Fast estimation of local magnitudes from non-standard Wood-Anderson, short period, seismograms

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RESUMEN

Se presenta un método para la evaluación rápida de magnitudes locales a partir de sismogramas (analógicos o digitales) distintos de los Wood-Anderson estándar. El método es una justificación de los métodos comúnmente usados para la evaluación aproximada de magnitudes a partir de sismogramas analógicos. Se ilustra el método mediante su aplicación a datos de microsismicidad local en Baja California, México.

PALABRAS CLAVE: Magnitudes locales.

ABSTRACT

We propose a method for fast estimation of local magnitudes from seismograms (analog or digital) different from the standard Wood-Anderson ones. The method provides a justification of the widely used methods for approximate evaluation of magnitudes from analog seismograms. The method is illustrated through an application to local microseismic data from Baja California, Mexico.

KEY WORDS: Local magnitudes.

INTRODUCTION

The local, or Richter, magnitude (Richter, 1935, 1958) is probably the best known and most widely used measure of the size of local earthquakes. Its linear relationship with the logarithm of the seismic energy is widely applied, and other magnitudes are expected to coincide with the local magnitude in overlapping ranges.

Richter (1958) defines local magnitude as

$$M = \log A^W - \log A_0 + S, \tag{1}$$

where logarithms are base 10, A^W is the maximum amplitude in mm measured on a standard torsion Wood-Anderson (W-A) seismograph (0.8s free period and 2800 static amplification, photographic recording, damping 0.8 of critical), A_0 is a correction calculated for the W-A (Richter, 1958, Table 22-1), that depends only on the epicentral distance Δ (in km), and S is a station (or instrument) correction.

In many cases, such as seismicity surveys with portable seismographs or operation of seismic telemetric networks, computing local magnitude present problems because the W-A seismograph is not common; it is not easily portable, and it does not have the advantages of the digital seismographs which are becoming more common every day. This problem is sometimes circumvented by constructing "equivalent" W-A seismograms from digital seismograms by deconvolving the instrument response and convolving with the theoretical W-A one. This is an involved process requiring good knowledge of the instrument response. Alternately, especially for analog seismographs, some other magnitude scale is used (e.g. coda), usually calibrated in some way to roughly agree with the local magnitude. A third approach is to define a magnitude scale using the maximum amplitudes of the non-W-A seismograms, related in some way to the local magnitude; this method is widely used, particularly for temporary networks (*e.g.* Gonzalez *et al*., 1984).

In this paper we propose a method to estimate the local magnitude in a simple way from non-W-A seismic records, based on a few known local magnitudes.

THE METHOD

Let M_R be a local magnitude, which will be called a reference magnitude, determined from a maximum W-A amplitude A_R^W, with an epicentral distance correction A_{0R}, and a station correction S. Then, from (1) we may compute magnitude for any other earthquake recorded at the same W-A station, from the corresponding maximum amplitude A^W and epicentral distance correction A_0, as:

$$M = M_R + log\, A_{0R} - log\, A_0 + log\, \frac{A^W}{A_R^W}\,. \qquad (2)$$

Note that the station correction no longer affects the magnitude determination.

Now, let A_R and A be the amplitudes measured by some other short-period instrument and, if necessary, integrated to correct for the effect of an inductive pickup (which the W-A does not have). If the earthquakes are measured at both instruments over more or less the same frequency range (usually a valid assumption for local events) and if the instruments act as linear filters, then

$$\frac{A^W}{A_R^W} \cong \frac{A}{A_R}\,. \qquad (3)$$

Figure 1 shows, as solid lines, the normalized amplitude responses for the standard Wood-Anderson seismograph and for some popular short-period, portable, seismometers, such as the Mark L22 (0.5075s free period and damping 0.737 of critical), Mark L28 (0.2026s free period and damping 0.740 of critical), and Lennartz LE3D (1.0s free period and damping 0.7 of critical). The vertical band labeled D indicates a wide frequency range for dominant frequencies in typical short-period local seismograms. Assuming a worst-case scenario where *AR* and *A* are measured at opposite ends of the dominant frequency band with an instrument whose response varies withing the band (some instruments, like the WA or the L28, have flat or approximately flat responses within that band), the right-hand ratio in (3) could be in error by a factor of up to 1.2. Thus even for the worst case, the error in the magnitude introduced by assumption (3) would be <0.08, which is well withing the uncertainty of magnitude determinations.

