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Probabilistic prediction of the next large earthquake in the Michoacán fault-segment of the Mexican subduction zone

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RESUMEN

La estimación del intervalo de tiempo Δt hasta el siguiente sismo fuerte, que es esperado en una región sísmica, es un problema difícil. Dentro del método más convencional de predicciones basadas en el intervalo de tiempo (Δt), dada una distribución probabilística de los intervalos de tiempo observados y el tiempo transcurrido (t) desde el último sismo fuerte, podemos estimar la probabilidad de un nuevo evento sísmico en un intervalo de tiempo, digamos los siguientes 25 años. En este artículo, invertimos la aproximación y estimamos el intervalo de tiempo (Δt) durante el cual ocurrirá el nuevo sismo fuerte, usando como criterio que la probabilidad condicional de ocurrencia sísmica sea un máximo, dada que el nuevo sismo fuerte no ha ocurrido en el tiempo (t) transcurrido desde el último sismo fuerte. Adoptamos como distribuciones probabilísticas la distribución de Weibull, la distribución de Rayleigh y la distribución de Pareto (power-law).

Seleccionamos una región limitada por 17.50 - 18.50° N y 101.7 – 103° W para definir el significado de Michoacán segmento de fractura. Usando una lista de sismos históricos (incluido el sismo de Petatlán) ocurridos en esta área, encontramos que usando el modelo de Pareto (power-law), se estima que un sismo destructor (M ≥ 7) puede ocurrir antes del año 2014.99, o equivalentemente antes de diciembre de 2014.

PALABRAS CLAVE: Predicción probabilística, probabilidad condicional de ocurrencia sísmica, modelos de Weibull, Rayleigh y Pareto, estimación de intervalos de tiempo de ocurrencia.

ABSTRACT

Estimation of the time interval Δt until the next strong earthquake to be expected in a seismic source region is a difficult problem. In the conventional method of time-interval prediction, given some distribution of observed interval times between large earthquakes and knowing the elapsed time t since the 1ast large earthquake, the probability of a new seismic event in an interval time Δt may be estimated. In this paper, we reverse the approach and we estimate the interval time for the occurrence of the next large seismic event assuming that the conditional probability of earthquake occurrence is a maximum, provided that a large earthquake has not occurred in the elapsed time t since the last large earthquake. We assume the Weibull distribution, the Rayleigh distribution or the Pareto distribution for the earthquake recurrence time intervals.

In the Michoacán seismic region and using a list of historical large earthquakes in this seismic area, we found that the Pareto model predicts a damaging earthquake (M ≥ 7) before the year 2014.99, or before December 2014 ± 1.76(yrs.).

KEY WORDS: Probabilistic prediction, conditional probability of earthquake occurrence, Weibull, Rayleigh and Pareto models, estimation of interval times of occurrence.

INTRODUCTION

The 19 September 1985 Michoacán (Mexico) earthquake (M ≥ 8.1) was the most severe natural disaster in Mexico's seismic history. It caused over 10 000 deaths in Mexico City and left an estimated 250 000 homeless (Astiz *et al.,* 1987). An imminent prediction of the next large earthquake under the Michoacán fault-segment would be useful. Such a prediction must rely en the observation of phenomena that relate to large earthquakes.

Sykes *et al*. (1999) use 10 to 30 years as warning time for long-term predictions characterized by average repeat times of 50 to 300 years. Predictions of this type are based on a knowledge of the average repeat time and the variations in individual repeat times for each segment, and of the time t that has elapsed since the last earthquake. The physics for this type of prediction is the slow buildup of stress. Predictions are usually probabilistic in nature to allow for observed differences in individual repeat times and uncertainties in the parameters used in the calculations.

In Mexico a 30-year prediction seems appropriate for active fault-segments in the Mexican subduction zone. The Mexican subduction zone extends over about 1000 km along the Middle-America trench, from the Jalisco-Colima region through the Michoacán-Guerrero region to the Oaxaca region. It appears to be segmented by the Rivera fracture zone, the East Pacific Rise, the Orozco and O'Gorman fracture zones, and the Tehuantepec ridge, (Singh and Mortera, 1991).

A method of long-term prediction, which has been studied extensively in connection with earthquakes, is the use of probability distributions of recurrence times on individual faults or fault segments. The difficulty lies in determining the correct distribution, given the scarcity of data of large seismic events on a given fault. By combining data from many different faults, Nishenko and Buland (1987) obtained a reasonably good fit to a lognormal distribution. McNally and Minster, (1981) have argued that a Weibull distribution is more appropriate.

Several stochastic earthquake generating models have been used for seismic hazard evaluation. The most common hazard model is the Poisson process model (Cornell, l968). Along a fault or within a seismic source zone, earthquakes are assumed to take place following a Poisson process. According to Cluff *et al.* (1980), a Poisson process model provides probabilities of earthquakes occurrences of any size up to the designed maximum size that is characteristic over an entire region, but the probabilities are independent of the size and the elapsed time since the last major earthquake event. The so-called "lack of memory" property of the Poisson process is generally found to be in agreement with the observed seismic activity related to moderate or large-magnitude earthquakes.

