibility of enhancement reaching values up to 10 times the equilibrium, also analysis of tidal data has indicated the existence of latitude dependencies as well as magnifications in shallow seas. In that light and by comparison the magnitude of enhancement obtained in this study does not appear altogether exorbitant, especially in the case of the IPMS data. Nevertheless the results should be considered as indicative of a possibility rather than as a quantitative determination of ocean behavior.

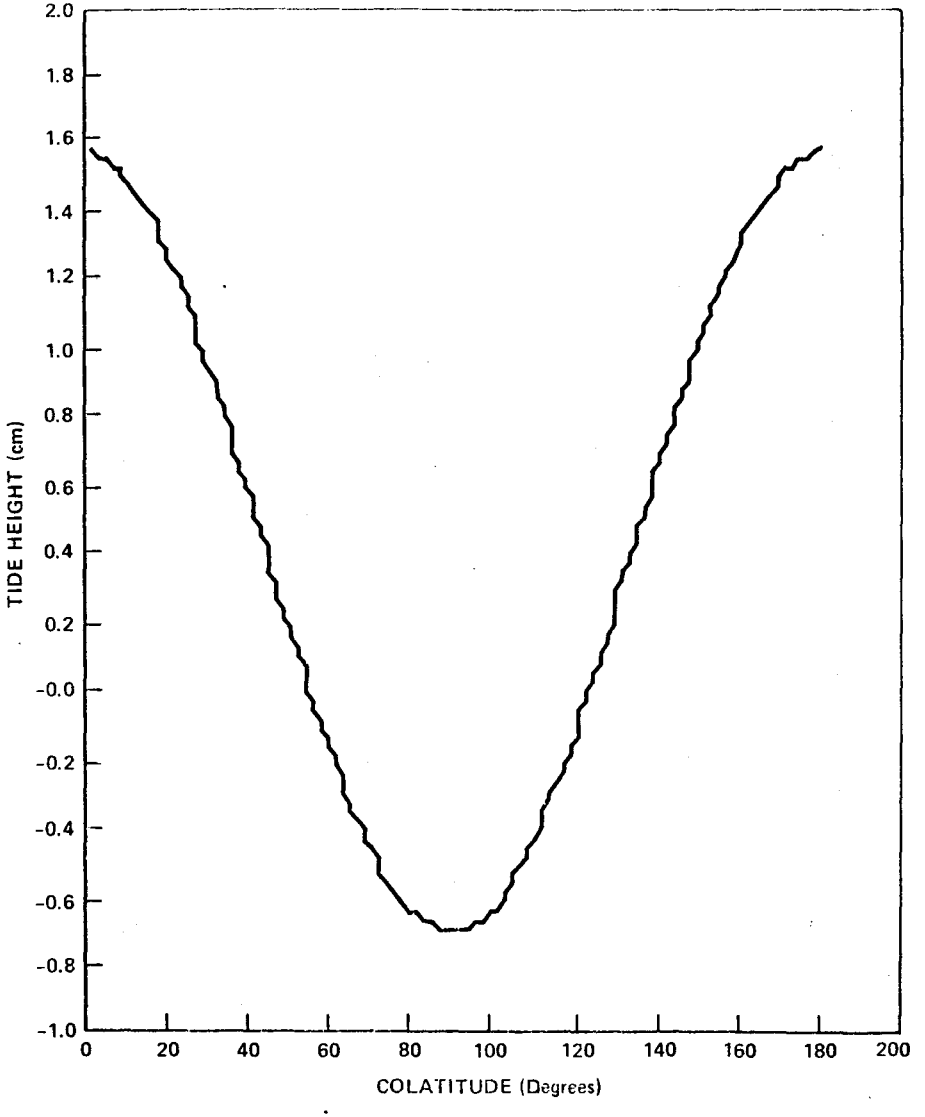

Figure 5.1. Unmodified Tide Height

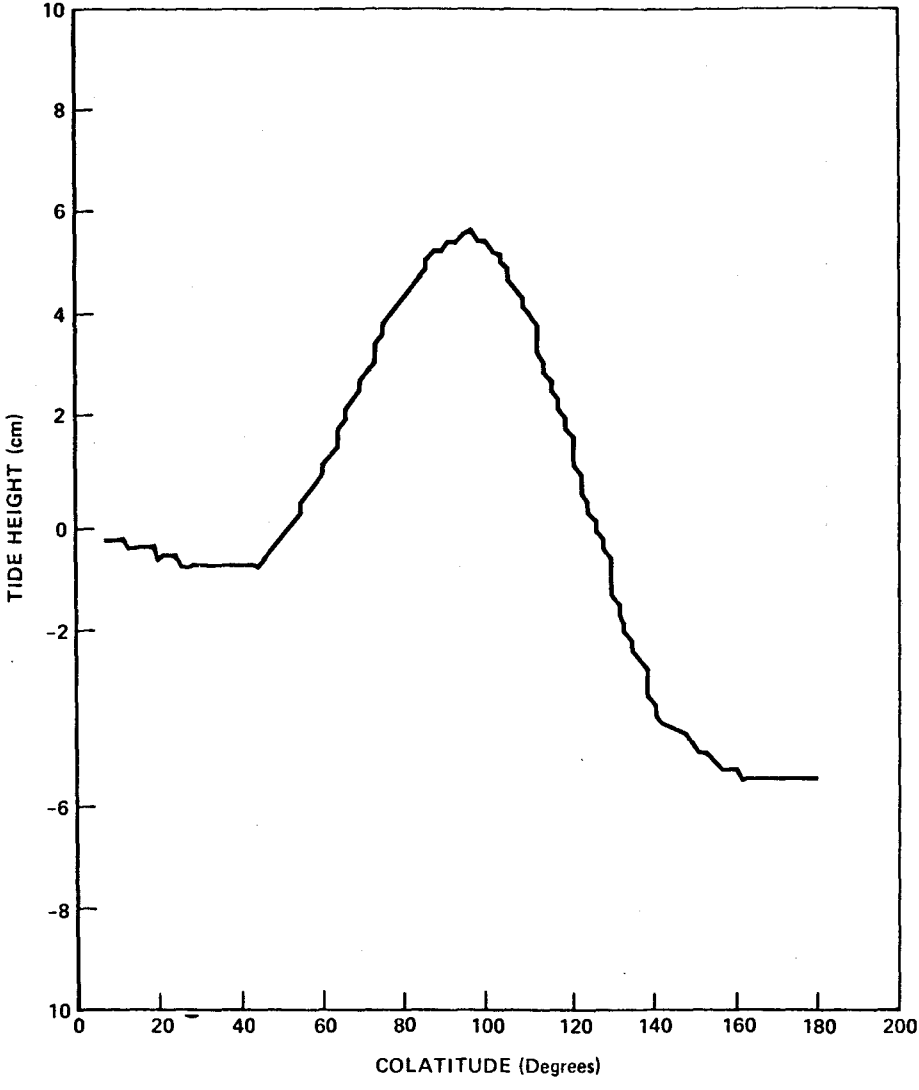

Figure 5.2. Modified Tide Height (ILS)