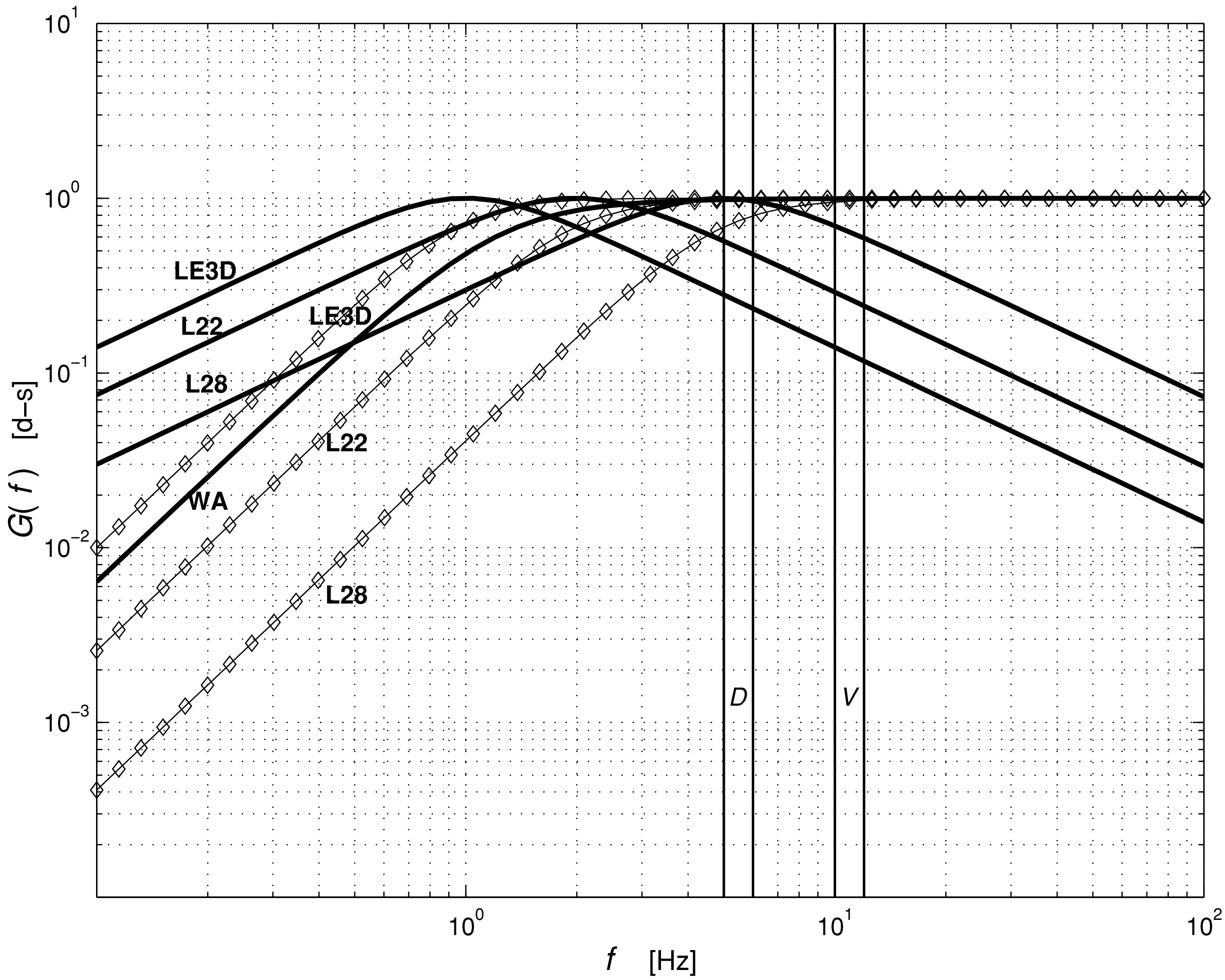

Fig. 1. Amplitude (thick lines) and velocity (diamond lines) response for the Wood-Anderson standard seismograph (WA), and for some typical short-period seismographs. Tbe bands labeled *D* and *V* are typical dominant frequency bands for displacement and velocity seismograms, respectively.