Although Poissonian behavior has been shown for seismic sequences of some regions (Gardner and Knopoff, 1974; Ferraes,1967), temporal dependence between earthquakes has been detected in several seismic regions or areas around the world (Bufe *et al.* 1977; Shimazaki and Nakata, 1980; Sykes and Quittmeyer, 1981)

Two kinds of time-dependent models have been proposed: time-predictable and slip-predictable (Shimazaki and Nakata,1980). In a time-predictable pattern the time between events is proportional to the size of the preceding event, and therefore the date but not the size of the next event can be predicted. In a slip-predictable model the time between events is proportional to the size of the following event, and the size of the next event can be predicted, but the date cannot be predicted. Stochastic models of earthquake occurrence have been developed, based on the time-predictable model (Anagnos and Kiremidjian, l984).

Following Schwartz and Coppersmith (l986), other stochastic models have been proposed for seismic hazard evaluation. Renewal models, which are referred to as real-time models, imply a time dependent accumulation of energy between major earthquakes. In this model, the likelihood of earthquake occurrence during a period of interest, which is referred to as conditional probability, is related to the elapsed time since the last major event and the average recurrence interval between major earthquakes. Renewal models have been widely used to describe earthquake occurrence (Veneziano and Cornell, 1976; Kameda and Ozaki, l979; Savy *et al.,* 1980; Grandori *et al.,* 1984).

More complex models that consider the nonrandom character of earthquake size (magnitude, moment, etc.) and recurrence time τ have also been proposed. For example, the semi-Markov model relates the probability of future earthquakes of particular sizes to the elapsed time since the last major event and the magnitude of the prior event (Patwardhan *et al.,* l980; Cluff *et al.,* 1980; Coppersmith, 1981).

Ferraes (1985) introduced the Bayesian technique to the field of earthquake prediction research. First, he used the discrete version of Bayes'theorem and the assumption that the earthquake process behaves as a Poisson process. Next, Ferraes (1986) employed Bayes'theorem and the assumption of a Bernoulli process. Further, Ferraes (1988) used the discrete form of Bayes'theorem in conjunction with the Poisson process and introduced the criterion of "optimum Bayesian conditional probability" to derive a predictive formula for the occurrence time of the next major earthquake within a given seismic region.

Finally, in order to include variations in magnitude in the prediction of earthquakes, Ferraes (1992) used the continuous form of Bayes'theorem in conjunction with the assumption that the earthquake process behaves as a lognormal Gaussian process. The criterion of "optimum Bayesian probability" was used to derive an equation to predict the recurrence time of the next major earthquake on the Ometepec fault-segment of the Mexican subduction zone.

In time-interval based prediction, given some assumed distribution of interval times and knowing the elapsed time since the last large event, the probability of a large event in an interval time Δt, say, the next 25 years, is estimated (Nishenko and Buland, 1987; Working Group on California Earthquake Probabilities, 1988, 1990; Davis *et al.,* l989). In this paper, we reverse the approach and we estimate the most probable interval time Δt for the occurrence of the next large event in the Michoacán fault-segment of the Mexican subduction zone.

CONDITIONAL PROBABILITY

The purpose of this section is to provide a brief synopsis of conditional probability of event occurrence, $P(\Delta t \mid t)$, and to discuss some applications of conditional probability. We use the conditional probability formula to predict the occurrence of the next large earthquake in the Michoacán fault-segment of the Mexican subduction zone.

Given an interval of t years since the occurrence of the previous event, we wish to determine the probability of failure before time $t + \Delta t$.

The conditional probability P(t < T ≤ t + Δt | T > t), which is the probability that an earthquake occurs during the next Δt interval, is

$$P(\Delta t|t) = \frac{P(t < T \le t + \Delta t)}{P(T \ge t)} . \quad (1)$$

In terms of the probability density of T, say f , we have

$$P(t < T \le t + \Delta t) = \int_{t}^{t+\Delta t} f(s)ds \quad (2)$$

and

$$P(T \ge t) = \int_{t}^{\infty} f(s)ds . \quad (3)$$

We substitute equation (2) and (3) in Equation (1). We get

$$P(\Delta t|t) = \frac{\int_{t}^{t+\Delta t} f(s)ds}{\int_{t}^{\infty} f(s)ds} . \quad (4)$$

Wesnousky *et al*. (1984) pointed out that Equation (4) provides a reasonable tool for estimating seismic hazard on a fault or fault-segment and made the assumption that the underlying probability distribution of earthquake recurrence time intervals is normal.

THE CRITERION OF MAXIMUM CONDITIONAL PROBABILITY

We have proposed that the system of fractures (or earthquakes) on any fault or fault-segment can be described in mathematical terms using the conditional probability of earthquake occurrence, equation (4), with probabilistic models for the recurrence times of large earthquakes.