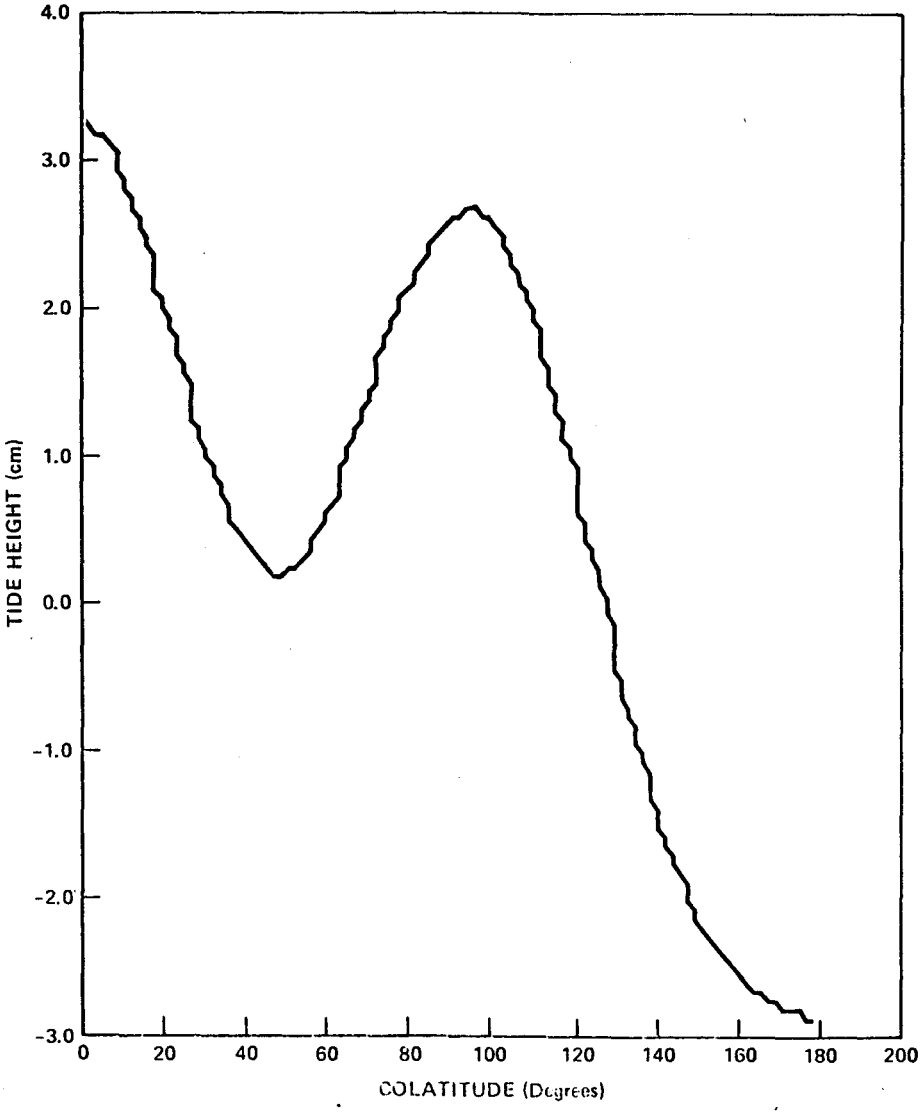

Figure 5.3. Modified Tide Height (IPMS)

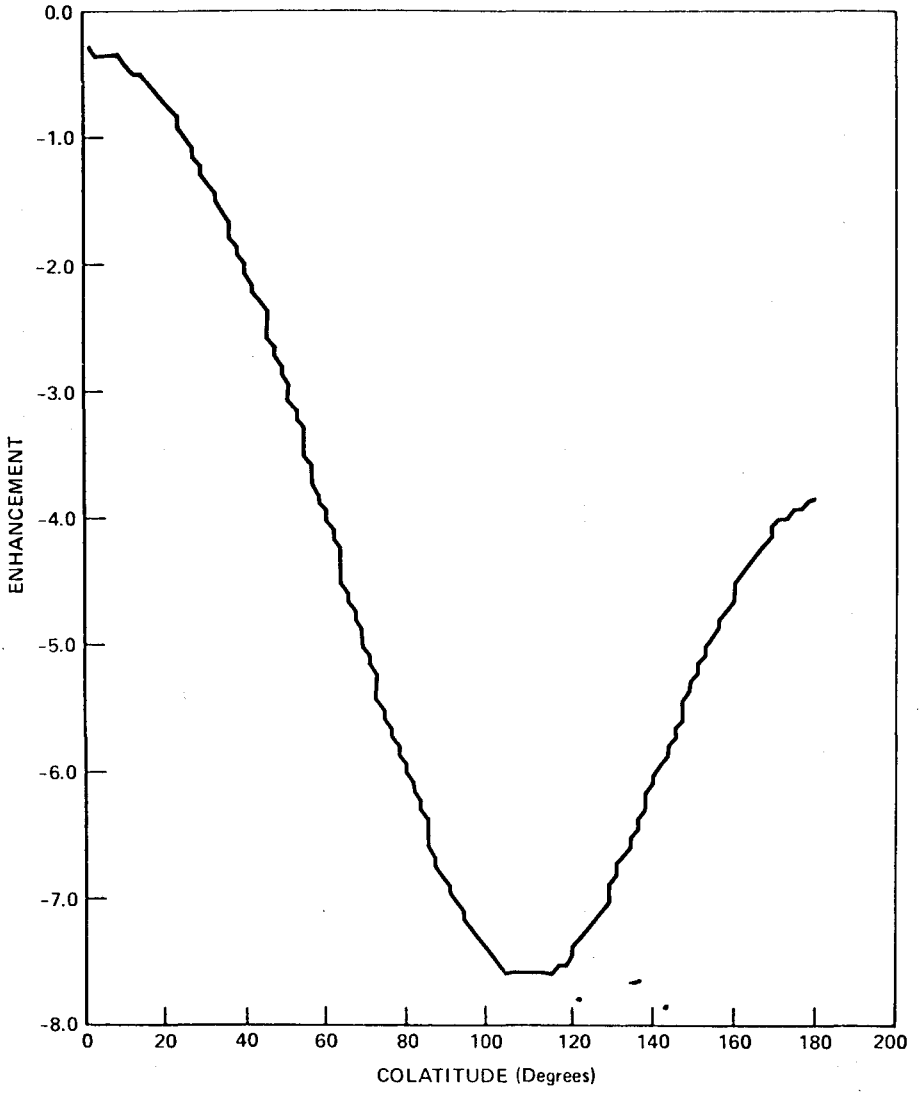

Figure 5.4. Enhancement Function (ILS)