Note that these are non-dimensional ratios. This obviates the necessity of working with any particular units, as long as the same units are used for all amplitudes. The ratios will be approximately independent of amplification and site effects.

Substituting (3) into (2), we can determine magnitudes approximately, often within the intrinsic error of magnitude determinations, based on the maximum amplitudes measured by other instruments

$$\begin{aligned} M &= M_R + \log A_{0R} - \log A_0 + \log\frac{A}{A_R} \\ &= M_R + \log A_{0R} - \log A_R - \log A_0 + \log A \\ &= C + \log A - \log A_0 \,, \end{aligned} \quad (4)$$

where all the information related to the reference magnitude is contained in the constant

$$C = M_R + \log A_{0R} - \log A_R \,. \quad (5)$$

Thus, a single constant quantity C will characterize the magnitude determination at each station. If the second instrument is also a W-A sited on the same type of ground as the reference instrument, constant C should equal the station correction, so that A^W and A are related by

$$A^W = 10^{C-S} A \cdot$$

TYPICAL DATA PROCESSING

Amplitudes

In what follows we describe how the method might be applied to typical data. Richter (1958) recommends evaluating the magnitude at a given station as the average of the magnitude estimations for both horizontal components:

$$M = \tfrac{1}{2}\left(M^{NS} + M^{EW}\right) = \tfrac{1}{2}\log\left(A^{NS}A^{EW}\right) - \log A_0 \,,$$

therefore, we use the log (amplitude)

$$\log A = \tfrac{1}{2}\log\left(A^{NS}A^{EW}\right). \quad (6)$$

Determination of *C* for each station

Since instrument and local soil response may be different for each station, we expect the constant C to be different as well. Let C_k be the constant for station k.

If the source radiation pattern for earthquakes were perfectly spherical and the epicentral distance corrections were the same in all directions, magnitudes could be estimated from a single record. In practice, the magnitude assigned to the ith earthquake, which we denote by M_i, is computed as an average:

$$M_i = \frac{1}{K}\sum_{k=1}^{K} M_{ik} \,, \quad (7)$$

where M_{ik} is the magnitude for event i determined at station k from amplitude A_{ik}, and K is the total number of stations. Due to effects of radiation pattern, directivity, path, site, etc., M_{ik} will, in general, be different at each station.

Thus, if M_{Ri}, $i = 1, N$ are local magnitudes assigned by a reference source to N earthquakes, we cannot expect all to yield, from (5), the same value for the constant C_k at station k as would be desirable. We denote by C_{ik} the estimate of C_k obtained at station k from earthquake i using a reference magnitude M_{Ri} and distance corrections and amplitudes - log A_{0ik} and log A_{ik}. The final estimate of C_k will be determined from the N estimates of C_{ik}.

This determination of C_k may be carried out in different ways. The scheme we use is as follows. We made a histogram of the C_{ik}, eliminating obvious outliers selected by eye. Since the C_{ik} usually constitute a sparse sample, it is convenient to use a spread histogram technique (Nava, 1998). The standard deviation for the normal distribution used for spreading was chosen as half the expected variation in C_{Rik} from the uncertainty in the reference magnitude ($\varepsilon \approx \pm 0.3 \Rightarrow \sigma \approx 0.15$). Using the sample mode estimated from the histogram $\hat{C}_k$ and the sample mean $\langle C_k \rangle$, a first estimate was obtained from $C_k = \tfrac{1}{2}\left(\hat{C}_k + \langle C_k \rangle\right)$. The largest reference magnitudes having the most reliable amplitude determinations were obtained by a best fit.