If earthquakes behaved in a purely periodic fashion, the conditional probability P (Δt | t) would always be unity. However, in Nature, significant stochastic fluctuations occur. Therefore, the forecasting can be obtained by maximizing the conditional probability P (Δt | t). In general, P (Δt | t) is a real-valued function of two real variables (Δt, t).

However, t, the elapsed time since the last large earthquake, can be measured. Thus, our fundamental problem is to predict the time interval Δt for the occurrence of the next large earthquake, given an observed elapsed time t since the last large earthquake.The criterion for a maximum conditional probability is

$$\frac{\partial}{\partial \Delta t} P(\Delta t|t) = 0 . \quad (5)$$

THREE PROBABILITY DENSITY MODELS

Assuming reasonable models for the probability density of interval times between earthquakes and using (4) for the conditional probability of earthquake occurrence, we may determine an expression for the conditional probability that a large earthquake occurs during a future time interval Δt for a given probability density model of earthquake recurrence time intervals, in a specific fault or fault-segment. Here we discuss three probability density models: (1) The Weibull probability density model, (2) The Rayleigh probability density model, and (3) The Pareto power-law probability density model.

(1) The Weibull Probability Density Model

Assuming that the exponential probability distribution is not the correct probability density for the Michoacán segment (Davis *et al.,* l989), we seek a distribution model with greater flexibility that the exponential distribution. The Weibull distribution provides more flexibility and variety of distribution shapes that some other models. Its two parameters are a scale parameter θ and a shape parameter η. For η = 1, the Weibull distribution becomes the single-parameter exponential distribution. The Weibull distribution is (Meyer, 1972)

$$f(T) = (\theta\eta)T^{\eta-1}e^{-\theta T^{\eta}} . \quad (6)$$

If the random variable T has a Weibull distribution with probability density function given by (6), we have the following expressions for the mean and the variance:

$$E(T) = \theta^{-\frac{1}{\eta}} \Gamma\left(\frac{1}{\eta}+1\right),$$
$$V(T) = \theta^{-\frac{2}{\eta}} \left\{\Gamma\left(\frac{2}{\eta}+1\right) - \left[\Gamma\left(\frac{1}{\eta}+1\right)\right]^2\right\}. \quad (7)$$

To derive the Weibull conditional probability $P_w(\Delta t | t)$ we substitute equation (6) in (4) and integrate. We get

$$P_w(\Delta t|t) = 1 - \frac{e^{-\theta(t+\Delta t)^{\eta}}}{e^{-\theta\ t^{\eta}}} . \quad (8)$$

Note that the exponent of the numerator in equation (8) can be written as follows:

$$\theta(t + \Delta t)^{\eta} = \theta\ t^{\eta}\left(1 + \frac{\Delta t}{t}\right)^{\eta}, \quad (9)$$

where

$$\left(1 + \frac{\Delta t}{t}\right)^{\eta} = 1 + \eta\left(\frac{\Delta t}{t}\right) + h(\Delta t). \quad (10)$$

If the elapsed time t since the last large earthquake (M ≥ 7) is large, the time interval Δt is small with respect to t and $h(\Delta t)$ becomes negligible. Substituting equation (10) in (9) we obtain

$$\theta(t+\Delta t)^{\eta} = \theta\ t^{\eta}\left[1+\eta\left(\frac{\Delta t}{t}\right)\right]. \tag{11}$$

Substituting equation (11) in (8) we get the approximate Weibull conditiona1 probability of earthquake occurrence

$$P_w(\Delta t|t) = 1 - e^{-\theta\eta t^{\eta-1}\Delta t}. \tag{12}$$

2) The Rayleigh Probability Density Model

The Rayleigh probability density appears frequently in communication problems. Sornette and Knopoff (1997) calculated the expected time to the next earthquake for several examples of statistical distributions with memory. They exhibit in their Figure A3a the interesting subcase of the Weibull distribution with m = 2, which is appropriate for rectified Gaussian noise, known as the Rayleigh distribution. According to Sornette and Knopoff (1997, p. 796), the Rayleigh distribution has a tail with similar properties to that of the Gaussian and decays faster than an exponential distribution.

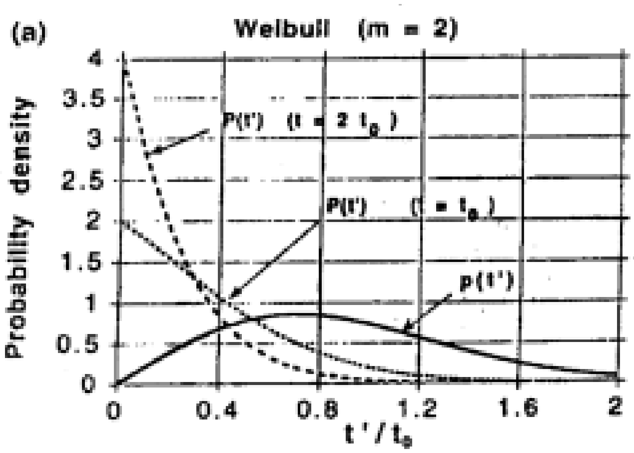

Fig. A3. Weibull distribution (a) m=2 (Rayleigh distribution), corresponding to tail decaying faster than an exponential $P(t^1)$ is shown for $t = t_0$ and $t = 2t_0$ together with $P(t^1)$. From Sornette and Knopoff (1997).