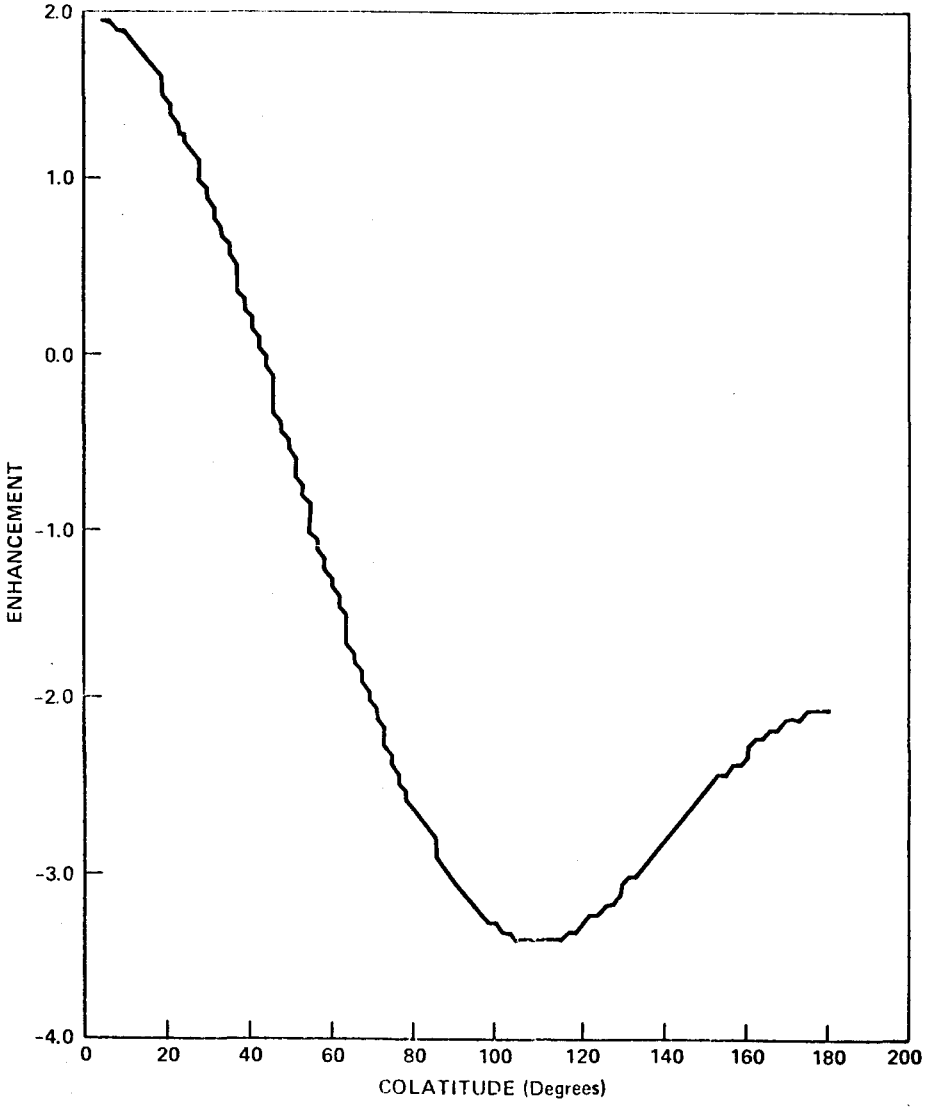

Figure 5.5. Enhancement Function (IPMS)

APPENDIX 1

Numerical values of the constants used in the investigation.

Moments of inertia.

$$A = 8.016(10^{44})\text{gm} - \text{cm}^2$$

$$C = 8.043(10^{44})\text{gm} - \text{cm}^2$$

$$\omega_z = 2\pi/86400\,\text{rad/sec}$$

$$R = 6.378(10^{8})\text{cm}$$

$$k_2 = 0.30$$

$$G = 6.67(10^{-8})\text{cm}^2\ \text{dynes/gm}^2$$

With the values above and making use of Equation (2.5) one obtains a 445 day Chandler period.

$$\frac{(1 + k_2 - h_2)}{1 - \alpha_2(1 + k_2' - h_2')} = 0.86465$$

$$\rho = 1.03\,\text{gm/cm}^3$$

$$g = 980\,\text{cm/sec}^2$$

The values of the coefficients a_n^m and b_n^m appearing in the ocean function, Equation (3.6) are those given by Balmino, Lambeck and Kaula (1973). The proper normalization factors have been applied in order to maintain consistency throughout the calculations.

$$G_D = 2.627723(10^{4})\text{cm}^2/\text{sec}^2$$

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