Displacement or velocity

In the common case where the instrument has an inductive pick-up and responds nearly proportionally to velocity rather than to displacement, it is necessary to first integrate the signal (which, for digital records, is easily done numerically), then measure the maximum amplitudes in (4), (5), and (6).

However, for short times and over a small frequency range (as in the case of a signal with a dominant period), displacements are fairly proportional to velocities, so that

$$\frac{A^W}{A_R^W} \cong \frac{A}{A_R} \cong \frac{A^V}{A_R^V} \,, \quad (8)$$

where A^V denotes maximum amplitudes on a velocity seismogram. The possibility is worth exploring. This may be done by following the above steps using A^V instead of A. Let C^A and M_A, and C^V and M_V be constants and magnitudes determined from amplitudes and velocities, respectively. As shown in Figure 1 (diamonds), the velocity response of short-period seismometers is essentially flat over the dominant frequency band labeled V, and instrument response would introduce no error in Eq. (8).

Obtaining A and A^V from inductive pick-up digital records

Let a digital seismic time series be $\{v\}=\{v_0, v_1, v_2,..., v_n\}$ in digital units. Amplitudes are supposed to be measured from the base level and numerical integration tends to amplify low frequencies. Therefore, a least-square straight line $v = b + m\,i$, incorporating a shift b and a trend m, is subtracted from the data. $A^V \equiv v_{max}$ is determined after de-trending.

Now the series may be integrated by a simple trapezoid rule to get the displacement time series $\{a\}$:

$$a_i = a_{i-1} + 0.5\left(v_i + v_{i-1}\right).$$

No normalization to the sampling interval is needed. If the integrated series presents contamination from low frequencies (periods larger than the signal duration), $\{v\}$ is filtered by a notch high-pass filter with a zero at (1,0) and a pole at (0.99,0) in the z plane, and the frequency response

$$G = 0.995\,\frac{1. - z^{-1}}{1. - 0.99\,z^{-1}}\,,$$

after which the process is repeated.

We proceed to find $a_{\max} = \mathrm{Max}\left\{|a_i|;\quad i = 0,n\right\} = \left|a_{i_{\max}}\right|$ and we construct a section $\{v^S\}=\{v_{i_1},...,v_{i_2}\}$, where $i_1 \le i_{max}$-I_1 and $i_2 \ge i_{max}$+I_2, and I_1 and I_2 are chosen depending on the sampling rate and the typical signal duration, so that elements close to a_{i_1} are at noise level and the maximum A is expected to occur between i_1 and i_2. The sequence $\{v^S\}$ is then integrated to obtain a new amplitude $\{a^S\}$ series, and from this series the maximum $A = a^s_{\max}$ is determined.

AN EXAMPLE

We test the validity of the key assumptions in Equation (3), and the applicability of the assumptions in Equation (8), with data from the SIERRA97 project. This microearthquake survey (Frez *et al*., 2000) monitored seismicity from May 20 to June 23, 1997 over an ~50×50 km^2 area in the Ojos Negros Valley region, in the Peninsular Ranges of northern Baja California, Mexico (Figure 2). A seismic array consisting of 13 portable REFTEK digital stations, kindly loaned to the SIERRA97 project by IRIS/PASSCAL, complemented by was two permanent digital stations from the Red Sísmica del Noroeste de México (RESNOM), five analog Sprengnether MEQ seismographs, and five Terra-Tech digital instruments.

We used a data subset of 12 events for which magnitudes had been reported by the Southern California Seismic Network and RESNOM. Local magnitudes, in the range of 1.4 to 4.4 (Table 1), had been determined from an empirical formula by Vidal and Munguía (1999) for this region. Each station featured a digital seismograph with a three-component, short-period seismometer and recording with 200 samples/s and 32bit words in the triggered mode. The earthquakes were recorded at seven REFTEK stations (Figure 2). Epicenters (Frez *et al*., 2000) covered an epicenter-station distance range from 1.7 to 35 km, and were well constrained.