Let us assume that the interval time between earthquakes on the Michoacán fault-segment is modeled by a Rayleigh distribution with parameter δ, that is, the probability density of T is

$$f(T) = \frac{T}{\delta^2} exp\left(-\frac{T^2}{2\delta^2}\right), \quad T > 0. \tag{13}$$

The mean and the variance of a Rayleigh - distributed variable are

$$E(T) = \delta\sqrt{\frac{\pi}{2}}, \qquad V(T) = \left(2-\frac{\pi}{2}\right)\delta^2 = 0.429\delta^2. \tag{14}$$

It is first necessary to estimate the parameter δ. To do this, equations (14) and the estimated sample mean μ and variance γ of our earthquake sample may be used (see Winkler and Hays, 1975, p.516)

$$\mu = \delta\sqrt{\frac{\pi}{2}} = 1.2533\delta \tag{15}$$

or equivalently

$$\delta = 0.7979\mu. \tag{16}$$

To determine the Rayleigh conditional probability of earthquake occurrence $P_r(\Delta t \mid t)$, we substitute equation (13) into equation (4), and integrate. We get

$$P_r(\Delta t|t) = 1 - \frac{e^{-\frac{(t+\Delta t)^2}{2\delta^2}}}{e^{-\frac{t^2}{2\delta^2}}}. \tag{17}$$

Next we expand $(t + \Delta t)^2$. We get

$$(t+\Delta t)^2 = t^2 + 2t\Delta t + (\Delta t)^2. \tag{18}$$

Substituting the above expression in equation (17). We get

$$P_r(\Delta t|t) = 1 - e^{-\frac{2t\Delta t+(\Delta t)^2}{2\delta^2}}. \tag{19}$$

3) The Pareto Model

The statistical mechanics of earthquake and fault populations have recently been suggested to be a self-organized critical phenomenon (Bak and Tang, 1989; Sornette and Sornette, 1989; Carlson and Langer,1989: Ito and Matsuzaki, 1990, Main, 1995). It is assumed that the Earth has approached a state where the local stress is near failure everywhere, and deformation occurs predominantly on large correlated faults (Sornette *et al.,* 1994). Individual earthquakes represent small perturbations of this critical state. According to Main (1995), this is consistent with the relatively small stress drop in individual earthquakes, of the order of 1 to 10

Mpa (Abercrombie and Leary, 1993), compared to the predicted frictional sliding stress for the upper crust of the order 100 to 400 MPa (Scholz, l990, p. 126) and to the ease with which earthquakes can be induced with relatively small stress perturbations of the order 0.2 MPa during fluid injection or withdrawal (Segall,1989). The distribution of ava1anches follows a power-law energy distribution. A fractal morphology of individual fault planes and a fractal correlation of hypocenters is consistent with observations (Kagan and Knopoff, 1980; Turcotte, l990).

A power-law increase in seismicity prior to a major earthquake was first proposed by Bufe and Varnes (1993). They considered the cumulative amount of Benioff strain (square root of seismic energy) in a specified seismic region and they showed that an accurate retrospective prediction of the Loma Prieta earthquake could be made by assuming a power-law increase in Benioff strain prior to the earthquake.

Assume a Pareto power-law probability distribution to represent the recurrence times on the Michoacán fault-segment. The Pareto distribution has been found to describe a wide variety of economic, social and physical phenomena. According to Taylor (1974), the probability density function for Pareto distribution is

$$f(T, \alpha, x_0) = \alpha x_0^{\alpha} T^{-(\alpha+1)} . \qquad (20)$$

The mean is

$$E(T) = \begin{cases} \dfrac{\alpha x_0}{\alpha - 1} & \text{if } \alpha < 1 \\ \infty & \text{if } \alpha = 1 \end{cases} . \qquad (21)$$

Thus, the mean of a Pareto distribution exists only for α > 1. The variance of the Pareto distribution is

$$V(T) = \frac{\alpha r_o}{(\alpha - 1)(\alpha - 2)} . \qquad (22)$$

The variance of the Pareto power-law distribution is infinite for α ≤ 2.