A program was written and implemented to measure amplitudes, both A and A^V, automatically from the digital records. Figures 3 and 4 are examples of signals and amplitude determinations for earthquakes recorded at stations SIE1 and LOSC, respectively.

Figure 5 shows the $C^A_{k\,i}$ and $C^V_{k\,i}$ values for station SDEC and the corresponding spread histograms. The vertical lines above the histograms indicate the corresponding mean (center) and the mean ± one standard deviation. Worst-case examples of $C_{k\,i}$ histograms for stations LOSC and RDEF are shown in Figure 6.

Final C_k values are shown in Table 2. A comparison between the reference and calculated magnitudes is shown in Table 1 and Figure 7. This figure also shows reference

Table 1

Reference magnitudes M_R, and calculated magnitudes M_A and M_V, computed from K records

M_r	M_A	M_V	K
3.0	3.0	3.1	2
2.2	2.2	2.3	5
2.2	2.1	2.1	5
1.8	1.9	1.9	4
1.5	1.4	1.5	6
1.4	1.7	1.8	7
2.0	1.9	1.9	6
1.7	1.4	1.6	5
1.9	1.9	1.9	5
2.1	2.5	2.7	4
4.4	4.4	4.3	2
3.4	3.5	3.3	4

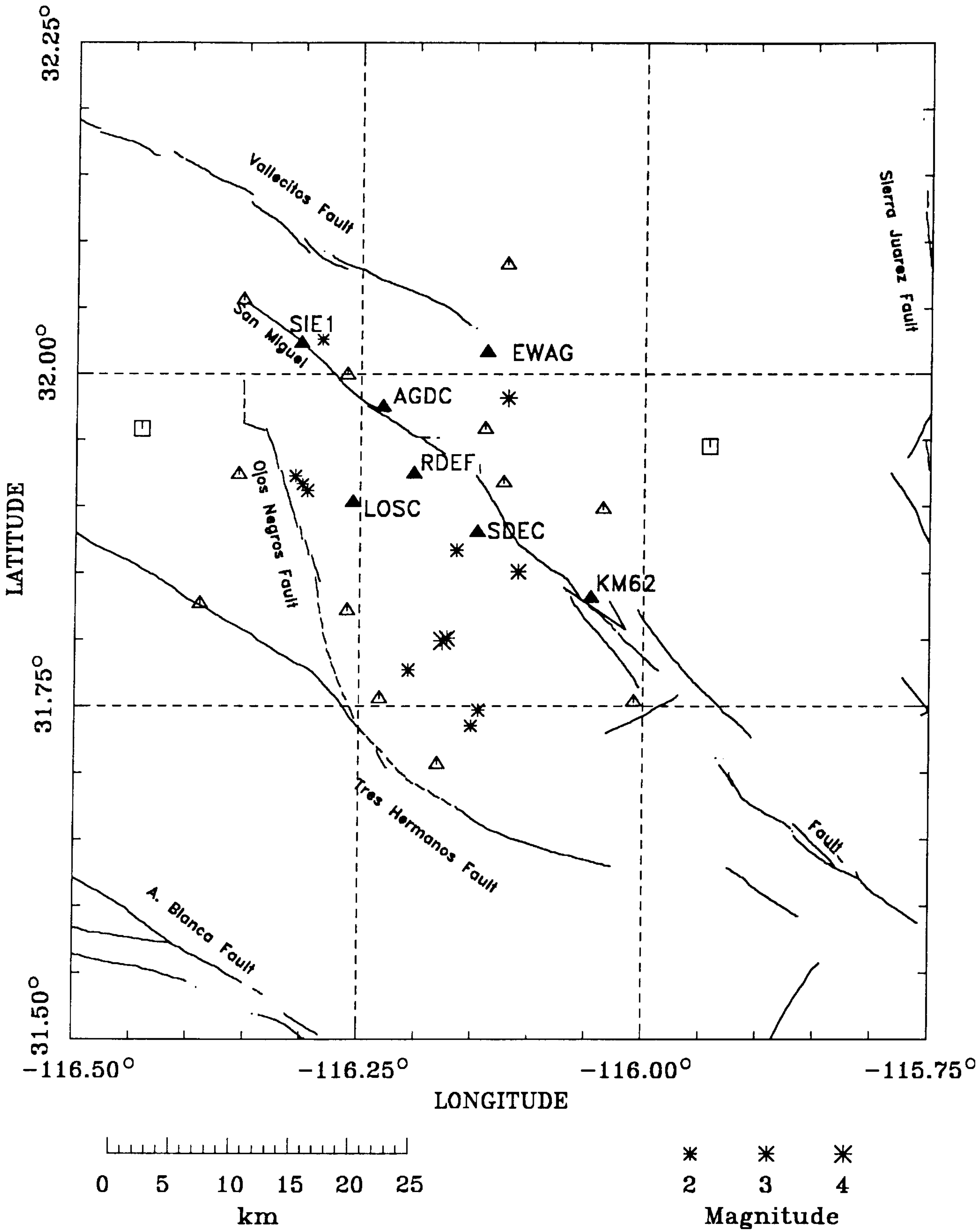

Fig. 2. SIERRA97 study area. Triangles and squares (RESNOM) indicate the seismic stations. Filled triangles and asterisks indicate seismic stations and epicenters of earthquakes used in this study.

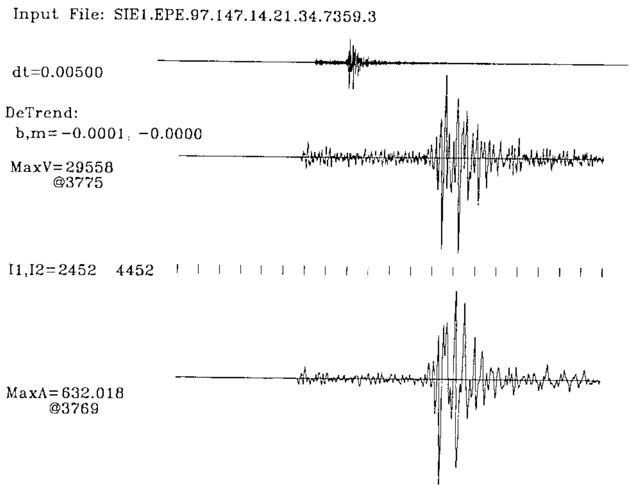

Fig. 3. Example of "velocity" seismogram recorded at SIE1. The complete record is shown on top, and below it is the chosen segment after de-trending with $b = -0.0001$ and $m = 0.000$, which has a maximum amplitude of 29558 counts. The bottom trace is the integrated "amplitude" trace obtained after one low-pass filtering, this trace has a maximum of 632.018 counts.

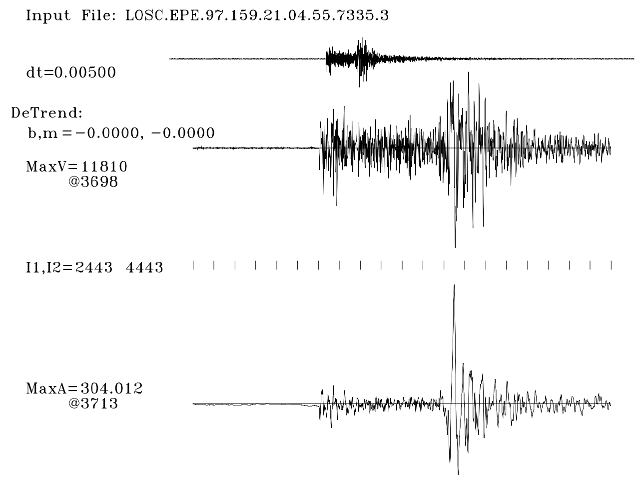

Fig. 4. Example of "velocity" seismogram recorded at LOSC. The complete record is shown on top, and below it is the chosen segment after de-trending with $b = -0.0000$ and $m = 0.0000$, which has a maximum amplitude of 11810 counts. The bottom trace is the integrated "amplitude" trace (no filtering), this trace has a maximum of 304.012 counts.