For the conditional probability of occurrence $P_p(\Delta t \mid t)$, we substitute Equation (20) in Equation (4) and integrate. We get

$$P_p(\Delta t|t) = 1 - \frac{t^{\alpha}}{(t+\Delta t)^{\alpha}} = 1 - t^{\alpha}(t+\Delta t)^{-\alpha} , \qquad (23)$$

or equivalently

$$P_p(\Delta t|t) = 1 - t^{\alpha}\left\{t\left(1+\frac{\Delta t}{t}\right)\right\}^{-\alpha} = 1 - \left\{1+\left(\frac{\Delta t}{t}\right)\right\}^{-\alpha} . \qquad (24)$$

If the elapsed time t since the last earthquake is large, we may assume that the time interval of interest Δt is small with respect to t. Then using the three first-order terms of the binominal series expansion,

$$P_p(\Delta t|t) = \frac{\alpha(\Delta t)}{t} - \frac{\alpha(\alpha+1)(\Delta t)^2}{2t^2} . \qquad (25)$$

IDENTIFICATION OF THE MICHOACÁN SEGMENTATION

Following Mikumo *et al*. (1998) for the purpose of statistical forecasting of large earthquakes (M ≥ 7) in the Michoacán fault-segment, we consider a sequence of five large earthquakes in the northern segments of the Mexican subduction zone between the Rivera and Orozco Fracture zones. Table 1 shows the distribution of recent major earthquakes in and around the Michoacán fault-segment.

Using the epicenters of Playa Azul (October 25, 1981), Michoacán (September 19, 1985) and the aftershock of April 30, 1986, we obtain a seismic region bounded by 17.50° - 18.50°N and 101.70° - 103°W as the Michoacán fault-seg-

Table 1

Recent major earthquakes in and around the Michoacán fault-segment of the Mexican subduction zone, after Mikumo *et al*. (1998)

Earthquakes	Date	Date (years)	Epicenter Lat N	Epicenter Lon W	Magnitude
1. Petatlán	03/14/1979	1979.21	17.46	101.46	7.62
2. Playa Azul	10/25/1981	1981.82	17.75	102.25	7.43
3. Michoacán	09/19/1985	1985.72	18.14	102.71	8.05
4. Zihuatanejo	09/21/1985	1985.72	17.62	101.82	7.66
5. Aftershock	04/30/1986	1986.33	18.42	102.99	6.99

ment. However, in Mikumo *et al*. (1998) it is shown that the Petatlán earthquake (March 4, 1979) and the September 21, 1985, earthquake ruptured complementary segments to the earthquakes occurred in September 19, 1985, October 25, 1981 and April 30, 1986. Thus, we consider two possible boundaries for the Michoacán fault-segment: (1) without the Petatlán and associated earthquakes (Table 2), and (2) containing the Petatlán and associated earthquakes (Table 3).

ANALYSIS OF THE CATALOGS

In order to evaluate the validity of the catalogs in Table 2 and Table 3, and to assess the correct probability density model for the Michoacán fault segment, we compare the three models (Weibull, Rayleigh, and Pareto) to estimate or predict the recurrence time intervals τ_e for the last three events in the segment.

To predict the occurrence interval for any large event already occurred we have the observed recurrence interval (τ_o). The problem is to estimate the time t elapsed since the last large earthquake and the time interval Δt.

One procedure consists in assuming t and Δt are random variables and to maximize the conditional probability of earthquake occurrence P(Δt | t) as follows.

P(Δt | t), has a maximum, if

$$\frac{\partial}{\partial \Delta t} P(\Delta t|t) = 0 \qquad (26)$$

and

$$\frac{\partial}{\partial t} P(\Delta t|t) = 0 \quad . \qquad (27)$$

(a) Weibull model

Using equation (12) and equation (26), we may write

$$\frac{\partial}{\partial t} P(\Delta t|t) = \frac{\partial}{\partial t}\left\{ 1 - e^{-\theta\eta(\Delta t)\ t^{\eta-1}} \right\} = 0 \,,$$

Table 2

Catalog of large earthquakes in the Michoacán fault-segment during the period 1911-1986 after Anderson et al. (1989). This table does not include the Petatlán earthquake

Event No.	**Date (years)**	**Latitude (°N)**	**Longitude (°W)**	**Recurrence** time	**Magnitude**
1	1911.43	17.5	102.5	——	7.7
2	1941.29	18.8	102.9	29.86	7.7
3	1973.08	18.4	103.2	31.79	7.5
4	1981.82	17.8	102.3	8.74	7.3
5	1985.72	18.1	102.7	3.98	8.1
6	1986.33	18.4	103.0	0.61	7.0

Table 3

Catalog of large earthquakes occurred in the Michoacán fault-segment during the period 1911-1986. as in Table 2, but including the Petatlán earthquake

Event No.	**Date (years)**	**Latitude (°N)**	**Longitude (°W)**	**Recurrence times**	**Magnitude**
1	1911.43	17.5	102.5	——	7.7
2	1941.29	18.8	102.9	29.86	7.7
3	1973.08	18.4	103.2	31.79	7.5
4	1979.21	17.46	101.46	6.13	7.62
5	1981.82	17.8	102.3	2.61	7.3
6	1985.72	18.1	102.7	3.9	8.21
7	1986.33	18.4	103.0	0.61	7.0

from which we obtain

$$\theta\eta(\Delta t)(\eta-1)t^{\eta-2}e^{-\theta\eta(\Delta t)t^{\eta-1}}=0\,. \quad (28)$$

Similarly using equation (12) and equation (27), we may write

$$\frac{\partial}{\partial\Delta t}P(\Delta t|t)=\frac{d}{d\Delta t}\left\{1-e^{-\theta\eta(\Delta t)\ t^{\eta-1}}\right\}=0$$

from which we obtain

$$e^{-\theta\eta(\Delta t)t^{\eta-1}}\left\{\theta\eta t^{\eta-1}\right\}=0\,. \quad (29)$$

From equations (28) and (29), we obtain the following equations for t and Δt

$$(\eta-1)\theta\eta(\Delta t)t^{\eta-2}=0 \quad (30)$$

and

$$\theta\eta t^{\eta-1}=0\,. \quad (31)$$

Solving (30) and (31) by subtraction, we obtain

$$t=(\eta-1)\Delta t\,. \quad (32)$$

For Equations (28) and (29) we can write

$$e^{-\theta\eta(\Delta t)t^{\eta-1}}=0\,. \quad (33)$$

Expanding the left-hand side of the above equation in a MacLaurin series, we get

$$1-\theta\eta(\Delta t)t^{\eta-1}+s(\Delta t)^2=0\ , \quad (34)$$

where $s(\Delta t)^2$ becomes negligible for small Δt. Thus

$$1-\theta\eta(\Delta t)t^{\eta-1}=0\,. \quad (35)$$

Substituting Equation (32) into Equation (35) we obtain

$$(\Delta t)^{\eta}=\frac{1}{\theta\eta(\eta-1)^{\eta-1}}\,. \quad (36)$$

For any large event already occurred, we do not know the elapsed time t. However, we know the observed recurrence time τ. Thus, in order to correlate t, Δt and τ, we may assume that the recurrence time τ is approximately equal to

$$\tau = t + \Delta t\ . \quad (37)$$

WEIBULL ANALYSIS OF THE CATALOGS

Now we consider the two sets of catalogues given in Table 2 and Table 3, and we apply the predictive Equations (36) and (37) to predict Δt, t and τ for the following last three earthquake events: The 1981 earthquake, the 1985 earthquake and the 1986 earthquake.

In Table 4 we carry out this analysis for the recurrence time τ (estimated and observed) associated to the earthquakes occurred in 1981, 1985 and 1986 for each catalog.

Inspection of Table 4 indicates that the use of the prediction Equations (32) and (36), of the Weibull model, would produce enormous errors of prediction in both catalogs. Therefore, we conclude that the Weibull probability density is not the correct probability density model to represent the interval times between earthquakes in the Michoacán fault–segment.

Table 4

Differences between observed and predicted (Weibull prediction) values of the recurrence times of events occurred in 1981, 1985 and 1986 and for each catalog (Table 2 and Table 3).

Catalog	Event	Predicted τ_e	Observed τ_0	Errors E
Table 2	Earthq. 1981	∞	8.74	∞
	Earthq. 1985	10.96	3.98	6.98
	Earthq. 1986	28.99	0.61	28.38
Table 3	Earthq. 1981	282.25	2.61	279.64
	Earthq. 1985	12.32	3.90	8.42
	Earthq. 1986	17.63	0.61	17.02

(b) Rayleigh model

In order to evaluate the Rayleigh model, we use equation (19) and equations (26) and (27) as follows:

Substituting Equations (19) into equations (27), we can write

$$\frac{\partial}{\partial t}P(\Delta t|t)=\frac{\partial}{\partial t}\left\{1-e^{-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}}\right\}=0,$$

from which we obtain

$$\frac{\Delta t}{\delta^2}e^{-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}}=0. \qquad (38)$$

Similarly substituting equation (19) into equation (26), we can write

$$\frac{\partial}{\partial \Delta t}P(\Delta t|t)=\frac{\partial}{\partial \Delta t}\left\{1-e^{-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}}=0\right\}$$

from which we obtain

$$\frac{t+(\Delta t)}{\delta^2}e^{-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}}=0 \ . \qquad (39)$$

From equations (38) and (39), we obtain the following equations in t and Δt

$$\frac{\Delta t}{\delta^2}=0 \qquad (40)$$

and

$$\frac{t+\Delta t}{\delta^2}=0. \qquad (41)$$

Solving (40) and (41) by addition, we obtain

$$t=-2\Delta t \ . \qquad (42)$$

From equations (38) and (39) we can write

$$e^{-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}}=0. \qquad (43)$$

Expanding the left-hand side of equation (43) in a MacLaurin series, we get

$$1-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}+Q(\Delta t)^2=0 \ , \qquad (44)$$

where $Q(\Delta t)^2$ becomes negligible for small Δt. Thus, for small Δt, we can write