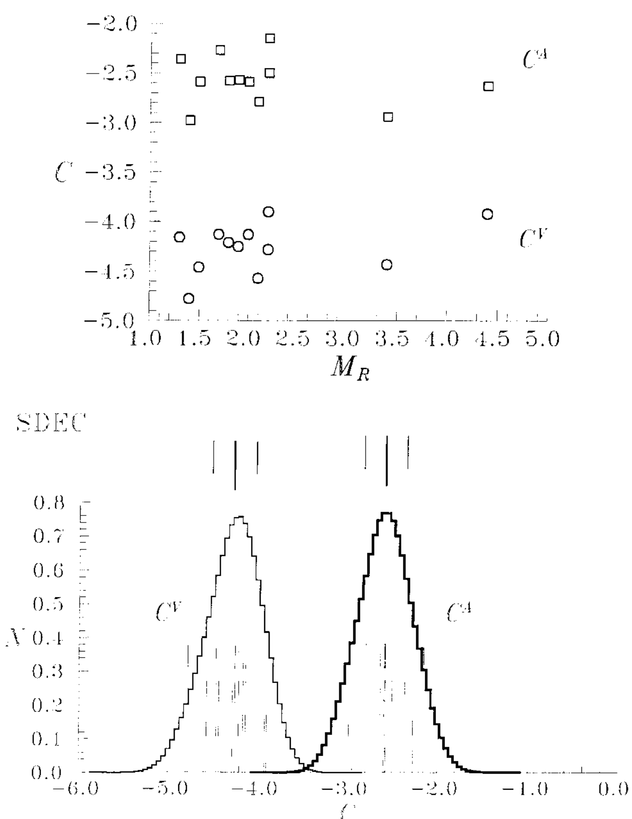

Fig. 5. Example of $C^A_{k\,i}$ (squares) and $C^V_{k\,i}$ (circles) values for station SDEC (top), and the corresponding spread histograms (bottom), with spreading $\sigma = 0.15$ and $\Delta C = 0.02$. The dashed lines under the histograms represent the data, and the vertical lines above them indicate the corresponding mean (center) and the mean ± one standard deviation.

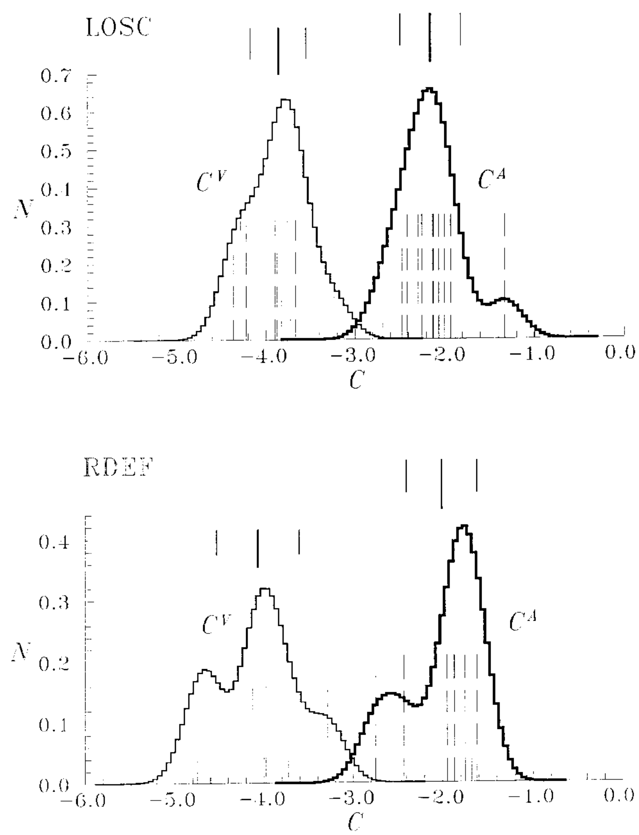

Fig. 6. Examples of worst case $C^A_{k\,i}$ and $C^V_{k\,i}$ histograms for LOSC (top) and RDEF (bottom). Conventions as in Figure 5.