$$1-\frac{2t(\Delta t)+(\Delta t)^2}{2\delta^2}=0 \ , \qquad (45)$$

or equivalently we can write

$$1-t\left(\frac{\Delta t}{\delta^2}\right)-\frac{(\Delta t)^2}{2\delta^2}=0.$$

From equation (40) we have $\left(\frac{\Delta t}{\delta^2}\right)$ equal zero. Therefore, we can write

$$1-\frac{(\Delta t)^2}{2\delta^2}=0 \ , \qquad (46)$$

from which we obtain

$$\Delta t=\pm\left(\sqrt{2}\right)\delta \ . \qquad (47)$$

RAYLEIGH ANALYSIS OF THE CATALOG

Now we consider the two sets of catalogs given in Table 2 and Table 3, and we apply the predictive equations (42) and (47) of the Rayleigh Model to predict Δt, t and τ for the following last three earthquake events: The 1981 earthquake, the 1985 earthquake and the 1986 earthquake.

In Table 5 we carry out this analysis for the recurrence time (estimated and observed) associated whit the last three earthquakes occurred in 1981, 1985 and 1986, and for each catalog.

Inspection of Table 5 indicates that use of the prediction Equations (42) and (47) of the Rayleigh model, would produce enormous errors of prediction. Therefore, we can conclude that the Rayleigh probability density is not the correct probability density model to represent the probability density of intervals times between earthquakes in the Michoacán fault-segment.

(c) Pareto model

In order to evaluate the Pareto model, we use equation (25) and equations (26) and (27) as follows.

Table 5

Difference between observed and predicted (Rayleigh prediction) values of the recurrence times of events occurred in 1981, 1985 and 1986, and for each catalogs (Table 2 and Table 3).

Catalog	Event	Predicted τ_e(yrs.)	Observed τ_o(yrs.)	Error ε(yrs.)
	Earthq. 1981	104.34	8.74	95.6
Table 2	Earthq. 1985	79.41	3.98	75.4
	Earthq. 1986	62.97	0.61	62.36
	Earthq. 1981	76.47	2.61	73.86
Table 3	Earthq. 1985	59.58	3.90	55.68
	Earthq. 1986	50.26	0.61	49.65

Substituting equation (25) into equation (27), we can write

$$\frac{\partial}{\partial t}P(\Delta t|t) = \frac{\partial}{\partial t}\left\{\frac{\alpha(\Delta t)}{t} - \frac{\alpha(\alpha+1)(\Delta t)^2}{2t^2}\right\} = 0$$

from which we obtain

$$-\frac{\alpha(\Delta t)}{t^2} + \frac{\alpha(\alpha+1)(\Delta t)^2}{t^3} = 0 \ . \qquad (48)$$

Similarly, substituting equation (25) into equation (26), we can write

$$\frac{\partial}{\partial \Delta t}P(\Delta t|t) = \frac{\partial}{\partial \Delta t}\left\{\frac{\alpha(\Delta t)}{t} - \frac{\alpha(\alpha+1)(\Delta t)^2}{2t^2}\right\} - = 0$$

from which we obtain

$$\frac{\alpha}{t} - \frac{\alpha(\alpha+1)(\Delta t)}{t^2} = 0. \qquad (49)$$

We solve equation (49) for Δt

$$\Delta t = \frac{t}{\alpha+1} \ \cdot \qquad (50)$$

Substituting this value for Δt in (48), we get

$$t = \alpha + 1 \cdot \qquad (51)$$

PARETO ANALYSIS OF THE CATALOGS

Here we consider the two sets of catalogs given in Table 2 and Table 3, and we apply the predictive equations (50) and (51) of the Pareto model to predict Δt, t and τ for the following last tree earthquake events: the 1981 earthquake, the 1985 earthquake and the 1986 earthquake.

In Table 6 we carry out this analysis for the recurrence time (estimated τ_e and the observed τ_o) associated to the last three earthquakes occurred in 1981, 1985 and 1986, and for each catalog.

Inspection of Table 6 indicates that use of prediction equations (50) and (51), of the Pareto model, would produce smaller errors of prediction than the Weibull model and Rayleigh model. Therefore, we can conclude that the Pareto probability density is a correct probability model for the probability density of interval times between large earthquakes in the Michoacán fault-segment of the Mexican subduction zone.

Differences between the recurrence time intervals (estimated and observed) associated with the last three large earthquake events for the catalog given in Table 3 are smaller than the errors determined using the catalog given in Table 2. As the catalog given in Table 3 includes the Petatlán earthquake, we can conclude that the Michoacán fault-segment is ruptured by the Petatlán earthquake, as pointed out by Mikumo (1998). Thus, our analysis confirms that the 1979 Petatlán earthquake ruptured the Michoacán fault-segment.

PARETO PREDICTION OF THE NEXT LARGE EARTHQUAKE

We proceed to find the time interval (Δt) which maximizes the Pareto (power-law) conditional probability of occurrence $P_p(\Delta t \mid t)$, Equation (25). We find the maxi-

Table 6

Difference between observed and predicted (Pareto prediction) values of the recurrence times of events occurred in 1981, 1985 and 1986, and for each catalog (Table 2 and Table 3).