Table 2

Station constants C_A and C_V.

Station	C_A	C_V
AGDC	-2.20	-3.80
EWAG	-2.00	-4.05
KM62	-2.00	-3.70
LOSC	-2.20	-3.80
RDEF	-1.95	-3.95
SDEC	-2.60	-4.25
SIE1	-2.20	-4.00

$M=M_R$ and $M=M_R \pm 0.2$ lines and the least-square regressions

$$M_A = 0.02 + 1.00\ M_R, \qquad M_V = 0.25 + 0.92\ M_R \tag{8}$$

indicating a very good coincidence between M_R and M_A, and an acceptable coincidence between M_R and M_V. Note that most discrepancies occur for the smaller magnitudes, which have a higher uncertainty.

Table 3 shows the mean and the standard deviation σ of the errors

$$\varepsilon_i = M_{Ri} - M_i\ , \tag{9}$$

and the linear correlation coefficients ρ between reference and calculated magnitudes, for both amplitude and velocity estimates.

DISCUSSION

Table 3 shows that the magnitudes calculated by our method are a good estimation of reference magnitudes. Magnitudes obtained from amplitudes and reference magnitudes have a high linear correlation; they agree within negligible mean error, and both rms error and standard deviation are well within the usual uncertainty (Hutton and Boore, 1987). It should be kept in mind that we are evaluating the fit and the errors assuming the reference magnitudes to be exact. A less strict approach, allowing for possible errors in the reference magnitudes, would yield even smaller errors.

Table 3

Means and standard deviation of errors, and linear correlation coefficient between reference and calculated magnitudes

	$\langle\varepsilon\rangle$	$\langle\lvert\varepsilon\rvert\rangle$	ε_{rms}	σ	ρ
A	-0.017	0.117	0.178	0.185	0.978
V	-0.058	0.158	0.229	0.231	0.967

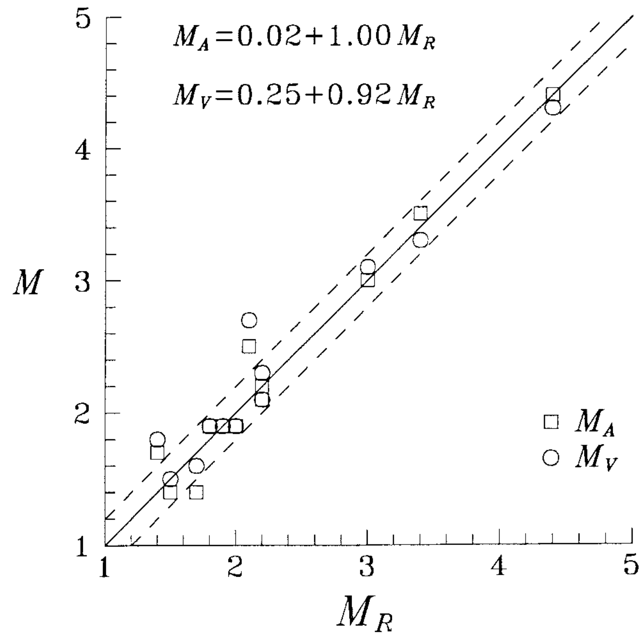

Fig. 7. Comparison between the reference and calculated magnitudes. Solid line is reference $M = M_R$, and dashed lines indicate $M = M_R \pm 0.2$.

As expected, since M_L is defined for amplitudes, the fit for magnitudes obtained from velocities is not as good as that from amplitudes. However, it is good within an acceptable uncertainty. We conclude that the method can be used for estimating magnitudes reliably from common seismograph data, including velocity data and analog seismograms.

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