Catalog	Event	Predicted τ_e(yrs.)	Observed τ_o(yrs.)	Error ε(yrs.)
Table 2	Earthq. 1981	2.99	8.74	-5.75
	Earthq. 1985	2.69	3.98	-1.29
	Earthq. 1986	1.79	0.61	1.18
Table 3	Earthq. 1981	2.65	2.61	0.04
	Earthq. 1985	2.46	3.90	-1.44
	Earthq. 1986	2.39	0.61	1.76

mum of P_p by examining its partial derivative and setting it equal to zero, as follows

$$\frac{\partial}{\partial \Delta t} P_p\left(\Delta t|t\right) = \frac{\alpha}{t} - \frac{\alpha(\alpha+1)(\Delta t)}{t^2} = 0\,, \qquad (52)$$

from which we obtain the predictive formula to estimate the time interval in the Pareto model

$$(\Delta\hat{t})_p = \frac{t}{\alpha+1}\ . \qquad (53)$$

In order to apply the Pareto model we have to estimate the parameter α. To do this, Equations (21) and (22), and the estimated sample mean μ and sample variance may be used (see Winkler and Hays, 1975, p. 516), as follows:

$$\frac{\alpha x_0}{x-1} = \mu \qquad (54)$$

$$\frac{\alpha x_0}{(\alpha-1)(\alpha-2)} = \nu\ . \qquad (55)$$

We find α and x_0, solving these two equations simultaneously, and we find $\alpha = 0.30976$. We also need to estimate the elapsed time t since the last earthquake. In order to do this, we use the occurrence date of the last large earthquake t_o = 1986.33, and the current date 2002.58 (July 2002) as follows:

t = 2002.58 (yrs) – 1986.33 (yrs) = 16.25 (yrs) .

To predict the time interval $(\Delta\hat{t})_p$ using the Pareto model, we substitute the value $\alpha = 0.30976$ and the elapsed time t = 16.25 (yrs) in equation (53) and we obtain

$$(\Delta\hat{t})_p = 12.41 \text{ (yrs.)}\ .$$

Next, we estimate the total time $\gamma = t + (\Delta\hat{t})_p$ for the occurrence of the next expected large earthquake.

$$\gamma_p = 16.25 \text{ (yrs.)} + 12.44 \text{ (yrs.)} = 28.66 \text{ (yrs.)}$$

It should be noted that γ_p is close to the recurrence time of the next expected future large earthquake.

Finally, we estimate the occurrence time of the next expected large earthquake ($M \geq 7$). To do this, we add to the predicted total time γ_p = 28.66 (yrs.) the occurrence time of the last observed earthquake t_o = 1986.33. Thus, we conclude that the next large earthquake event may occur approximately before the year 2014.99, or equivalently before December 2014.

ERROR OF PARETO PREDICTION OF NEXT EVENT

According to Sterling and Pollock (1986, p. 338), a realistic estimate of the error most likely to be incurred by using a particular fit to a set of experimental data is obtained from the mean of the deviation between observed τ_{io} and predictive τ_{ie} values. We use the absolute rather than the signed values for the deviations:

$$\left|\bar{d}\right| = \frac{\sum_{i=1}^{\eta}\left|\tau_{io} - \tau_{ie}\right|}{n}\ . \qquad (56)$$

From Table 6 the errors are $\varepsilon_1 = 0.04$, $\varepsilon_2 = -1.44$ and $\varepsilon_3 = 1.76$. Substituting these values into equation (56) we obtain the prediction error

$$\varepsilon = \pm\, 1.76 \text{ (yrs.)}\ .$$

This is the error for the total time γ_p = 28.66 (yrs). Recall that γ_p is the approximate recurrence time for the occurrence of the next large event in the Michoacán fault-segment.

CONCLUSIONS

We determine a time interval for the occurrence of the next large earthquake in the Michoacán fault-segment using the maximum of the conditional probability $P(\Delta t \mid t)$ of earthquake occurrence.

As a guideline for a logical decision between models we compare the difference between observed τ_o and predicted τ_e recurrence times values, for the last three large earthquakes in the Michoacán fault-segment. We conclude that the distribution of time intervals between events in the Michoacán fault-segment is well represented by a Pareto probability distribution, and we also conclude that the most appropriate catalog is given by Table 3.

Using the Pareto model we conclude that a large earthquake of magnitude $M \geq 7$ may occur in the next time interval $\Delta\hat{t}$ = 12.41 (yrs.) counting from July, 2002, or before December 2014 ± 1.76 (yrs.).

Following Main (1995), if the distribution of time intervals between large earthquakes in the Michoacán fault-segment follows a Pareto (power-law) distribution, the Michoacán fault-segment has already approached a state where the local stress is near failure and deformation occurs predominantly on large correlate faults. This is consistent with observations (Mikumo *et al.,* 1998)

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